

Augmented Reality II

- Camera Calibration -

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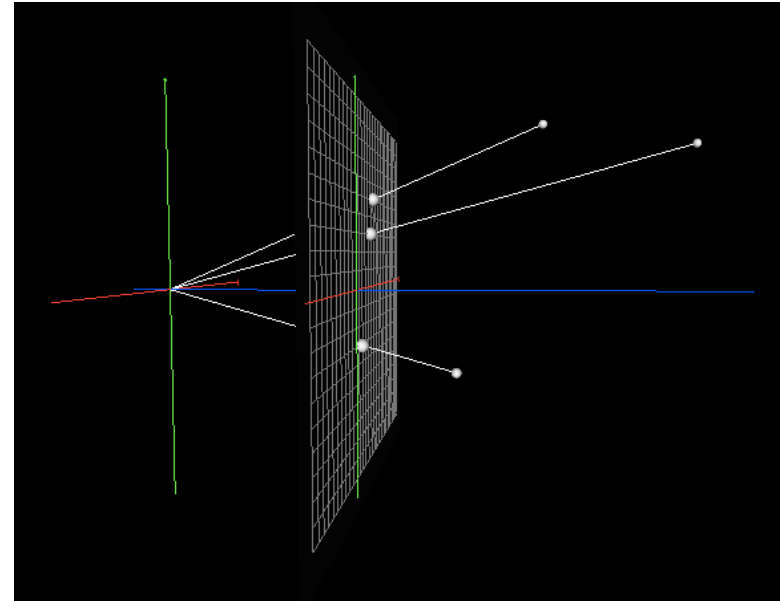
May 11, 2004

Literature

- Richard Hartley and Andrew Zisserman, “Multiple View Geometry in Computer Vision”, Cambridge University Press, 2000. (Section 5, pp. 139-161)

Camera Calibration

- 3D-to-2D Projection:



$$\mathbf{x} = \mathbf{P}\mathbf{X}$$

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Outline

- Finite camera model
(Definition of camera parameters)
- Projective camera model
(Properties of the projection matrix P ,
relationship to camera properties)
- Models of cameras at infinity
- Computation of P

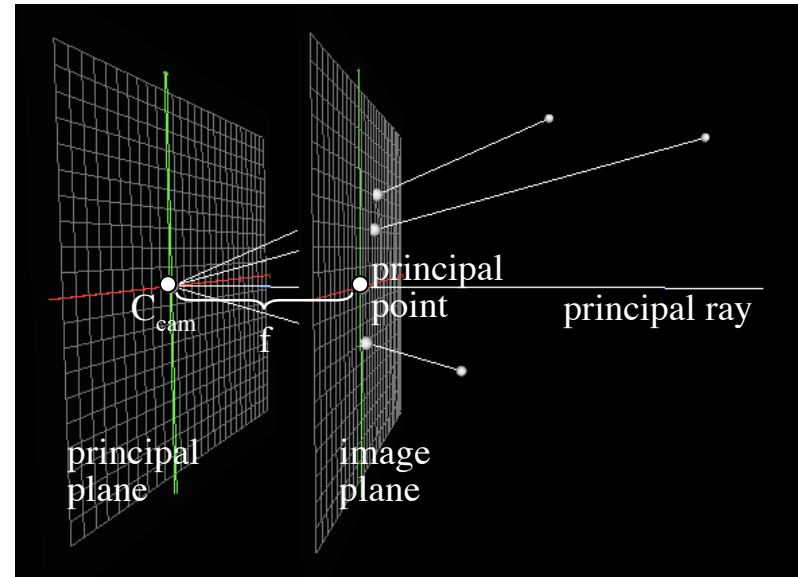
Finite Camera Model

Finite Camera Model

- Pinhole camera geometry
- Central projection using homogeneous coordinates
- Principal point offset
- Camera rotation and translation
- CCD cameras
- Skew parameter

Pinhole Camera Geometry

- Projection of 3D-Points onto a plane
- Center of projection: origin of Euclidean coordinate system C_{cam}
- Image plane at distance $z = f$ (focal length)



$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \rightarrow \begin{bmatrix} f \cdot X / Z \\ f \cdot Y / Z \end{bmatrix}$$

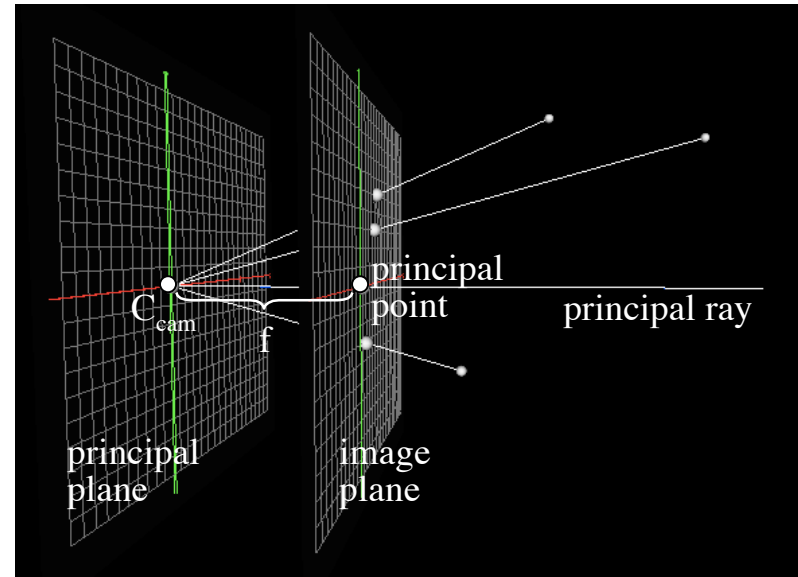
Central Projection Using Homogeneous Coordinates

$$\begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} \rightarrow \begin{bmatrix} f X/Z \\ f Y/Z \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} fX \\ fY \\ Z \\ 1 \end{bmatrix} = f \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$\mathbf{x} = \mathbf{P}\mathbf{X}$$

$$\mathbf{P} = \text{diag}(f, f, 1) \begin{bmatrix} \mathbf{I} & \mathbf{0} \end{bmatrix}$$



Principal Point Offset

- Principal point at (p_x, p_y) rel. to image origin

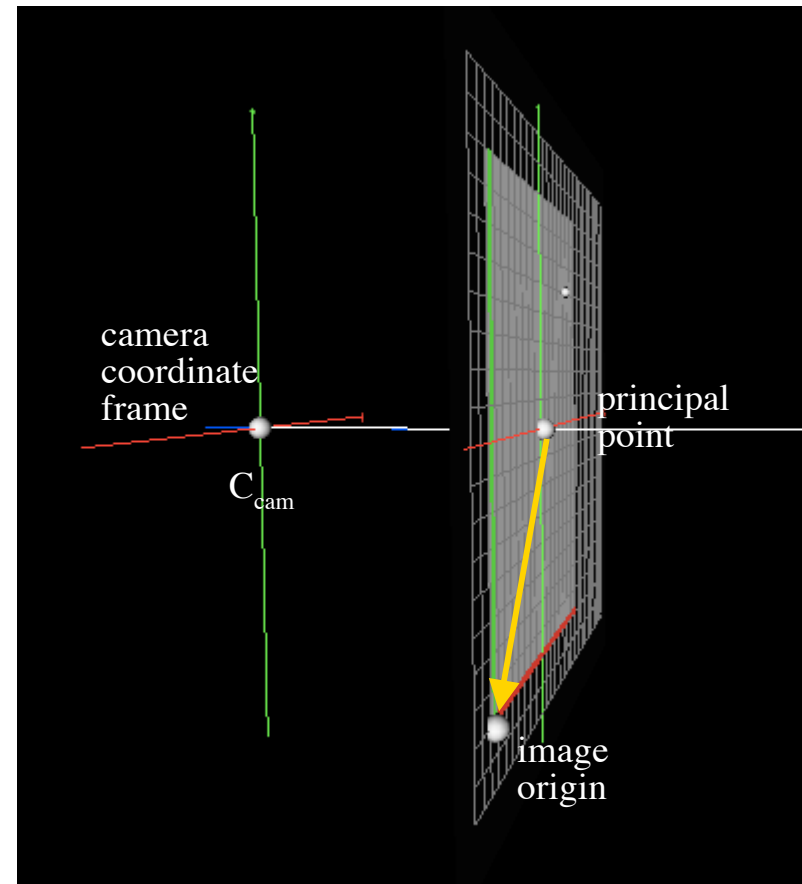
$$\begin{bmatrix} fX + Zp_x \\ fY + Zp_y \\ Z \\ 1 \end{bmatrix} = \begin{bmatrix} f \\ f \\ Z \\ 1 \end{bmatrix} \quad \begin{matrix} p_x \\ p_y \\ 1 \end{matrix} \quad \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$\mathbf{x} = \mathbf{K}[\mathbf{I} \quad | \quad \mathbf{O}] \mathbf{X}_{\text{cam}}$$

$$\mathbf{K} = \begin{bmatrix} f & 0 & 0 & p_x \\ 0 & f & 0 & p_y \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

camera calibration matrix

- 3 degrees of freedom



Camera Rotation and Translation

- inhomogeneous (3x3):

$$\begin{aligned}\tilde{\mathbf{X}}_{\text{cam}} &= \mathbf{R}(\tilde{\mathbf{X}} \square \tilde{\mathbf{C}}) \\ &= \mathbf{R}\tilde{\mathbf{X}} \square \mathbf{R}\tilde{\mathbf{C}}\end{aligned}$$

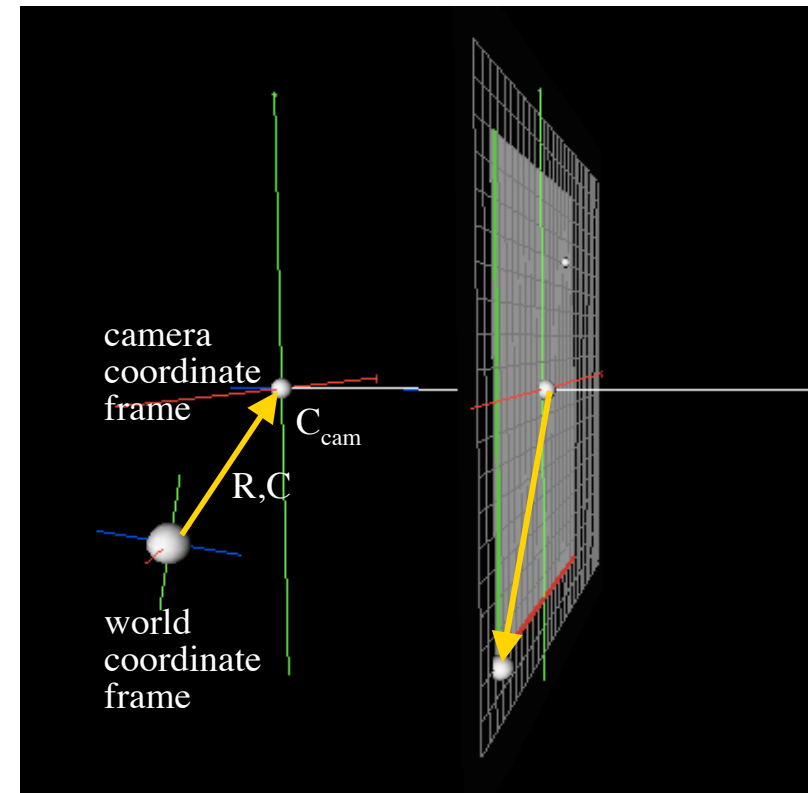
- homogeneous (4x4):

$$\mathbf{X}_{\text{cam}} = \begin{bmatrix} \mathbf{R} & -\mathbf{R}\tilde{\mathbf{C}} \\ \mathbf{0} & 1 \end{bmatrix} \mathbf{X}$$

- 3D-to-2D (3x4):

$$\mathbf{x} = \mathbf{K}\mathbf{R}[\mathbf{I} \quad | \quad \square \tilde{\mathbf{C}}] \mathbf{X}$$

- 9 degrees of freedom;
internal parameters \mathbf{K} , external parameters \mathbf{R} , \mathbf{C}



Camera Rotation and Translation

- Alternative Representation -

- inhomogeneous (3x3):

$$\mathbf{t} = \mathbf{R}\mathbf{C}$$

$$\tilde{\mathbf{X}}_{\text{cam}} = \mathbf{R}\tilde{\mathbf{X}} + \mathbf{t}$$

- homogeneous (4x4):

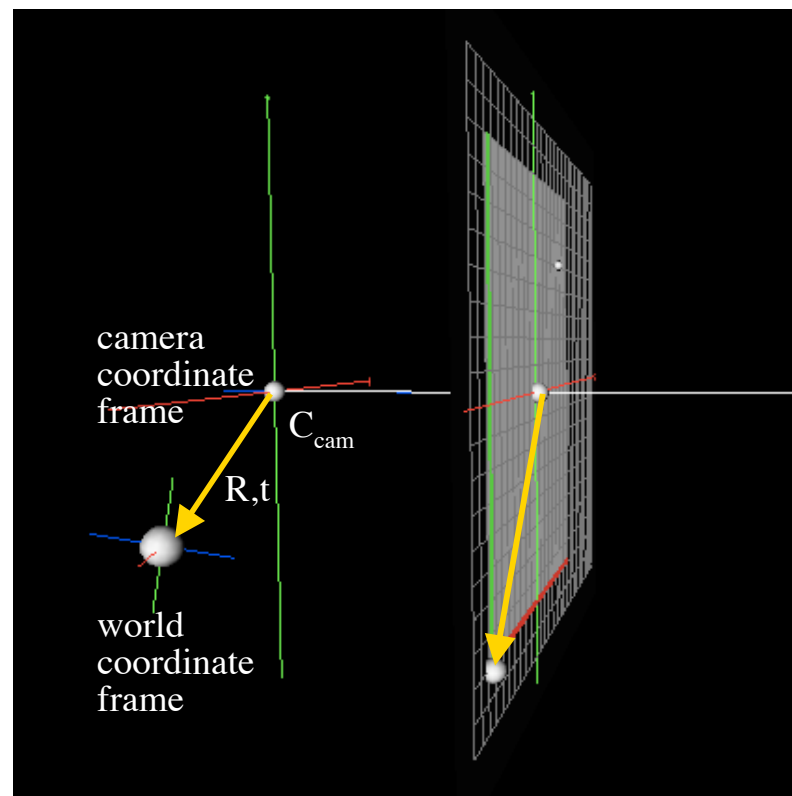
$$\mathbf{X}_{\text{cam}} = \begin{bmatrix} \mathbf{R} & \mathbf{t} \\ \mathbf{0} & 1 \end{bmatrix} \mathbf{X}$$

- all-together (3x4):

$$\mathbf{x} = \mathbf{K}[\mathbf{R} \quad | \quad \mathbf{t}] \mathbf{X}$$

- 9 degrees of freedom;

internal parameters \mathbf{K} , external parameters \mathbf{R} , \mathbf{t}



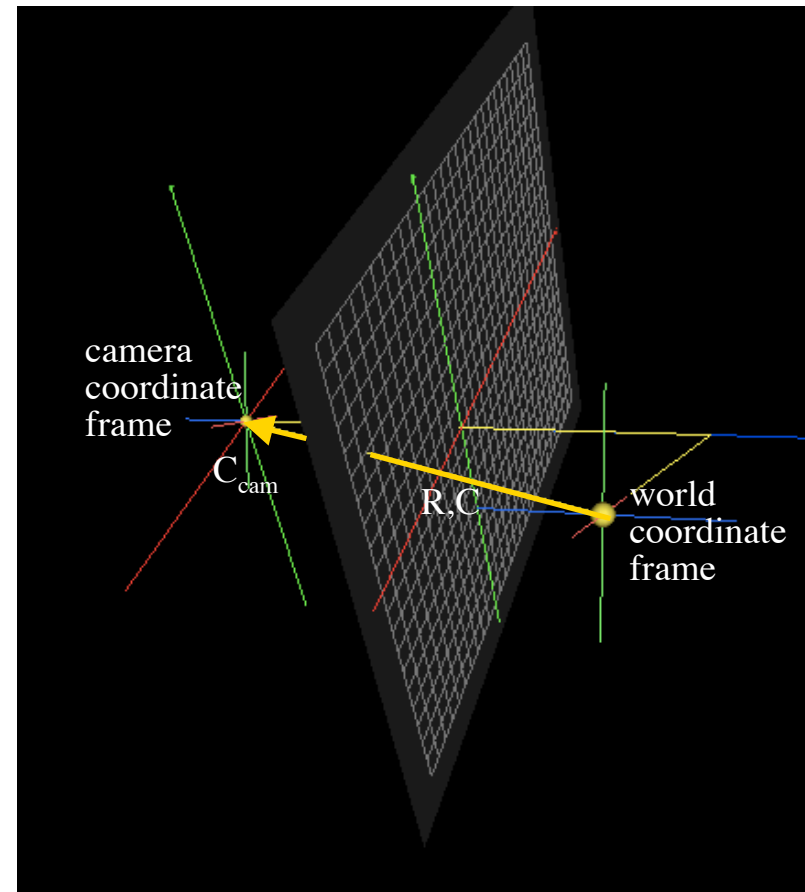
Example

$$\tilde{\mathbf{C}} = \begin{bmatrix} 2.0 & 0.0 \\ 0.0 & 2.0 \end{bmatrix}$$

$$\mathbf{R}_z(30^\circ) = \begin{bmatrix} 0.866 & 0.5 & 0.0 \\ -0.5 & 0.866 & 0.0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{x}_{\text{cam}} = \begin{bmatrix} \mathbf{R}_z & | & -\mathbf{R}_z \tilde{\mathbf{C}} \end{bmatrix} \mathbf{x}$$

$$= \begin{bmatrix} 0.866 & 0.5 & 0 & -1.732 \\ -0.5 & 0.866 & 0 & 1.0 \\ 0 & 0 & 1 & 2.0 \end{bmatrix} \mathbf{x}$$



CCD Cameras

- non-square pixels
- scale factors m_x , m_y , aspect ratio m_x/m_y

$$\mathbf{K} = \begin{bmatrix} m_x & 0 & 0 & 0 \\ 0 & m_y & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} f \\ f \\ f \\ 1 \end{bmatrix} = \begin{bmatrix} p_x & 0 & 0 & x_0 \\ 0 & p_y & 0 & y_0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

- all-together (3x4):

$$\mathbf{x} = \mathbf{KR}[\mathbf{I} \quad | \quad \mathbf{t}] \mathbf{X}$$

- 10 degrees of freedom

Skew Parameter (Finite Projective Camera)

- skew parameter s

$$\mathbf{K} = \begin{bmatrix} \square_x & s & x_0 \\ \square & & \\ \square & & \\ \square & & \end{bmatrix} \begin{bmatrix} y_0 \\ 1 \end{bmatrix}$$

- all-together:

$$\mathbf{x} = \mathbf{KR}[\mathbf{I} \quad | \mathbf{t}] \mathbf{X}$$

- 11 degrees of freedom

Finite Camera Model

- Summary -

- Pinhole camera geometry (f)
- Central projection
- Principal point offset (p_x, p_y)
- Camera rotation and translation (R, \mathbf{t})
- CCD cameras (m_x, m_y) $\rightarrow (a_x, a_y, x_0, y_0)$
- Skew parameter (s)

$$\mathbf{x} = \begin{bmatrix} \square_x \\ \square \\ \square \\ \square \end{bmatrix} = \begin{bmatrix} s & x_0 \\ \square_y & y_0 \\ & 1 \end{bmatrix} \mathbf{R} + \mathbf{t} \mathbf{X}$$

Projective Camera Model

Projective Camera Model

Given some matrix \mathbf{P}

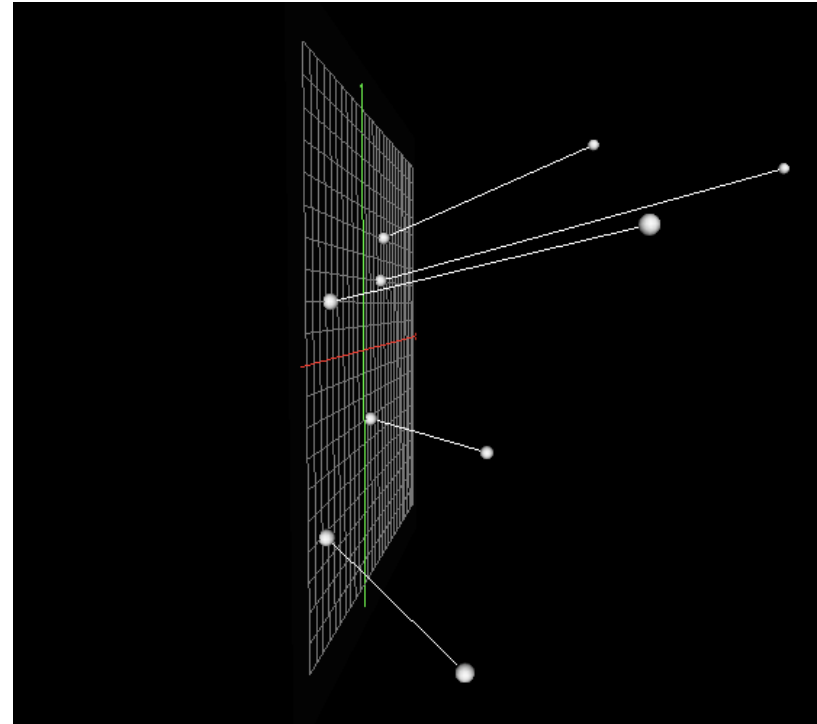
Notations for \mathbf{P} :

$$\mathbf{P} = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix}$$

$$= \begin{bmatrix} \mathbf{p}^{1T} \\ \mathbf{p}^{2T} \\ \mathbf{p}^{3T} \end{bmatrix}$$

$$= [\mathbf{p}_1 \quad \mathbf{p}_2 \quad \mathbf{p}_3 \quad \mathbf{p}_4]$$

$$= [\mathbf{M} \quad | \mathbf{p}_4]$$



Camera Center

- P is 3x4 matrix, rank = 3
1-dimensional null-space

$$PC = \mathbf{0}$$

$$[M \quad | \quad \mathbf{p}_4]C = \mathbf{0}$$

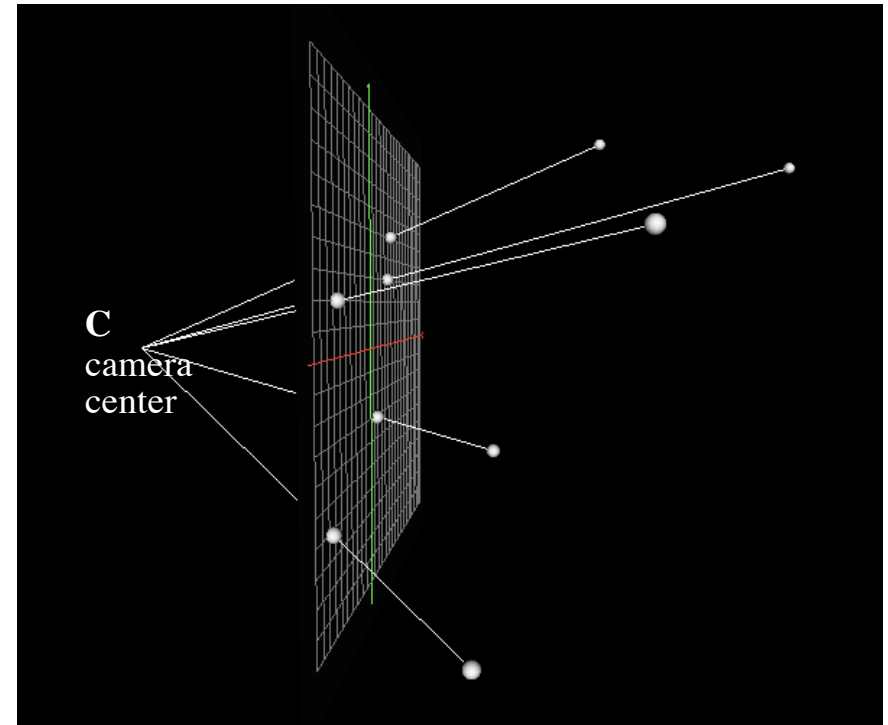
- Ray through the camera center

$$\mathbf{X}(\lambda) = \lambda \mathbf{A} + (1 - \lambda) \mathbf{C}$$

$$\mathbf{x} = P\mathbf{X}(\lambda)$$

$$= \lambda P\mathbf{A} + (1 - \lambda) P\mathbf{C}$$

$$= \lambda P\mathbf{A}$$



Camera Center

- 1-dimensional null-space

$$PC = \mathbf{0}$$

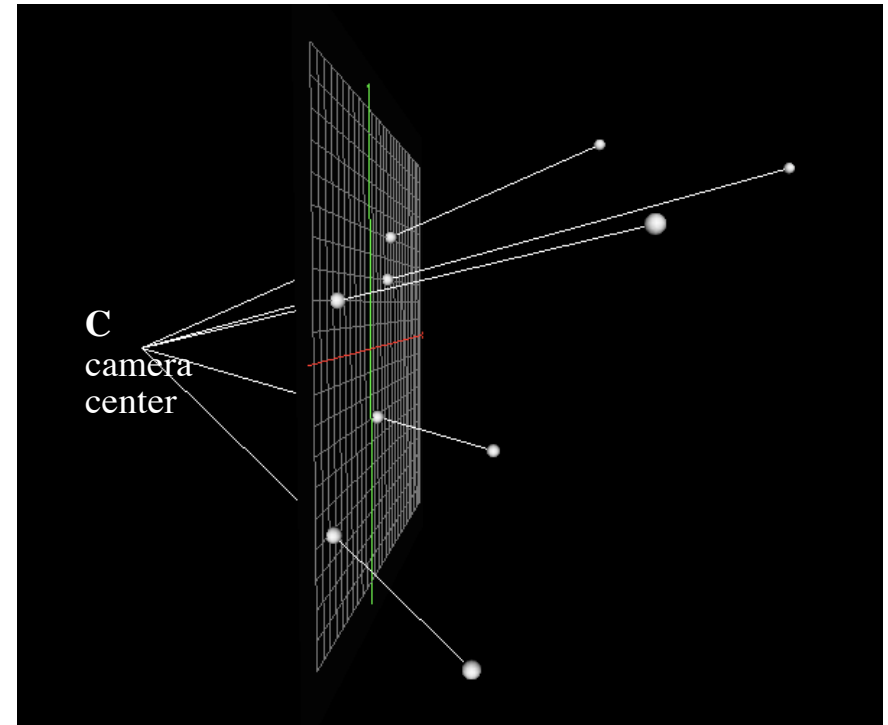
$$[M \quad | \mathbf{p}_4]C = \mathbf{0}$$

- Ray through the camera center

- Hom. representation of camera center

$$KR[I \quad | \quad \tilde{C}]C = \mathbf{0}$$

$$C = \begin{bmatrix} \tilde{C}^T \\ 1 \end{bmatrix} = \begin{bmatrix} M^{-1} \mathbf{p}_4 \\ 1 \end{bmatrix}$$



Example

$PC = 0$

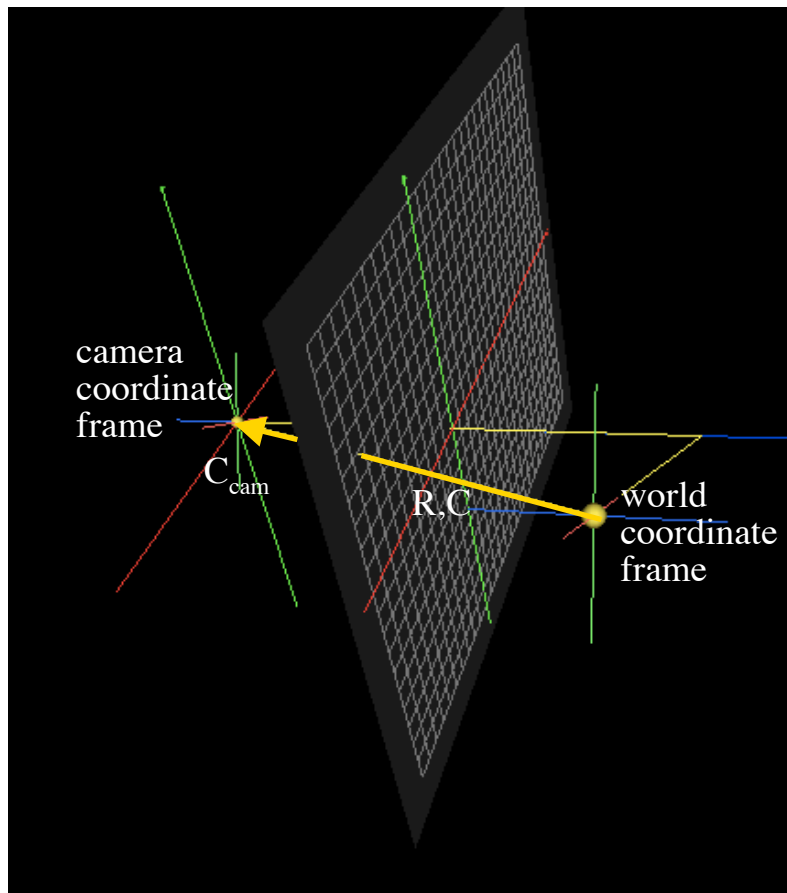
$$\begin{bmatrix} 0.866 & 0.5 & 0 \\ 0.5 & 0.866 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1.732 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 1.732 \\ 1.732 \\ 2 \end{bmatrix}$$

$$[R_z \mid \tilde{R}_z \tilde{C}] C = 0$$

$$[M \mid p_4] C = 0$$

$$C = \begin{bmatrix} M^{-1} p_4 \\ 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 0.866 & 0.5 & 0 \\ 0.5 & 0.866 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1.732 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 1.732 \\ 1.732 \\ 2 \end{bmatrix}$$



Summary

- Camera Center -

	M		$-M^{-1}\mathbf{p}_4$ camera center
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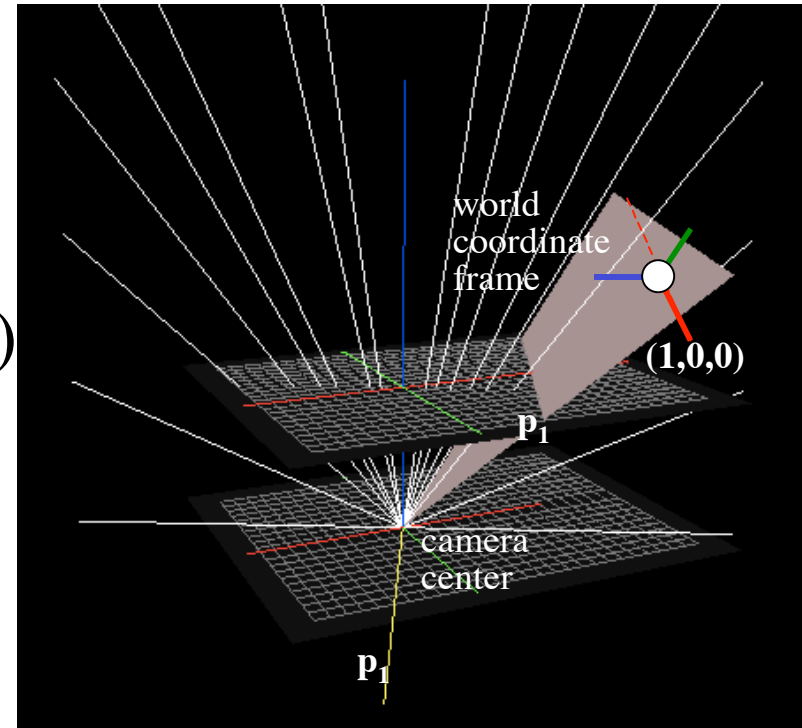
Column Vectors of P

$$P = [\mathbf{p}_1 \quad \mathbf{p}_2 \quad \mathbf{p}_3 \quad \mathbf{p}_4]$$

- $\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3$: vanishing points of x-, y-, z-axis, (e.g.: direction of x-axis, \mathbf{D})

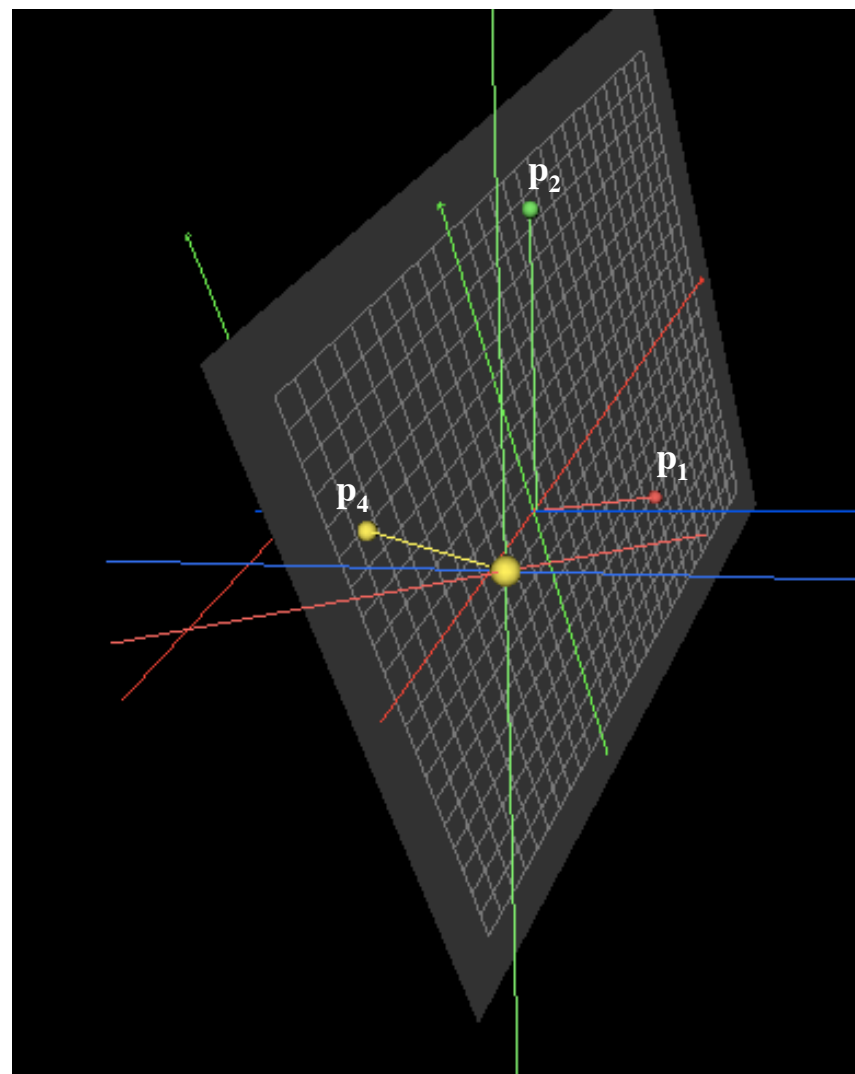
$$\mathbf{p}_1 = P\mathbf{D} = P \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

- \mathbf{p}_4 : image of world origin $(0,0,0 \ 1)^T$



Example

$$\begin{array}{ccc}
 \begin{array}{|c|} \hline 0.866 \\ \hline \end{array} & 0.5 & 0 & \begin{array}{|c|} \hline 1.732 \\ \hline \end{array} & \begin{array}{|c|} \hline 0.866 \\ \hline \end{array} \\
 \begin{array}{|c|} \hline 0.5 \\ \hline \end{array} & \begin{array}{|c|} \hline 0.866 \\ \hline \end{array} & 0 & \begin{array}{|c|} \hline 1.0 \\ \hline \end{array} & \begin{array}{|c|} \hline 0.5 \\ \hline \end{array} \\
 \begin{array}{|c|} \hline 0 \\ \hline \end{array} & 0 & 1 & \begin{array}{|c|} \hline 2.0 \\ \hline \end{array} & \begin{array}{|c|} \hline 0 \\ \hline \end{array} \\
 \hline \\
 \begin{array}{|c|} \hline 0.866 \\ \hline \end{array} & \begin{array}{|c|} \hline 0.5 \\ \hline \end{array} & 0 & \begin{array}{|c|} \hline 1.732 \\ \hline \end{array} & \begin{array}{|c|} \hline 0.5 \\ \hline \end{array} \\
 \begin{array}{|c|} \hline 0.5 \\ \hline \end{array} & \begin{array}{|c|} \hline 0.866 \\ \hline \end{array} & 0 & \begin{array}{|c|} \hline 1.0 \\ \hline \end{array} & \begin{array}{|c|} \hline 0.866 \\ \hline \end{array} \\
 \begin{array}{|c|} \hline 0 \\ \hline \end{array} & 0 & 1 & \begin{array}{|c|} \hline 2.0 \\ \hline \end{array} & \begin{array}{|c|} \hline 0 \\ \hline \end{array} \\
 \hline \\
 \begin{array}{|c|} \hline 0.866 \\ \hline \end{array} & 0.5 & \begin{array}{|c|} \hline 0 \\ \hline \end{array} & \begin{array}{|c|} \hline 1.732 \\ \hline \end{array} & \begin{array}{|c|} \hline 0 \\ \hline \end{array} \\
 \begin{array}{|c|} \hline 0.5 \\ \hline \end{array} & \begin{array}{|c|} \hline 0.866 \\ \hline \end{array} & 0 & \begin{array}{|c|} \hline 1.0 \\ \hline \end{array} & \begin{array}{|c|} \hline 0 \\ \hline \end{array} \\
 \begin{array}{|c|} \hline 0 \\ \hline \end{array} & 0 & 1 & \begin{array}{|c|} \hline 2.0 \\ \hline \end{array} & \begin{array}{|c|} \hline 1 \\ \hline \end{array} \\
 \hline \\
 \begin{array}{|c|} \hline 0.866 \\ \hline \end{array} & 0.5 & 0 & \begin{array}{|c|} \hline 1.732 \\ \hline \end{array} & \begin{array}{|c|} \hline 1.732 \\ \hline \end{array} \\
 \begin{array}{|c|} \hline 0.5 \\ \hline \end{array} & \begin{array}{|c|} \hline 0.866 \\ \hline \end{array} & 0 & \begin{array}{|c|} \hline 1.0 \\ \hline \end{array} & \begin{array}{|c|} \hline 1.0 \\ \hline \end{array} \\
 \begin{array}{|c|} \hline 0 \\ \hline \end{array} & 0 & 1 & \begin{array}{|c|} \hline 2.0 \\ \hline \end{array} & \begin{array}{|c|} \hline 2.0 \\ \hline \end{array}
 \end{array}$$



Summary

- Column Space -

image of x-axis	image of y-axis	image of z-axis	image of world origin
			“distance” w of world origin from camera ctr

Row Vectors of P

$$P = \begin{bmatrix} \mathbf{P}^{1T} \\ \mathbf{P}^{2T} \\ \mathbf{P}^{3T} \end{bmatrix}$$

- \mathbf{P}^{3T} : principal plane

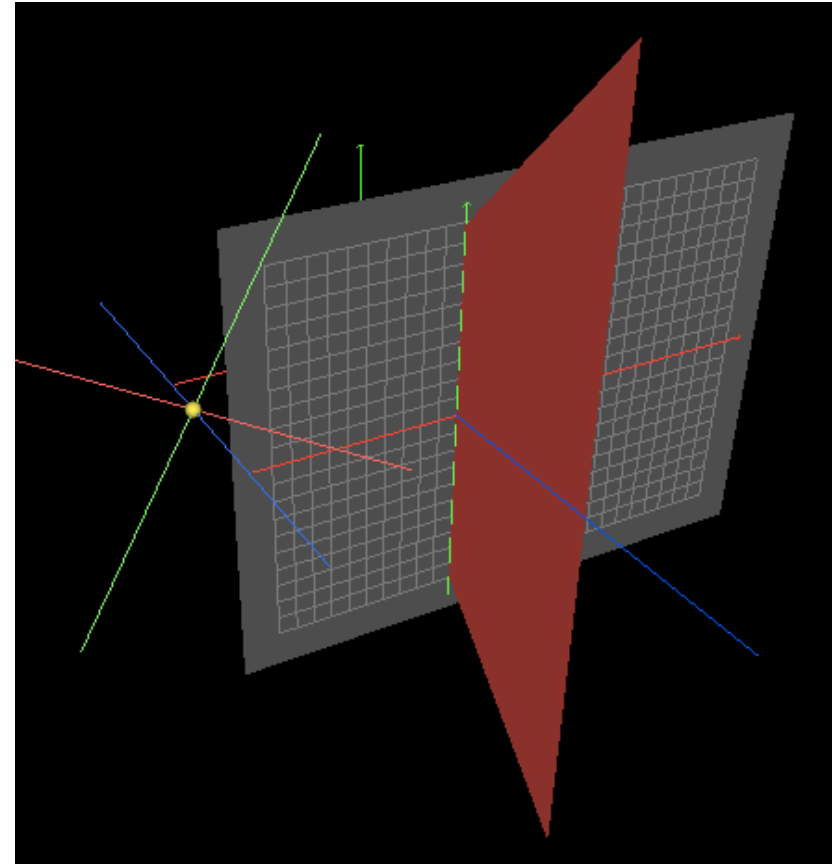
$$PX = \begin{bmatrix} x \\ y \\ 0 \end{bmatrix} \quad \text{i.e.,} \quad \mathbf{P}^{3T} \mathbf{X} = 0$$

(line at infinity)

- $\mathbf{P}^{1T}, \mathbf{P}^{2T}$: axis planes

$$PX = \begin{bmatrix} 0 \\ y \\ w \end{bmatrix} \quad \text{i.e.,} \quad \mathbf{P}^{1T} \mathbf{X} = 0$$

(plane P^1)



Example

- \mathbf{P}^{3T} : principal plane

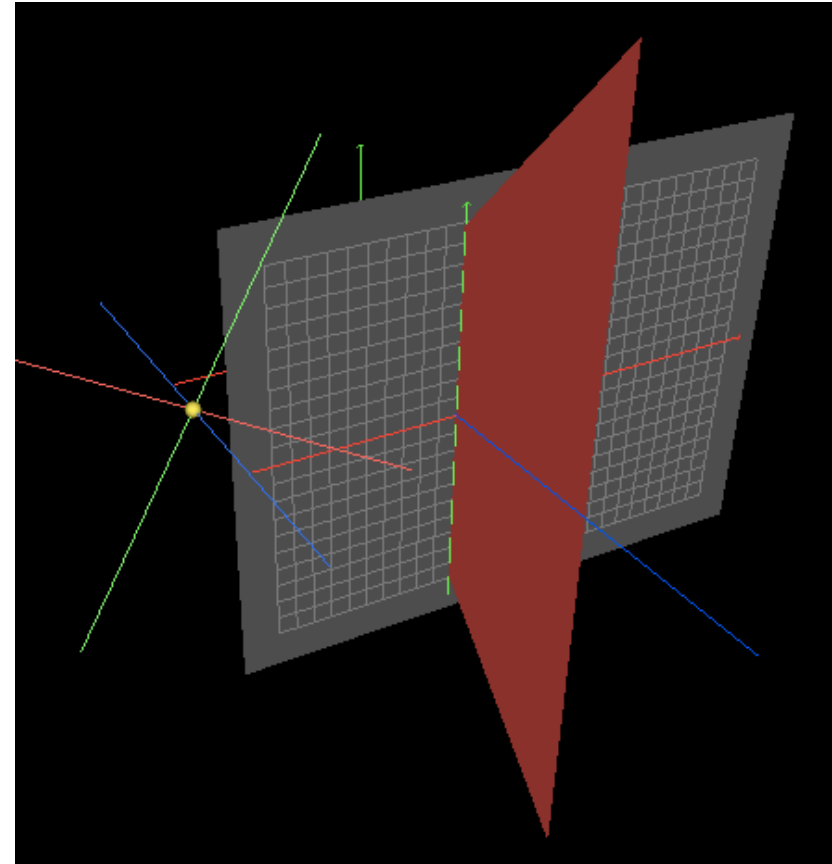
$$\begin{bmatrix} 0.866 & 0.5 & 0 & -1.732 \\ 0.5 & 0.866 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix} \mathbf{x} = \begin{bmatrix} x \\ y \\ 0 \end{bmatrix}$$

$(0 \ 0 \ 1 \ 2)\mathbf{x} = 0$
 $z = -2$

- axis plane \mathbf{P}^{1T}

$$\begin{bmatrix} 0.866 & 0.5 & 0 & -1.732 \\ 0.5 & 0.866 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 0 \\ y \\ w \end{bmatrix}$$

$(0.866 \ 0.5 \ 0 \ -1.732)\mathbf{x} = 0$
 $0.866x + 0.5y = 1.732$
 $y = 2(1.732 - 0.866x)$



Summary

- Row Space -

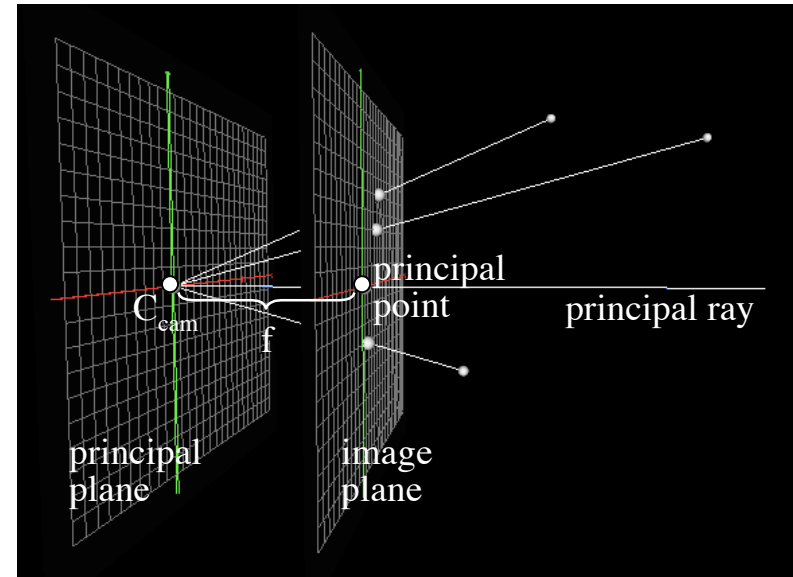
normal to plane through
y-axis and camera center

normal to plane through
x-axis and camera center

normal to principal plane
through camera center
“principal ray”

Principal Point

- Principal axis: ray from C , perp. to principal plane P^3
- Normal to $P^3: (p_{31}, p_{32}, p_{33}, 0)^T$ (on plane at infinity)
- Principal point:
 - intersection of principal axis with image plane
 - Projection of $(p_{31}, p_{32}, p_{33}, 0)^T$ onto the image plane
 $\mathbf{x}_0 = P(p_{31}, p_{32}, p_{33}, 0)^T = M\mathbf{m}^3$
(with $P = [M \mid p_4]$)



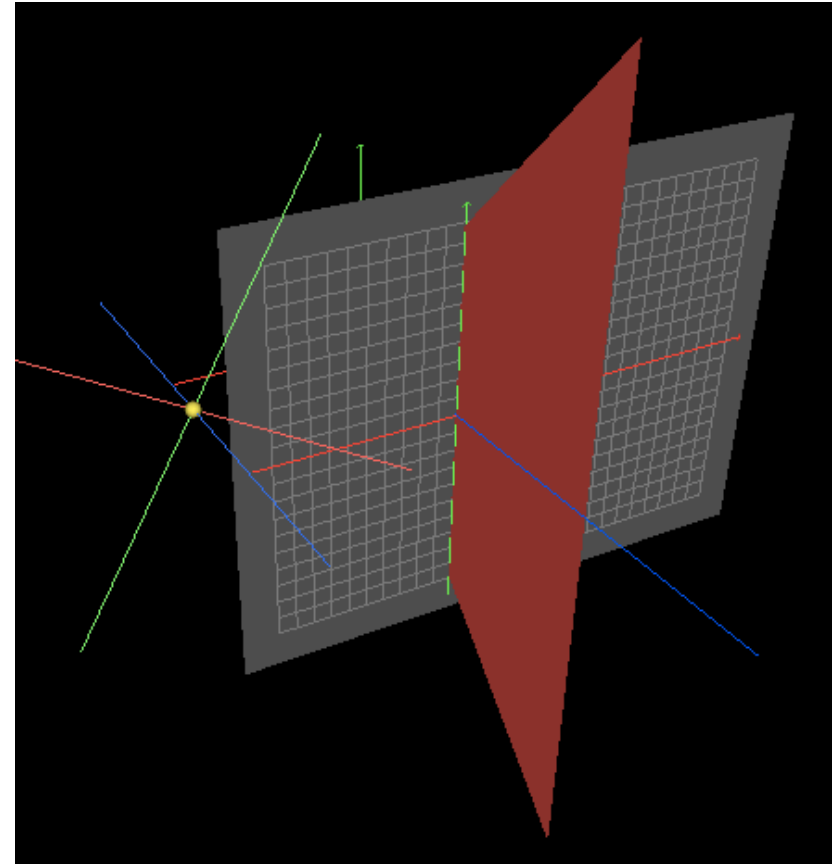
Example

- principal point

$$P = \begin{bmatrix} 0.866 & 0.5 & 0 \\ 0.5 & 0.866 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1.732 \\ 1 \\ 2 \end{bmatrix}$$

$$\mathbf{x}_0 = \mathbf{M}\mathbf{m}^3$$

$$= \begin{bmatrix} 0.866 & 0.5 & 0 \\ 0.5 & 0.866 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$



Summary

- Principal Ray -

	image of world origin
principal ray direction	“distance” w of world origin from camera center

Depth of Points

- Perpendicular distance w of a point \mathbf{X} from the principal plane (along the principal ray)

$$\begin{aligned}w &= \mathbf{P}^{3T} \mathbf{X} \\ &= \mathbf{P}^{3T} (\mathbf{X} - \mathbf{C})\end{aligned}$$

(since $\mathbf{P}\mathbf{C}=0$ for the camera center)

$$= \mathbf{m}^{3T} (\tilde{\mathbf{X}} - \tilde{\mathbf{C}})$$

- Dot product of the ray from the camera center to \mathbf{X} with the principal ray direction

Cameras at Infinity

Cinematography

- Move camera backwards along the optical axis (with increasing z)
- Rescale picture by increasing f

$$x' = f \frac{x}{z}$$

$$dx' = f \left(\frac{x}{z} - \frac{x}{z+1} \right) = f \left(\frac{x}{z^2 + z} \right)$$

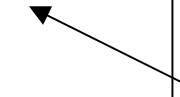
- small z and f : perspective
- large z and f : weak perspective

Cameras at Infinity

Moving camera along the principal ray:

\mathbf{r}^{1T}	$-\mathbf{r}^{1T}\mathbf{C}$
\mathbf{r}^{2T}	$-\mathbf{r}^{2T}\mathbf{C}$
\mathbf{r}^{3T}	$d_{0..t} = -\mathbf{r}^{3T}\mathbf{C}$

“distance” w
of world origin
from camera
center



Cameras at Infinity

Zooming (by factor $k = d_t/d_0$):

	\mathbf{r}^{1T}	$-\mathbf{r}^{1T}\mathbf{C}$
$d_t/d_0 \mathbf{K}$	\mathbf{r}^{2T}	$-\mathbf{r}^{2T}\mathbf{C}$
	$\mathbf{r}^{3T}d_0/d_t \rightarrow \mathbf{0}^T$	d_0

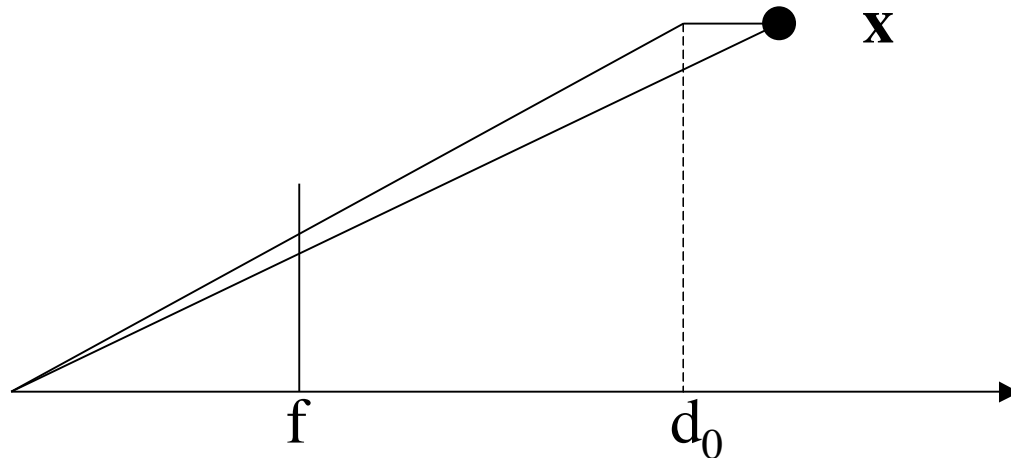
\mathbf{P} , affine Camera

“distance” w of initial world origin from camera center

Cameras at Infinity

- Weak Perspective -

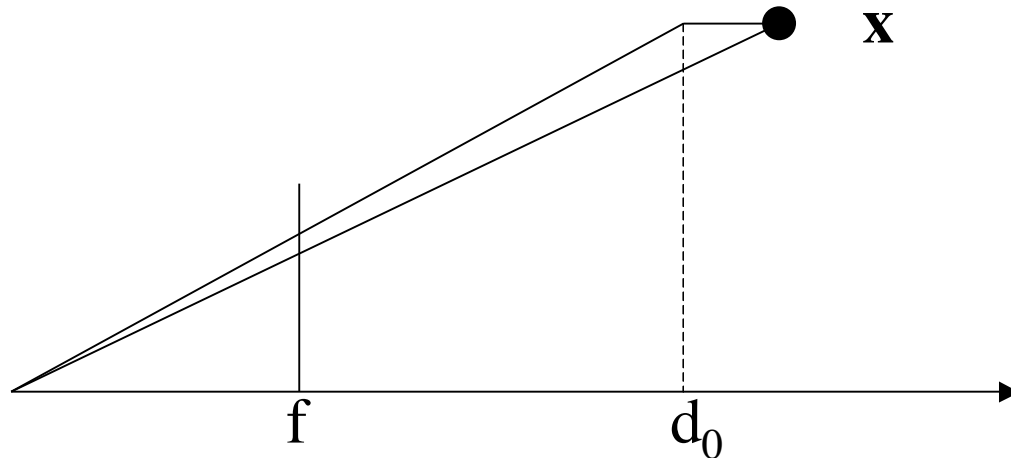
- The action of a weak perspective camera is equivalent to orthographic projection onto a plane at $Z=d_0$, followed by a perspective projection from the plane.



Cameras at Infinity

- Affine Approximation -

- The effect of the affine approximation P to the true camera matrix P_0 is to move the image of a point X radially towards or away from the principal point x_0 by a factor equal to $1 + \frac{\Delta}{d_0}$.



Cameras at Infinity

- The distance between the true perspective image position and the position obtained using the affine camera approximation is small, if
 - the depth relief is small compared to the average depth (d_0)
 - the distance of the point from the principal ray is small

Parallel Projection versus Perspective Projection

Reminder

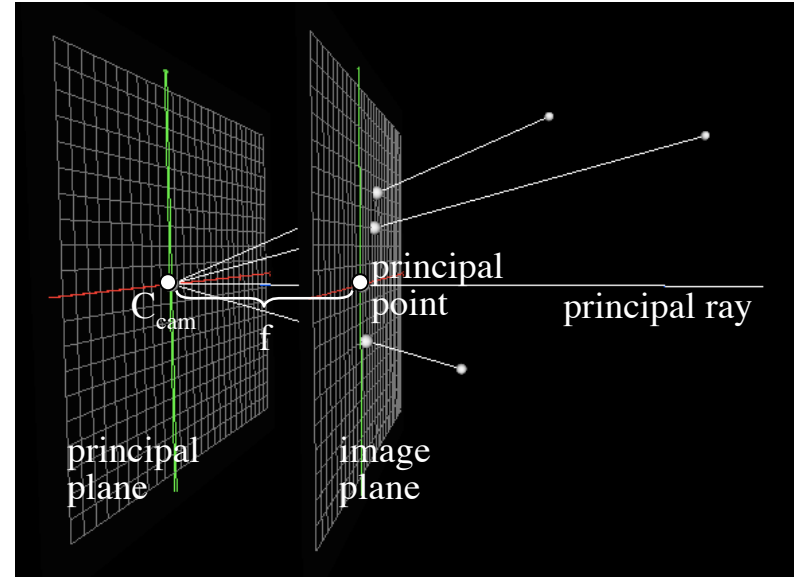
Central Projection Using Homogeneous Coordinates

$$\begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} \rightarrow \begin{bmatrix} f X/Z \\ f Y/Z \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} fX \\ fY \\ Z \\ 1 \end{bmatrix} = f \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$\mathbf{x} = \mathbf{P}\mathbf{X}$$

$$\mathbf{P} = \text{diag}(f, f, 1) [\mathbf{I} \quad | \quad \mathbf{O}]$$



Reminder

Principal Point Offset

- Principal point at (p_x, p_y) rel. to image origin

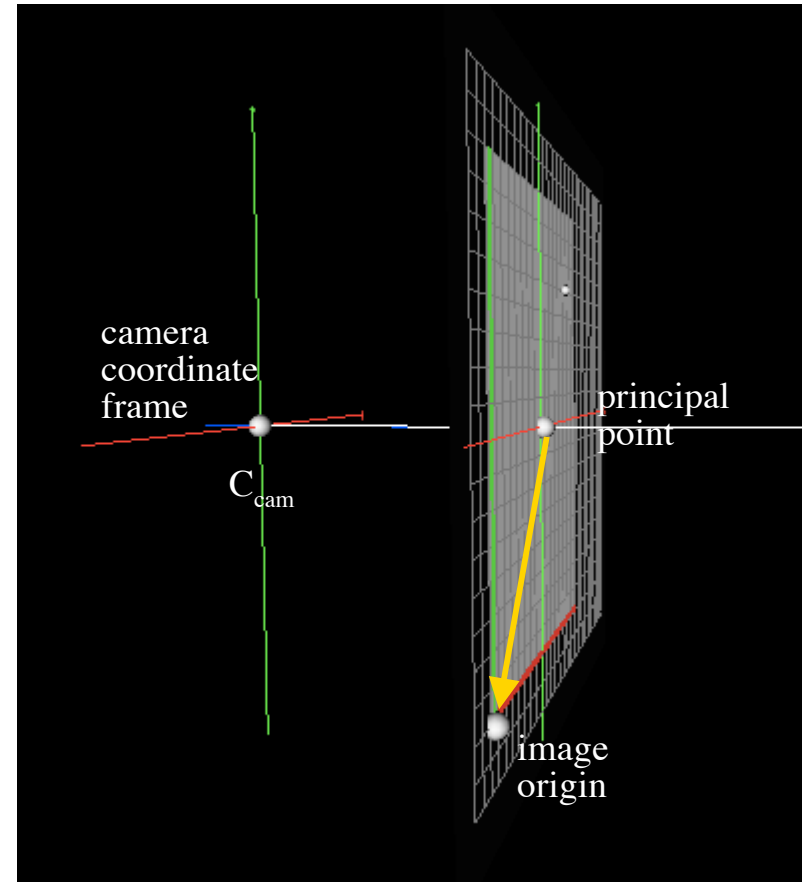
$$\begin{bmatrix} fX + Zp_x \\ fY + Zp_y \\ Z \\ 1 \end{bmatrix} = f \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} \quad \begin{matrix} p_x \\ p_y \\ 1 \end{matrix}$$

$$\mathbf{x} = \mathbf{K}[\mathbf{I} \mid \mathbf{O}]\mathbf{X}_{\text{cam}}$$

$$\mathbf{K} = \begin{bmatrix} f & p_x \\ 0 & f & p_y \\ 0 & 0 & 1 \end{bmatrix}$$

camera calibration matrix

- 3 degrees of freedom



Cameras at Infinity

- Parallel Projection -

- parallel projection matrix:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- calibration matrix

$$\begin{bmatrix} \mathbf{K}_{2 \times 2} & \hat{\mathbf{0}} \\ \hat{\mathbf{0}}^T & 1 \end{bmatrix}$$

- principal point is not defined

Hierarchy of Affine Cameras

Hierarchy of Affine Cameras

- orthographic projection

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

- with Euclidean transformations (5 DOF)

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} \mathbf{R} \\ \mathbf{0}^T \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} \mathbf{r}^{1T} \\ \mathbf{r}^{2T} \\ \mathbf{0}^T \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} + \begin{bmatrix} t_1 \\ t_2 \\ 1 \end{bmatrix}$$

Hierarchy of Affine Cameras

- scaled orthographic projection (6 DOF)

$$\begin{array}{c} \square \\ \square \\ \square \\ \square \\ \square \end{array} \begin{array}{c} x \\ y \\ 1 \end{array} = \begin{array}{c} \square \\ \square \\ \square \\ \square \\ \square \end{array} \begin{array}{c} k \\ \\ \\ \\ \\ \end{array}$$

$$\begin{array}{c} \square \\ \square \\ \square \\ \square \\ \square \end{array} \begin{array}{c} \mathbf{r}^{1T} \\ \mathbf{r}^{2T} \\ \mathbf{0}^T \\ 1 \end{array}$$

$$\begin{array}{c} t_1 \\ t_2 \\ 1 \end{array} \begin{array}{c} \square \\ \square \\ \square \\ \square \\ \square \end{array} \begin{array}{c} x \\ y \\ z \\ 1 \end{array} = \begin{array}{c} \square \\ \square \\ \square \\ \square \\ \square \end{array} \begin{array}{c} \mathbf{r}^{1T} \\ \mathbf{r}^{2T} \\ \mathbf{0}^T \\ 1 \end{array}$$

$$\begin{array}{c} t_1 \\ t_2 \\ 1/k \\ 1 \end{array} \begin{array}{c} \square \\ \square \\ \square \\ \square \\ \square \end{array} \begin{array}{c} x \\ y \\ z \\ 1 \end{array}$$

- weak perspective projection (7 DOF)

$$\begin{array}{c} \square \\ \square \\ \square \\ \square \\ \square \end{array} \begin{array}{c} x \\ y \\ 1 \end{array} = \begin{array}{c} \square \\ \square \\ \square \\ \square \\ \square \end{array} \begin{array}{c} \square_x \\ \\ \\ \\ \\ \end{array}$$

$$\begin{array}{c} \square \\ \square \\ \square \\ \square \\ \square \end{array} \begin{array}{c} \mathbf{r}^{1T} \\ \mathbf{r}^{2T} \\ \mathbf{0}^T \\ 1 \end{array}$$

$$\begin{array}{c} t_1 \\ t_2 \\ 1 \end{array} \begin{array}{c} \square \\ \square \\ \square \\ \square \\ \square \end{array} \begin{array}{c} x \\ y \\ z \\ 1 \end{array}$$

Hierarchy of Affine Cameras

- affine camera (8 DOF)

$$\begin{array}{c} \square \\ \square \\ \square \\ \square \\ \square \\ \square \\ \square \\ \square \\ \square \\ \square \end{array} \begin{array}{c} x \\ y \\ 1 \end{array} = \begin{array}{c} \square \\ \square \\ \square \\ \square \\ \square \\ \square \\ \square \\ \square \\ \square \\ \square \end{array} \begin{array}{c} x \\ y \\ 1 \end{array}$$

S
 \square_y

$$\begin{array}{c} \square \\ \square \\ \square \\ \square \\ \square \\ \square \\ \square \\ \square \\ \square \\ \square \end{array} \begin{array}{c} \mathbf{r}^{1T} \\ \mathbf{r}^{2T} \\ \mathbf{0}^T \end{array}$$

$$\begin{array}{c} \square \\ \square \\ \square \\ \square \\ \square \\ \square \\ \square \\ \square \\ \square \\ \square \end{array} \begin{array}{c} x \\ y \\ z \\ 1 \end{array}$$

$$\mathbf{P}_A = \begin{array}{c} \square \\ \square \\ \square \\ \square \\ \square \\ \square \\ \square \\ \square \\ \square \\ \square \end{array} = \begin{array}{c} m_{11} \\ m_{12} \\ m_{13} \\ m_{21} \\ m_{22} \\ m_{23} \\ 0 \\ 0 \\ 0 \\ 1 \end{array} \begin{array}{c} t_1 \\ t_2 \\ 1 \end{array}$$