# Note for coordinate system conversion 

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Figure 1: Coordinate systems of Ubitrack, OpenCV and OpenGL ${ }^{1}$.

## 1 Camera parameter conversions

Perspective projection with radial and tangential distortion ${ }^{2}$ :

$$
\begin{aligned}
x^{\prime} & =x / z \\
x^{\prime \prime} & =x^{\prime} S+2 p_{1} x^{\prime} y^{\prime}+p_{2}\left(r^{2}+2 x^{\prime 2}\right) \\
u & =f_{x} x^{\prime \prime}+c_{x} \\
y^{\prime} & =y / z \\
y^{\prime \prime} & =y^{\prime} S+p_{1}\left(r^{2}+2 y^{\prime 2}\right)+2 p_{2} x^{\prime} y^{\prime} \\
v & =f_{y} y^{\prime \prime}+c_{y}
\end{aligned}
$$

where $S=\frac{1+k_{1} r^{2}+k_{2} r^{4}+k_{33} r^{6}}{1+k_{4} r^{2}+k_{5} r^{4}+k_{6} r^{6}} . S$ is constant regardless of the signs of $x$ and $y$.

[^0]
### 1.1 From OpenCV (image top-left) to Ubitrack

The $v$-axis of the image plane and $y$-axis and $z$-axis of the camera coordinates have to be flipped. Namely, $-y$ has to be projected to $h-1-v$, and $x$ to $u$, this follows

$$
\left\{\begin{array}{l}
\hat{p_{1}}=p_{1} \\
\hat{p_{2}}=-p_{2} \\
\hat{f_{x}}=-f_{x} \\
\hat{f_{y}}=-f_{y} \\
\hat{c_{x}}=c_{x} \\
\hat{c_{y}}=h-1-c_{y}
\end{array}\right.
$$

Note that $f_{y}$ should stay the same. The same conversion applies from Ubitrack to OpenCV.

### 1.2 From OpenCV (image bottom-left) to Ubitrack

The $y$-axis and $z$-axis of the camera coordinates have to be flipped. Namely, $-y$ has to be projected to $v$, and $x$ to $u$

$$
\begin{aligned}
u & =\hat{f_{x}}\left(\hat{x^{\prime}} S+2 \hat{p_{1}} \hat{x^{\prime}} \hat{y^{\prime}}+\hat{p_{2}}\left(r^{2}+2 \hat{x}^{2}\right)\right)+\hat{c_{x}} \\
u & =\hat{f_{x}}\left(-x^{\prime} S-2 \hat{p_{1}} x^{\prime} y^{\prime}+\hat{p_{2}}\left(r^{2}+2 x^{\prime 2}\right)\right)+\hat{c_{x}} \\
f_{x}\left(x^{\prime} S+2 p_{1} x^{\prime} y^{\prime}+p_{2}\left(r^{2}+2\right)\right)+c_{x} & =-\hat{f_{x}}\left(x^{\prime} S+2 \hat{p_{1}} x^{\prime} y^{\prime}-\hat{p_{2}}\left(r^{2}+2\right)\right)+\hat{c_{x}} \\
v & =\hat{f_{y}}\left(\hat{y^{\prime}} S+\hat{p_{1}}\left(r^{2}+2 \hat{y^{\prime}}{ }^{2}\right)+2 \hat{p_{2}} \hat{x^{\prime}} \hat{y^{\prime}}\right)+\hat{c_{y}} \\
v & =\hat{f}_{y}\left(y^{\prime} S+\hat{p_{1}}\left(r^{2}+2 y^{\prime 2}\right)-2 \hat{p_{2}} x^{\prime} y^{\prime}\right)+\hat{c_{y}} \\
f_{y}\left(y^{\prime} S+p_{1}\left(r^{2}+2 y^{\prime 2}\right)+2 p_{2} x^{\prime} y^{\prime}\right)+c_{y} & =\hat{f_{y}}\left(y^{\prime} S+\hat{p_{1}}\left(r^{2}+2 y^{\prime 2}\right)-2 \hat{p_{2}} x^{\prime} y^{\prime}\right)+\hat{c_{y}} \\
f_{y}\left(y^{\prime} S+p_{1}\left(r^{2}+2 y^{\prime 2}\right)+2 p_{2} x^{\prime} y^{\prime}\right)+c_{y} & =\hat{f_{y}}\left(y^{\prime} S+\hat{p_{1}}\left(r^{2}+2 y^{\prime 2}\right)-2 \hat{p_{2}} x^{\prime} y^{\prime}\right)+\hat{c_{y}}
\end{aligned}
$$

, this follows

$$
\left\{\begin{array}{l}
\hat{p_{1}}=p_{1} \\
\hat{p_{2}}=-p_{2} \\
\hat{f_{x}}=-f_{x} \\
\hat{f_{y}}=f_{y} \\
\hat{c_{x}} \\
\hat{c_{y}}
\end{array}=c_{x} .\right.
$$

Note that $f_{y}$ should stay the same. The same conversion applies from Ubitrack to OpenCV.

### 1.3 From Ubitrack to OpenGL

The $v$-axis of the image plane has to be flipped

$$
\begin{aligned}
h-1-v & =\hat{f}_{y} y^{\prime \prime}+\hat{c_{y}} \\
h-1-f_{y} y^{\prime \prime}-c_{y} & =\hat{f}_{y} y^{\prime \prime}+\hat{c_{y}} \\
-f_{y} y^{\prime \prime}+h-1-c_{y} & =\hat{f}_{y} y^{\prime \prime}+\hat{c_{y}} .
\end{aligned}
$$

This follows

$$
\begin{cases}\hat{f_{y}} & =-f_{y} \\ \hat{c_{y}} & =h-1-c_{y}\end{cases}
$$


[^0]:    ${ }^{1}$ Under a typical OpenGL programming convention before converting to Normalized Device Coordinates, which is left-handed
    ${ }^{2}$ http://docs.opencv.org/modules/calib3d/doc/camera_calibration_and_3d_reconstruction.html

