Note for coordinate system conversion

Yuta Itoh





Figure 1: Coordinate systems of Ubitrack, OpenCV and OpenGL¹.

1 Camera parameter conversions

Perspective projection with radial and tangential distortion²:

$$\begin{aligned} x' &= x/z \\ x'' &= x'S + 2p_1 x'y' + p_2 \left(r^2 + 2x'^2\right) \\ u &= f_x x'' + c_x \end{aligned}$$

$$y' = y/z y'' = y'S + p_1(r^2 + 2y'^2) + 2p_2x'y' v = f_yy'' + c_y$$

where $S = \frac{1+k_1r^2+k_2r^4+k_3r^6}{1+k_4r^2+k_5r^4+k_6r^6}$. *S* is constant regardless of the signs of *x* and *y*.

 $^{^{\}rm l}$ Under a typical OpenGL programming convention before converting to Normalized Device Coordinates, which is left-handed

²http://docs.opencv.org/modules/calib3d/doc/camera_calibration_and_3d_reconstruction.html

1.1 From OpenCV (image top-left) to Ubitrack

The *v*-axis of the image plane and *y*-axis and *z*-axis of the camera coordinates have to be flipped. Namely, -y has to be projected to h - 1 - v, and *x* to *u*, this follows

$$\begin{cases} \hat{p}_1 &= p_1 \\ \hat{p}_2 &= -p_2 \\ \hat{f}_x &= -f_x \\ \hat{f}_y &= -f_y \\ \hat{c}_x &= c_x \\ \hat{c}_y &= h - 1 - c_y \end{cases}$$

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Note that f_y should stay the same. The same conversion applies from Ubitrack to OpenCV.

1.2 From OpenCV (image bottom-left) to Ubitrack

The *y*-axis and *z*-axis of the camera coordinates have to be flipped. Namely, -y has to be projected to *v*, and *x* to *u*

$$u = \hat{f}_x \left(\hat{x}'S + 2\hat{p}_1 \hat{x}' \hat{y}' + \hat{p}_2 \left(r^2 + 2\hat{x}'^2 \right) \right) + \hat{c}_x$$
$$u = \hat{f}_x \left(-x'S - 2\hat{p}_1 x' y' + \hat{p}_2 \left(r^2 + 2x'^2 \right) \right) + \hat{c}_x$$
$$f_x \left(x'S + 2p_1 x' y' + p_2 \left(r^2 + 2 \right) \right) + c_x = -\hat{f}_x \left(x'S + 2\hat{p}_1 x' y' - \hat{p}_2 \left(r^2 + 2 \right) \right) + \hat{c}_x$$

$$\begin{aligned} v &= \hat{f}_y \left(\hat{y}' S + \hat{p}_1 \left(r^2 + 2 \hat{y}^2 \right) + 2 \hat{p}_2 \hat{x}' \hat{y}' \right) + \hat{c}_y \\ v &= \hat{f}_y \left(y' S + \hat{p}_1 \left(r^2 + 2 y'^2 \right) - 2 \hat{p}_2 x' y' \right) + \hat{c}_y \\ f_y \left(y' S + p_1 \left(r^2 + 2 y'^2 \right) + 2 p_2 x' y' \right) + c_y &= \hat{f}_y \left(y' S + \hat{p}_1 \left(r^2 + 2 y'^2 \right) - 2 \hat{p}_2 x' y' \right) + \hat{c}_y \\ f_y \left(y' S + p_1 \left(r^2 + 2 y'^2 \right) + 2 p_2 x' y' \right) + c_y &= \hat{f}_y \left(y' S + \hat{p}_1 \left(r^2 + 2 y'^2 \right) - 2 \hat{p}_2 x' y' \right) + \hat{c}_y \end{aligned}$$

, this follows

$$\begin{cases} \hat{p_1} &= p_1 \\ \hat{p_2} &= -p_2 \\ \hat{f_x} &= -f_x \\ \hat{f_y} &= f_y \\ \hat{c_x} &= c_x \\ \hat{c_y} &= c_y \end{cases}.$$

Note that f_y should stay the same. The same conversion applies from Ubitrack to OpenCV.

1.3 From Ubitrack to OpenGL

The *v*-axis of the image plane has to be flipped

$$\begin{array}{rcl} h-1-v &=& \hat{f}_y y''+\hat{c}_y \\ h-1-f_y y''-c_y &=& \hat{f}_y y''+\hat{c}_y \\ -f_y y''+h-1-c_y &=& \hat{f}_y y''+\hat{c}_y. \end{array}$$

This follows

$$\begin{cases} \hat{f}_y &= -f_y \\ \hat{c}_y &= h-1-c_y \end{cases}.$$