

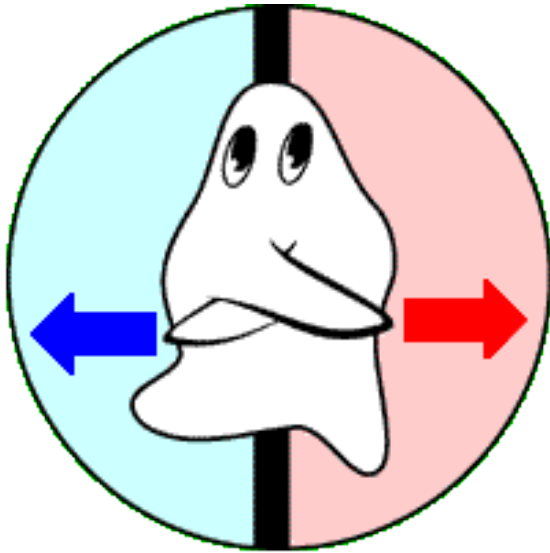
Demons: A Deformable Registration Toolbox

Presented by Tom Vercauteren
Mauna Kea Technologies
Paris, France



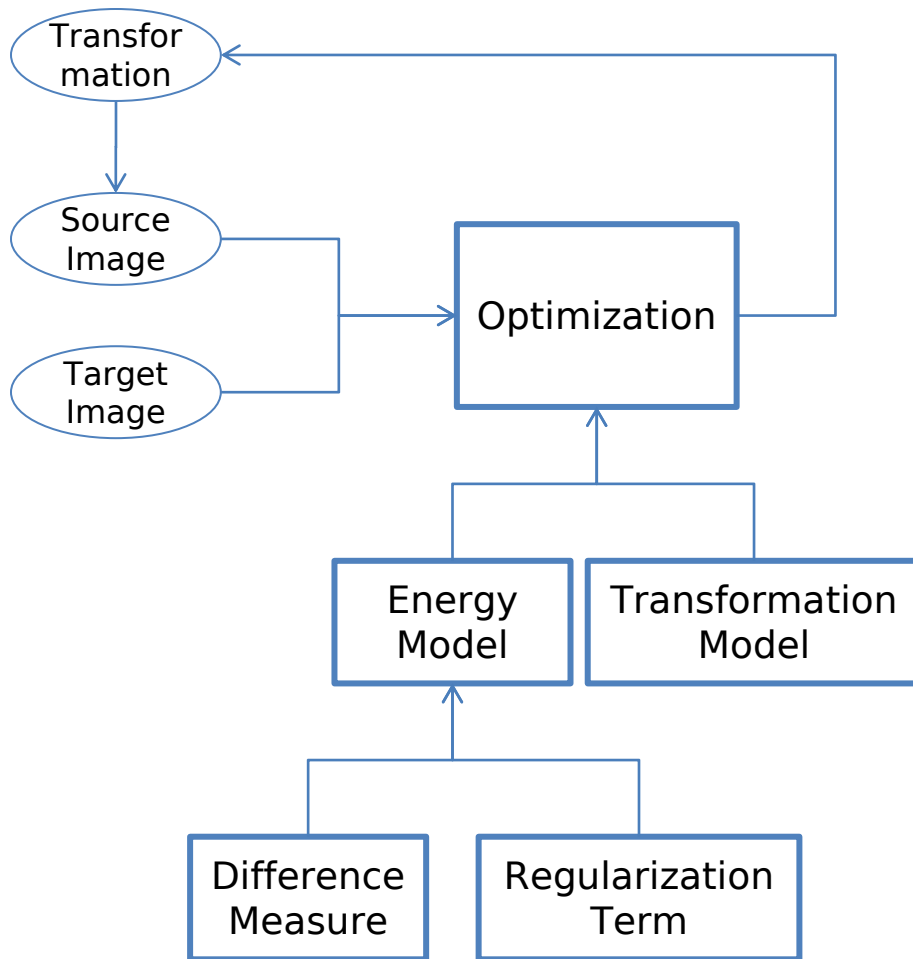
Goal of this Session

- Provide key concepts behind the demons algorithm
 - Alternate optimization
- Show that the demons is more of a framework than an algorithm
 - Can handle different application constraints
- Intuition on why demons is useful
 - Largely available, easy to code, decent accuracy and efficiency



HISTORICAL BACKGROUND

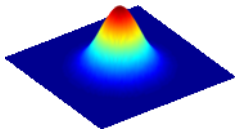
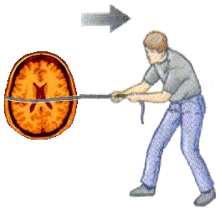
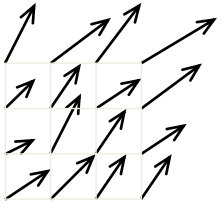
Yet Another Framework?



- Well-posed intensity-based framework existed, did we need another one?
- Devil in optimization & numerics
 - N degrees of freedom on transformation \Rightarrow N by N system to solve
 - Huge on 3D data

Demons Rationale *ca* 1995

- Trade theory for efficiency
- Transformation: Each pixel has a displacement vector
- Intuitive image forces to compute independent pixel displacements
- Regularize displacements by Gaussian smoothing
- (Thirion, 1995)



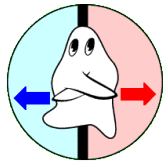
Optical flow as Image Forces

- View images to register as in motion
 - Fixed image $I(x,t)$
 - Moving image $I(x, t+dt)$
- Brightness consistency constraint
 - $I(x+dx, t+dt) = I(x,t)$
- Taylor expansion
 - $I(x+dx, t+dt) \approx I(x,t) + \nabla_x I \cdot dx + \nabla_t I \cdot dt$
- Optical flow equation (e.g. Horn and Schunk, 1981)
 - $\nabla_x I \cdot dx = - \nabla_t I \cdot dt = - (I(x, t+dt) - I(x,t))$



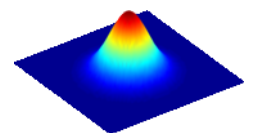
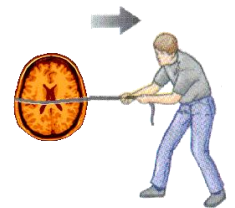
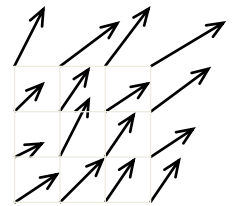
Optical flow as Image Forces

- No unique solution to optical flow equation
 - Aperture problem
 - \Rightarrow Rely on the minimum norm solution
 - $dx = (I(x,t) - I(x, t+dt)) \cdot \nabla_x I / \|\nabla_x I\|^2$
 - Stabilize it in a somewhat ad-hoc manner
- Intuition behind the image force
 - Demons push according to image gradient if pixel value is lower than target value
 - Resp. opposite of the image gradient if pixel value is higher than target value



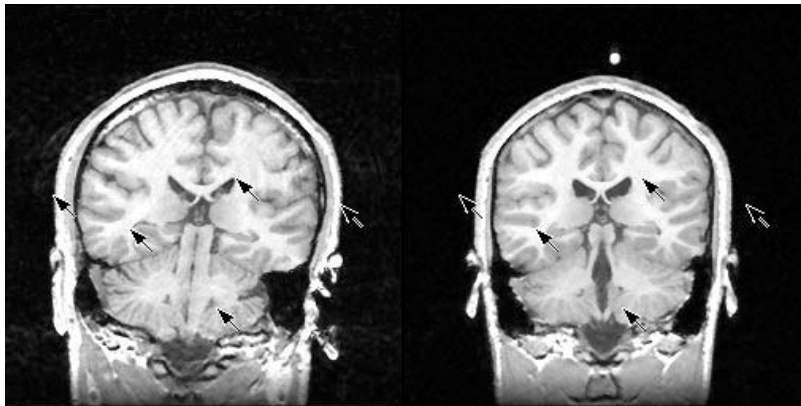
Classical Demons Algorithm

- Using more convenient notation
 - F , M resp. fixed and moving images
- Demons iterations
 - Initial displacement field s
 - Compute image forces u to push $M \circ s$ towards F
i.e. make $M \circ (s+u)$ more similar to F
 - $s \leftarrow$ Gaussian smoothing on $s+u$

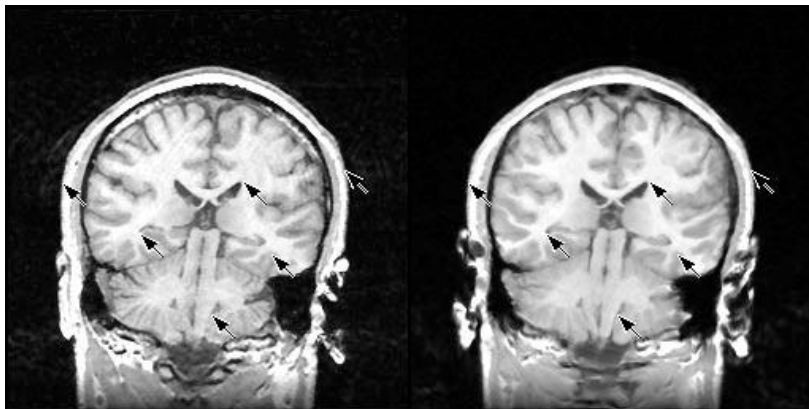


Examples by Thirion *ca* 1995

Pairwise registration problem



Demons result



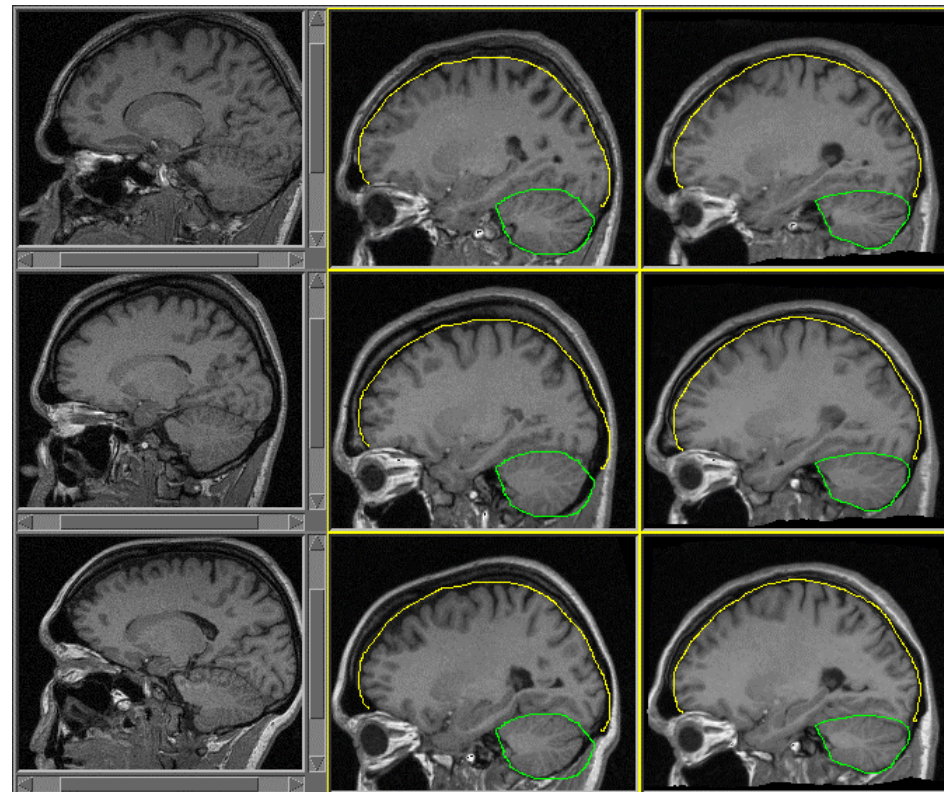
Courtesy R. Kikinis

**Segmentation through
Atlas registration**

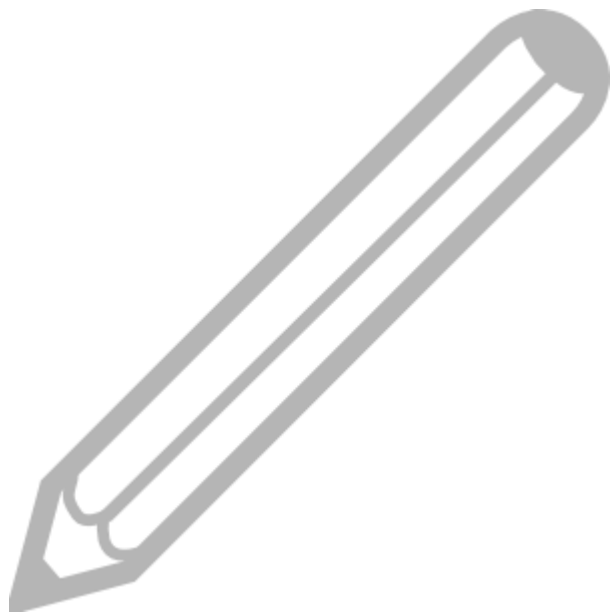
Original

Affine

Demons



Courtesy D. Vandermeulen



THEORETICAL INSIGHT



Beyond Thirion's View

- Demons are efficient but have shortcomings
 - No strong theoretical analysis
 - E.g. Not presented as energy minimization
 - \Rightarrow Difficult to generalize
 - Similarity measures, transformation constraints, etc.
 - \Rightarrow No insight on convergence
 - Why does it work?

Alternate Optimization

- Several attempts to interpret the demons
 - Modersitzki et al., Cathier & Pennec
 - Refer to Darko's talk for *fluid* demons
- Focus on Cathier & Pennec approach
 - Hidden variable $c \Rightarrow$ demons as minimization of a global criterion

$$- E(c, s) = \text{Sim}(F, M \circ c) + \text{dist}(c, s)^2 + \text{Reg}(s)$$

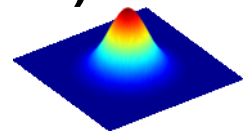
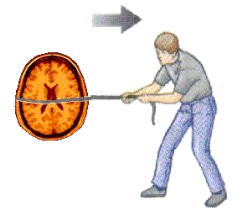
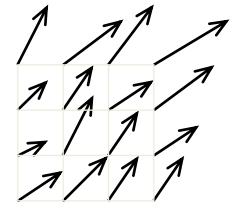
1: Correspondence energy

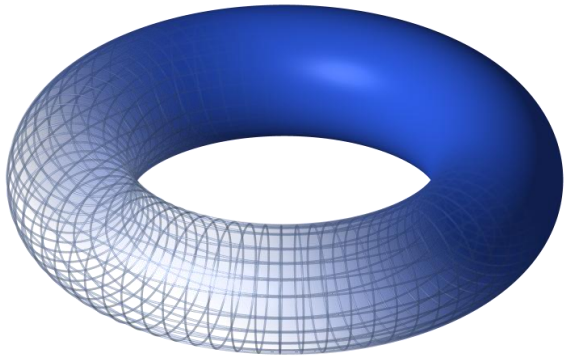
2: Regularization term



Classical Demons Revisited

- Demons iterations
 - s (displacement field) given
 - SSD Forces u to optimize
$$E_{\text{corr}}(u) = \|F - M \circ (s+u)\|^2 + \|u\|^2$$
 - Additive update: $c \leftarrow s + u$
 - Diffusion-like regularization (Gaussian):
$$s \leftarrow Id + K * (c - Id)$$





A geometric approach to extend the demons

INTRODUCING Diffeomorphisms

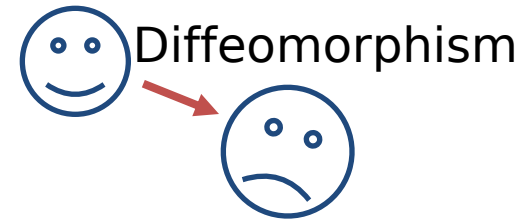
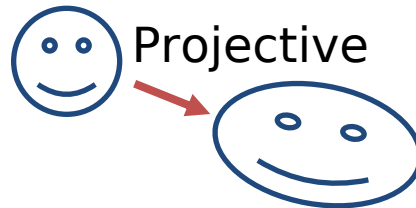
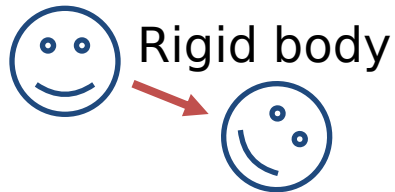


What's with Diffeomorphisms?

- Deformable registration not only about intensity differences
 - Need to address the transformations
- Diffeomorphic registration
 - One to one, invertible, mapping
 - No foldings
 - Preserves topology
 - Essential for computational anatomy
 - Sound assumption if no privileged direction
- Not all problems benefit from diffeomorphisms!
 - Different applications \Rightarrow different constraints
 - E.g. topology changes from tumor resections
- Should leverage a relevant representation of the transformation space



Spatial Transformations



- Spatial transformations do not form vector spaces
- Addition: no geometric meaning

$$- s1, s2 \in G \Rightarrow s = s1 + s2 \notin G$$



- Natural operation: composition

$$- s1, s2 \in G \Rightarrow s = s1 \circ s2 \in G, \quad s: p \in \Omega, s(p) = s1(s2(p))$$



- *Lie group* structure

Walking on Lie Groups

- Inconsistency of additive optimization steps

- Ad-hoc or slow

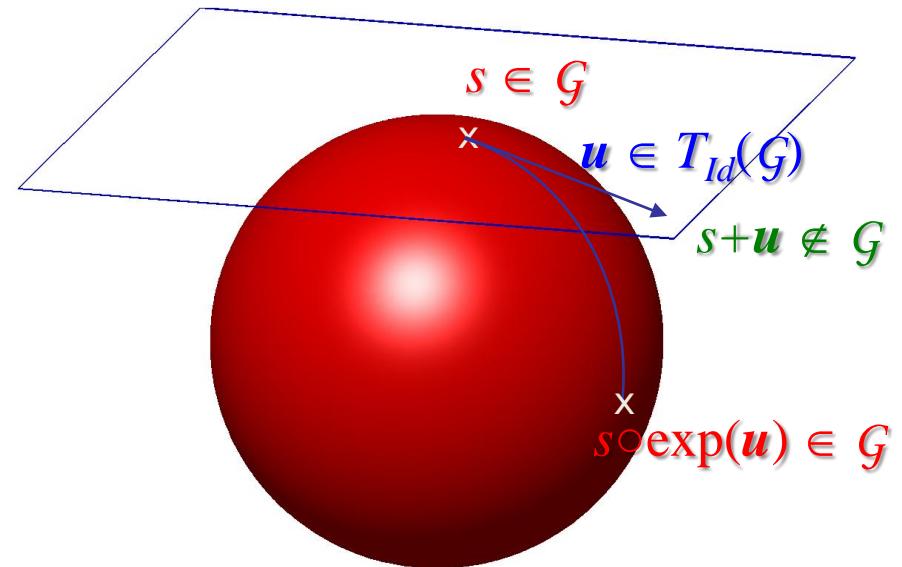
$$~~s \leftarrow s + u~~$$

- Lie group Geometry

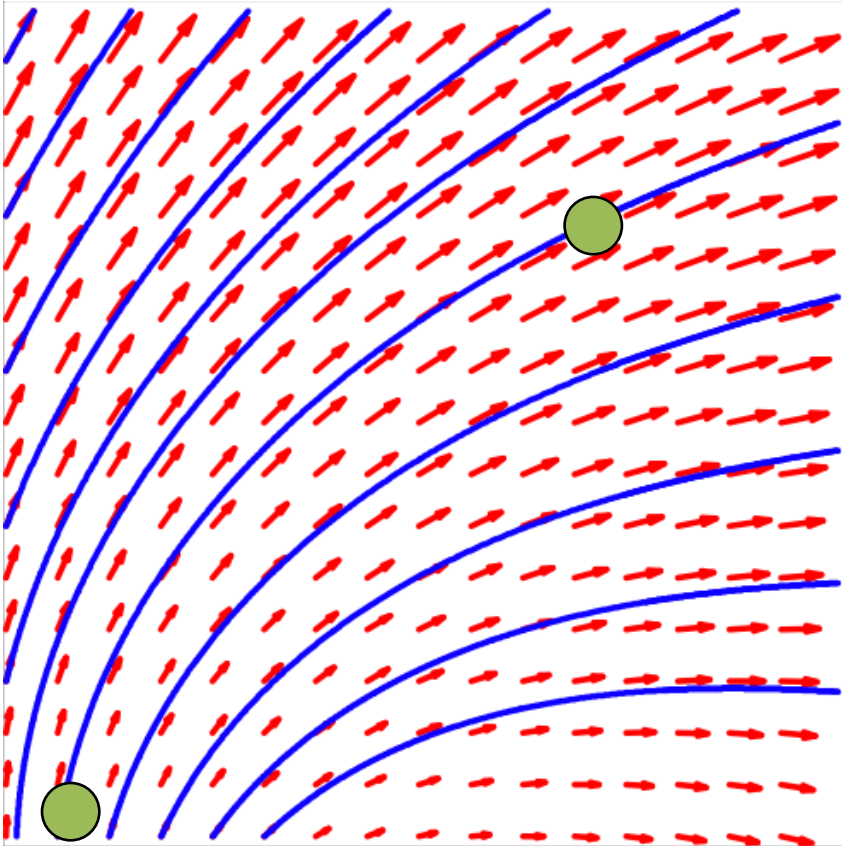
- Intrinsic steps

- No constrain

$$s \leftarrow s \circ \exp(u)$$



Exponential & Diffeomorphisms: Flows of Vector Fields

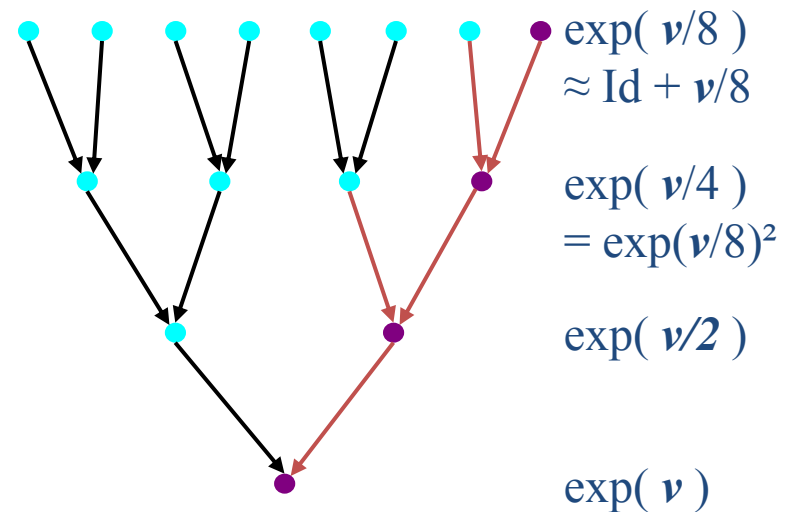


$$\frac{\partial x}{\partial t} = v(x)$$

$$x(0) = x_0$$

$$x(1) = \int_0^1 v(x(t)) dt$$

$$\triangleq \exp(v)$$

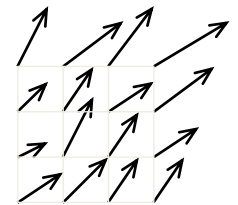


Diffeomorphic Demons

- Vercauteren et al., NeuroImage 09

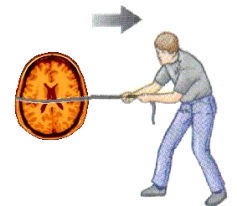
- Demons iterations

- s (displacement field) given



- SSD Forces u to optimize

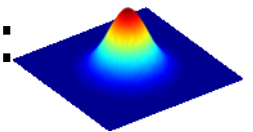
$$E_{\text{corr}}(\mathbf{u}) = \| F - M \circ \underline{s \circ \exp(\mathbf{u})} \|^2 + \|\mathbf{u}\|^2$$



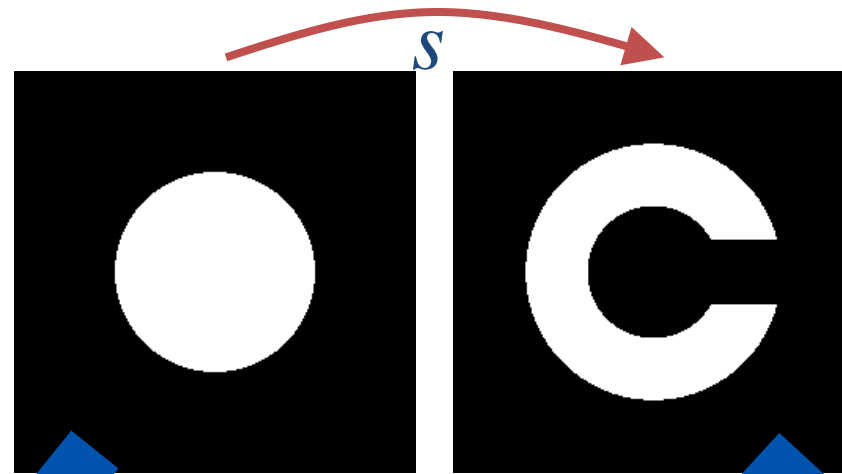
- Diffeomorphic update: $c \leftarrow \underline{s \circ \exp(\mathbf{u})}$

- Diffusion-like regularization (Gaussian):

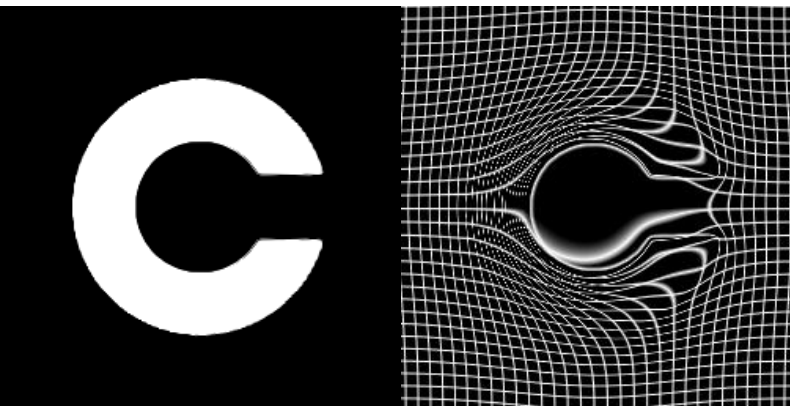
$$s \leftarrow Id + K * (c - Id)$$



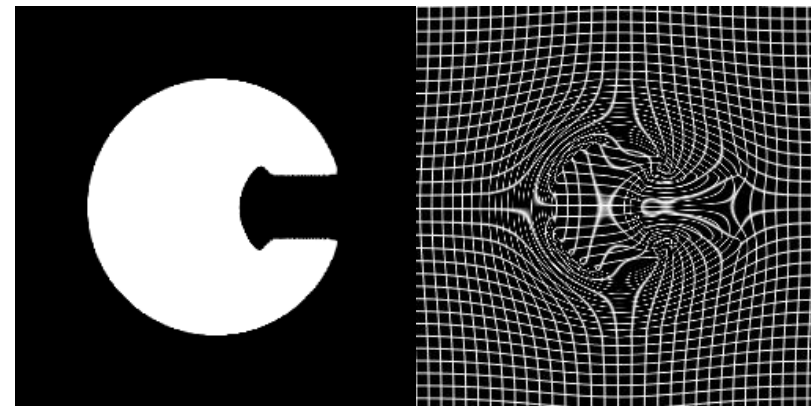
Synthetic Example



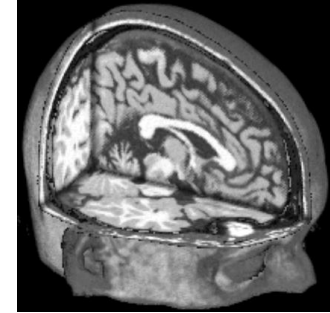
Diffeomorphic
Demons



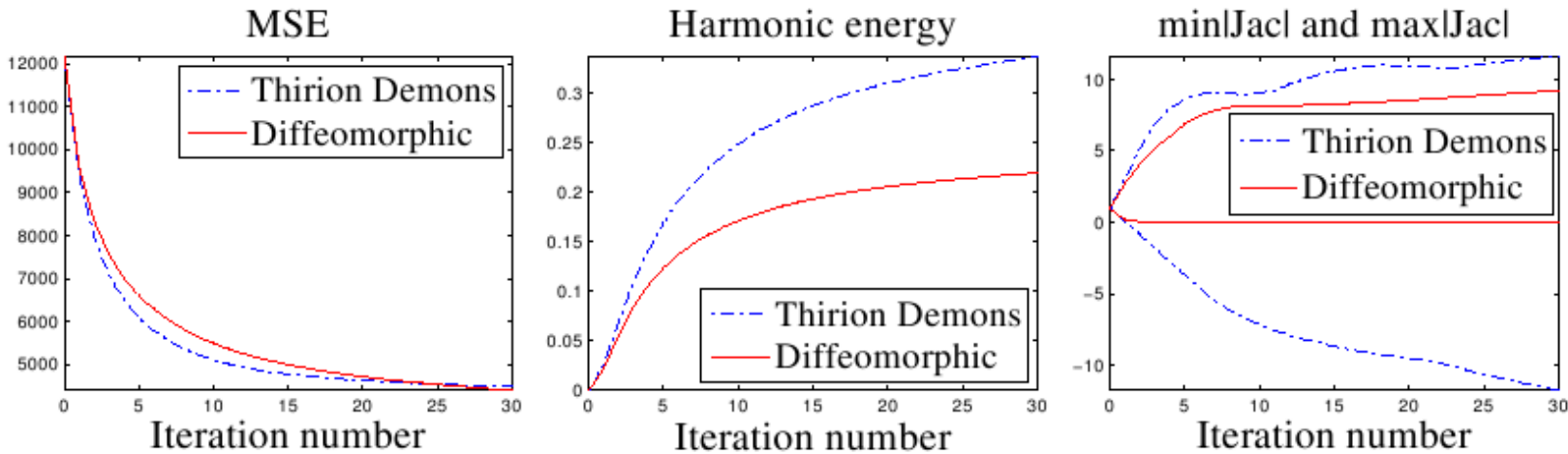
Additive
Demons



Brain Web Registration



- Similar visual appearance
- Similar segmentation agreement
 - Dice index = $2 |X \cap Y| / (|X| + |Y|)$
- Smoother and diffeomorphic transformations



| | CSF | GM | WM | Fat | Muscle | Skin | Skull | Vessels | Fat2 | Dura | Marrow |
|----------------|-------|-------|-------|-------|--------|-------|-------|---------|-------|-------|--------|
| Initial | 41.73 | 63.06 | 61.51 | 19.30 | 20.14 | 66.65 | 42.75 | 14.26 | 6.06 | 14.74 | 28.19 |
| Thirion | 63.41 | 78.99 | 79.23 | 47.74 | 36.40 | 78.57 | 64.91 | 27.21 | 14.75 | 23.13 | 45.05 |
| Diffeo | 64.37 | 78.94 | 78.43 | 47.22 | 36.11 | 79.39 | 65.02 | 27.25 | 14.70 | 24.56 | 43.92 |



When square lattices are just not enough

WORKING ON DIFFERENT GEOMETRIES

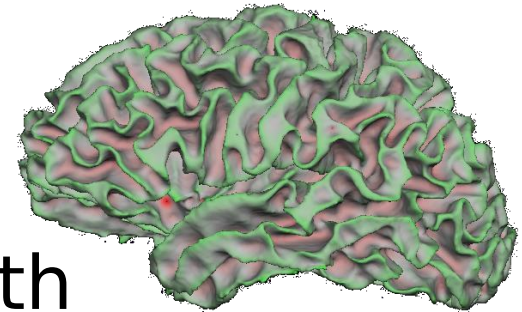


First author
B.T. Thomas Yeo

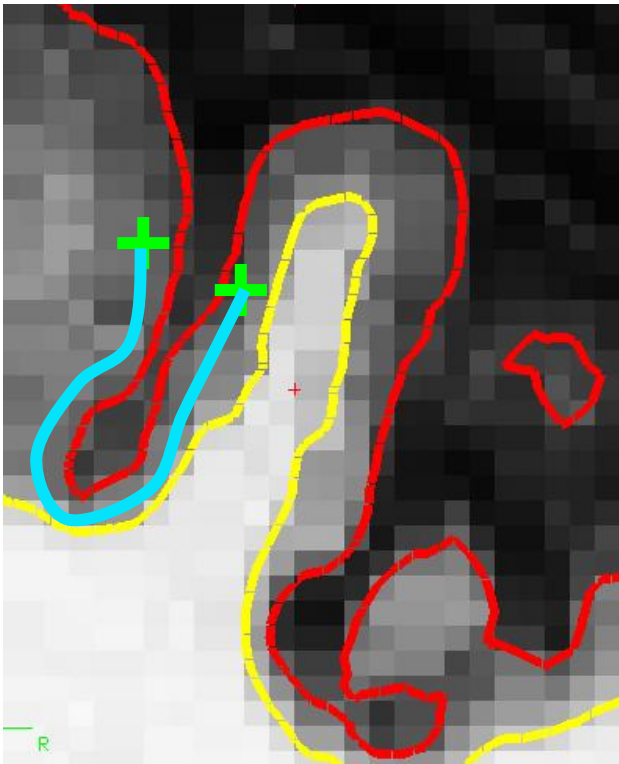


Cortical Surfaces Registration

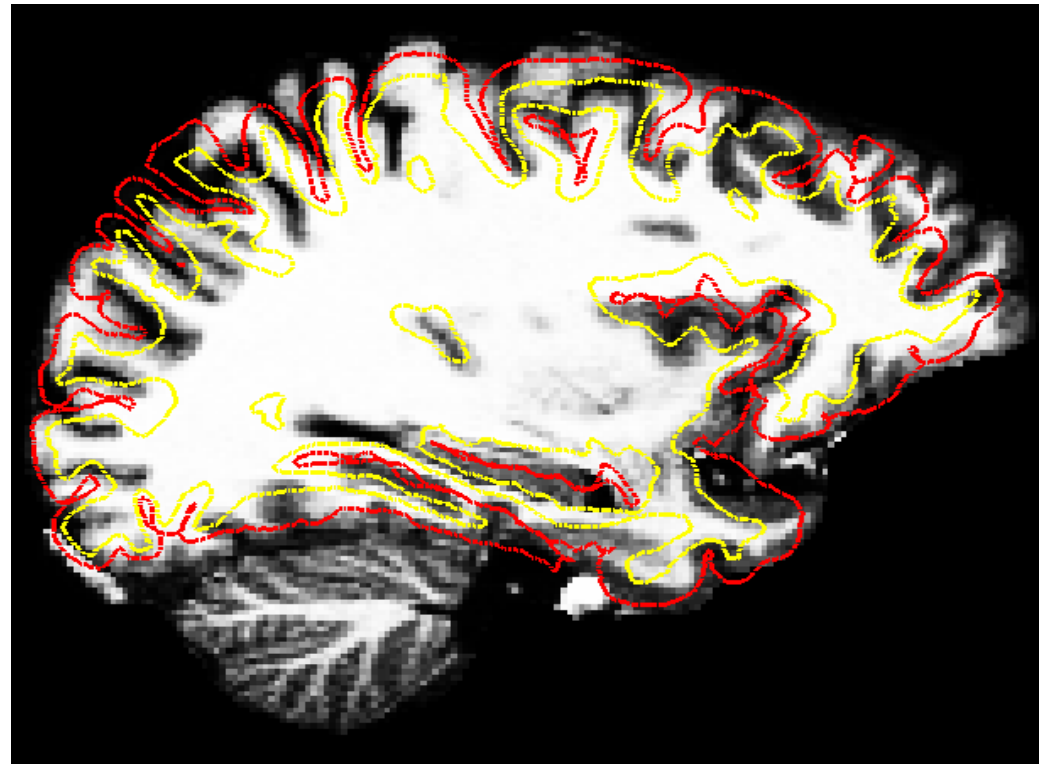
- Human Cerebral Cortex
 - Highly convoluted surface
 - Folding pattern correlated with cytoarchitectonics & function
 - Functional organization largely 2D!
- Modern MRI: Closed 2D mesh in 3D
- Population studies requires registration of these 2D meshes



Aligning Cortical Folds Directly in the Volume is Hard



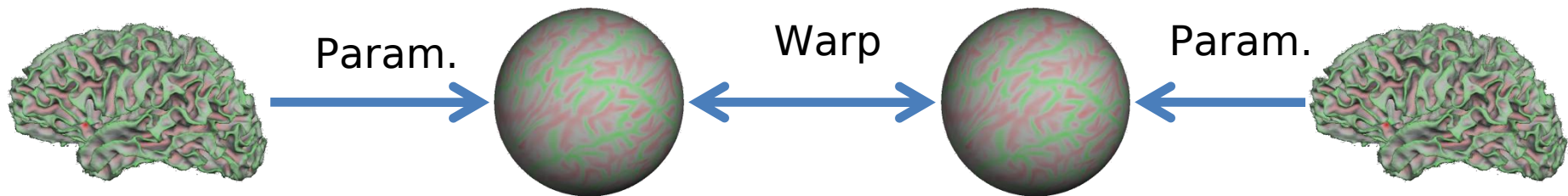
Cortical surface
extraction example



Affine transform of surfaces from one subject
mapped to another. Courtesy L. Zollei.

Spherical Demons Surface Registration Strategy

- Landmark-free
- Intrinsic use of the surface
- Spherical parameterization
- Intensities on surfaces are typically geometric attributes, e.g. curvature



Extending the Diffeomorphic Demons to the Sphere

- Yeo et al., MICCAI 08
- Same diffeomorphism tools
 - Exponential of velocity fields
 - Scaling and squaring
- Same SSD-based objective function
 - $E(c,s) = \text{Sim}(F, M \circ c) + \text{dist}(c,s)^2 + \text{Reg}(s)$
- Requires workable definitions of
 - Transformation distances: $\text{dist}(c,s)$
 - Transformation regularity: $\text{Reg}(s)$



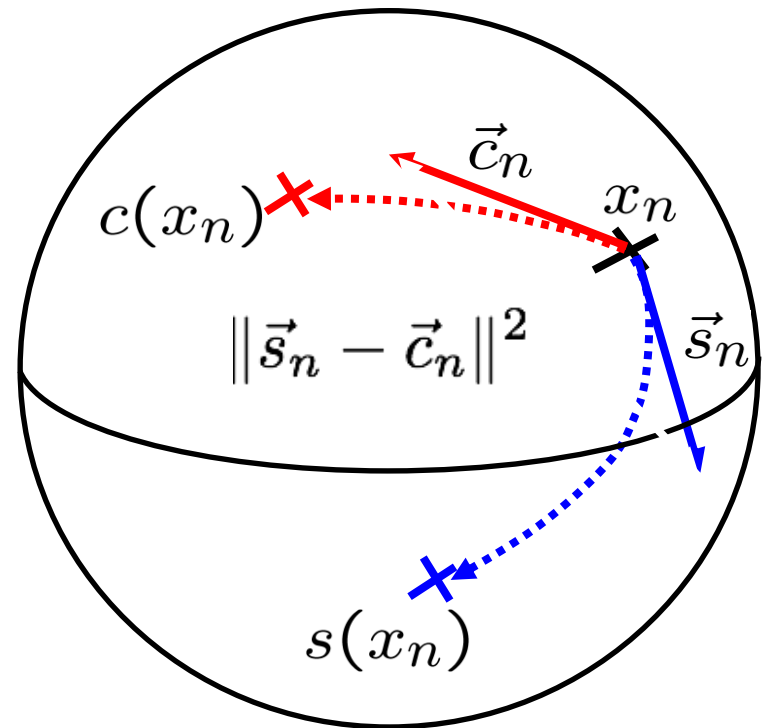
Choice of $\text{dist}(s, c)$

- Represent deformations as tangent vectors

- $c \triangleq \{\vec{c}_n\}$

- $s \triangleq \{\vec{s}_n\}$

- $\text{dist}(s, c) = \sum_{n=1}^N \|\vec{s}_n - \vec{c}_n\|^2$



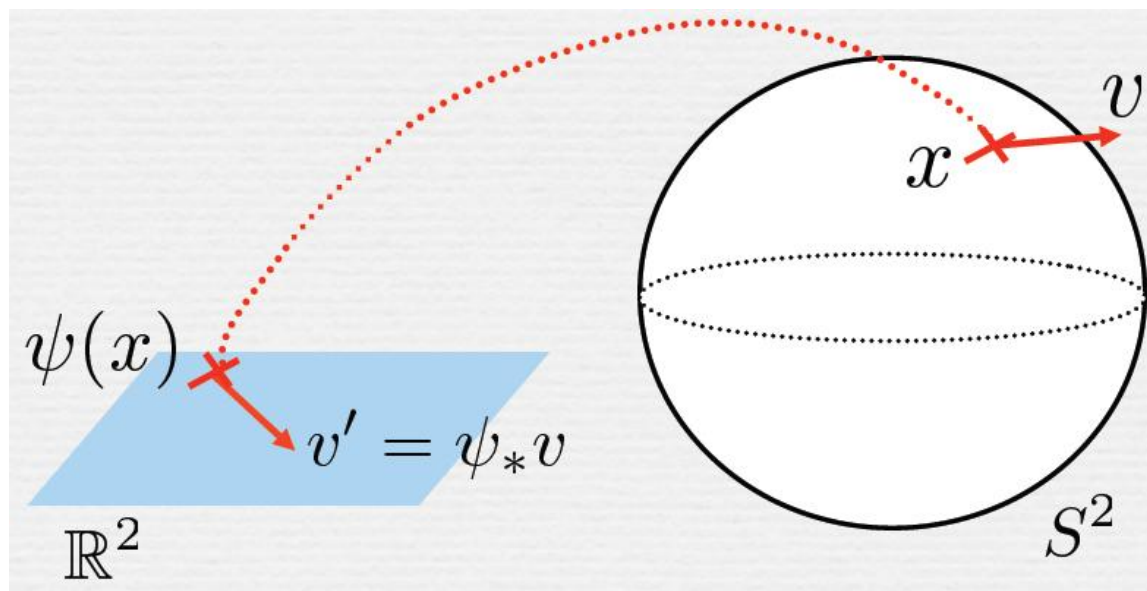
Spherical Demons: Step 1

$$\min_v \|F - M \circ s \circ \exp(v)\|^2 + \sum_n \|\vec{s}_n - \vec{c}_n(v)\|^2$$

Chart $\psi : S^2 \mapsto \mathbb{R}^2$

\implies Flow of v' on $\mathbb{R}^2 = \psi(\text{Flow of } v \text{ on } S^2)$

- Invert 2 x 2 matrix per vertex
- Independent of the chart



Spherical Demons: Step 2

$$\min_s \frac{1}{\sigma_x^2} \sum_{n=1}^N \|\vec{s}_n - \vec{c}_n\|^2 + \frac{1}{\sigma_T^2} \text{Reg}(s)$$

- Choice of regularization $\text{Reg}(s) = \|\vec{s}\|_H^2$
- Approximate optimum (Trouvé '98, Glaunès '04)
 - Spherical convolution

$$\vec{s}_n = \sum_{i=1}^N \lambda(\|x_i - x_n\|) T_{x_i, x_n}(\vec{c}_i)$$

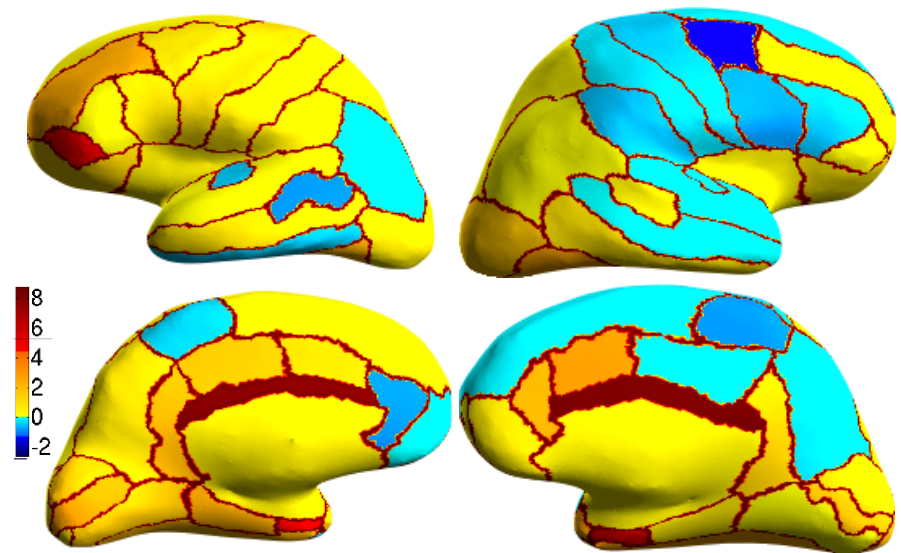
← Parallel transport

- Practice: *Iterated smoothing* with close neighbors
 - Reduce parallel transport burden

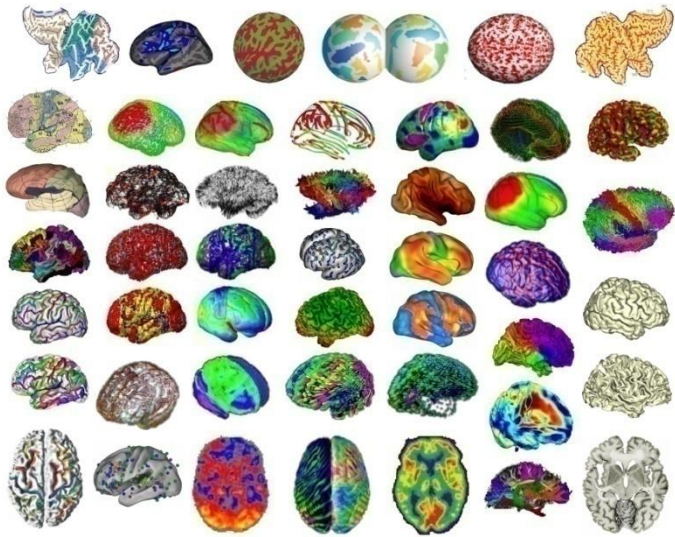


Automatic Parcellation

- 39 cortical surface meshes
- Spherical Demons vs. FreeSurfer
- Atlas-based parcellation into sulci/gyri
- Dice score, leave-n-out
- Statistical test
 - 24 better structures on 70
 - No structure is worse

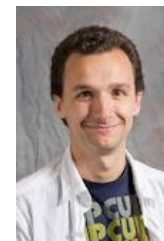


Percentage of improvement on Dice score



Need more than SSD?

USING OTHER SIMILARITY METRICS



NMI work with
Marc Modat

DTI work with
B.T. Thomas Yeo

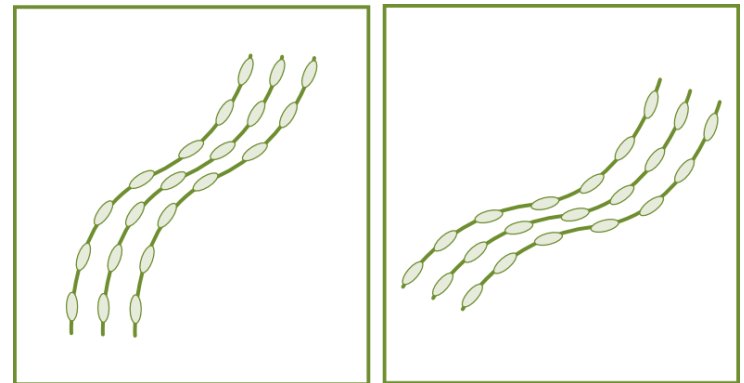
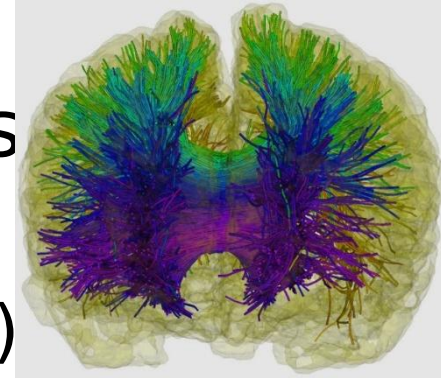


Demons Similarity Metrics

- Previous slides relied on SSD and focused on optimization strategy or transformation spaces
- Optimization strategy also dependent on the similarity metric
 - Can we extend the demons framework to something else than SSD?
 - Yes, see e.g. the work by Cathier, Guimond or Stefanescu
 - In this presentation, focus on DTI similarity
 - Not presented in the previous talks
 - Highlights the interdependence between similarity, transformation and optimization

DTI vs. Scalar Registration

- DTI captures white matter fibers information
 - Pixel: 3×3 positive matrix (tensors)
- Tensors intrinsically linked to geometry
 - Warping changes the geometry
 - Tensor reorientation required
- Similarity metric
 - Influenced by reorientation



DTI Similarity Criterion

- Local rotation component (Finite Strain)
 - Local linear approximation: $s(p) \approx \text{Jac}(p) \cdot p$
 - Closest rotation:
$$R(p) = (\text{Jac}(p) \text{Jac}(p)^T)^{-1/2} \text{Jac}(p)$$
- Similarity using full tensor
 - $\text{Sim}(F, M, s) = \sum \underline{\text{dist}}(F(p), R(p)^T \underline{M(s(p))} R(p))^2$
 - Log-Euclidean framework used

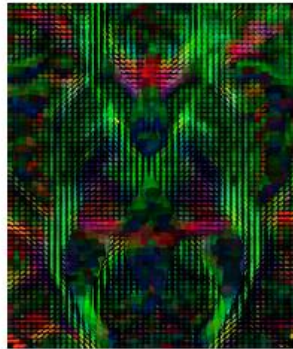
$$\text{Sim} = \|\log(\text{img}_1) - \text{rot}(\log(\text{img}_2))\|_{\log}^2$$

DTI Demons Forces

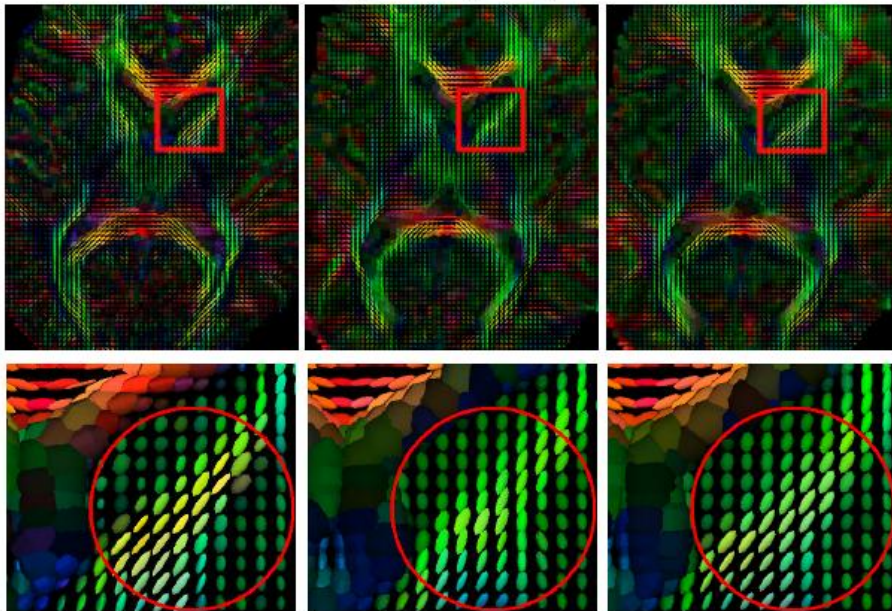
- Yeo et al., ISBI 08
- Requires derivative of the similarity
 - FS uses matrix inverse square-root
 - Complex closed-form
 - Finite difference: poor man's solution
- Happily we are in 3D!
 - Use 3D cross-product
 - $dR = -R [R^T (\text{tr}((JJ^T)^{1/2})I - (JJ^T)^{1/2})^{-1} \sum (R^T)_i \otimes (dJ^T)_i]^\oplus$



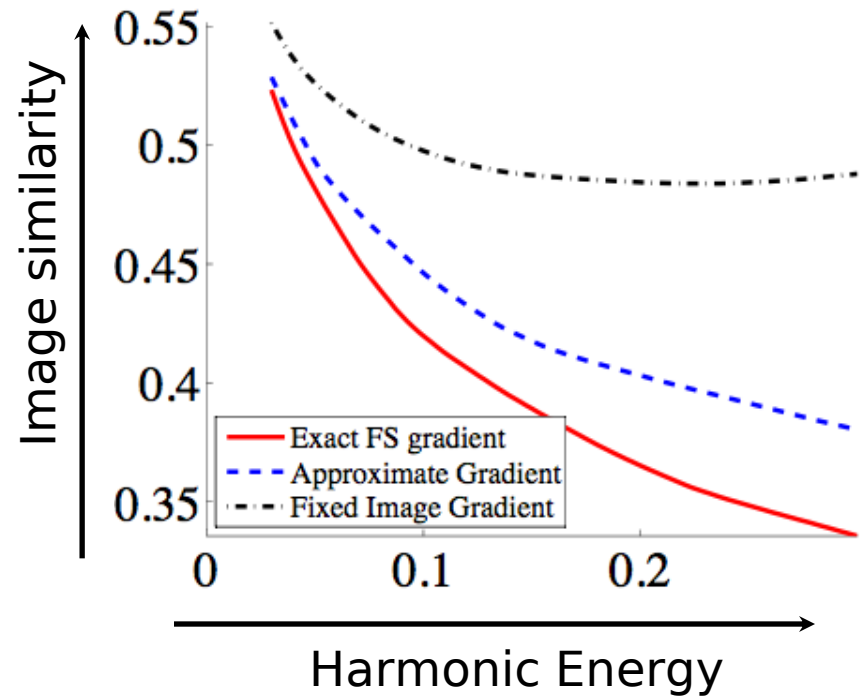
DTI Results



(a) Moving Image

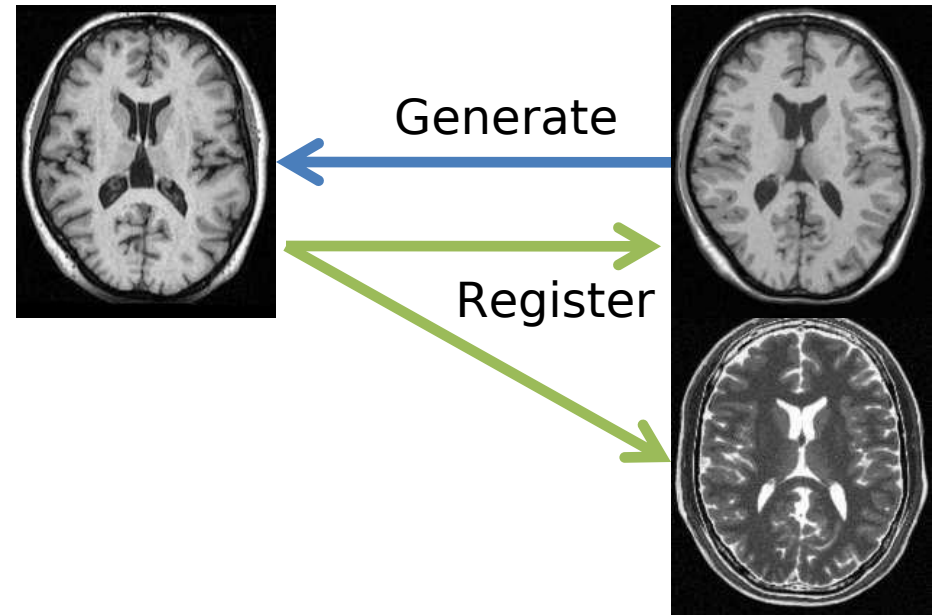


(b) Fixed image and zoom (c) Registered image (approx. grad.) (d) Registered image (exact grad.)



Sneak Peak at NMI Demons

- Collaboration with UCL
 - Marc Modat
- Uses mutual information derivative
 - Unlike SSD-based metrics:
 - No 2nd order info
 - Conjugate gradient approach to control step size
- NMI demons
 - Better NMI than SSD demons
 - Reverse holds



Multimodal Atlas

| | MSE-demons (T1w-T1w) | NMI-demons (T1w-T1w) | NMI-demons (T1w-T2w) |
|---------------------------|-------------------------|-------------------------|-------------------------|
| Mean absolute error (std) | 0.4677 (0.0235) | 0.1025 (0.0318) | 0.5473 (0.0255) |

Table 2. Mean absolute error between the ground truth and the recovered deformation fields



Ensuring diffeomorphism is not enough? You also need the inverse transformation?

LOG-DOMAIN DEMONS

Back to Spatial Constraints

- From previous slides: Demons is indeed a framework
 - As such there is room for improvements and adaptations
- As an example, we present a symmetric extension of the demons
 - Symmetry is sound for subject-subject registration
- Other extensions not presented in this tutorial
 - Subject-template registration (with M. Sabuncu)
 - Symmetry not advocated by probabilistic models
 - Spatio-temporal constraints
 - Cyclicity by J.M. Peyrat
 - Incompressibility of heart muscle by T. Mansi

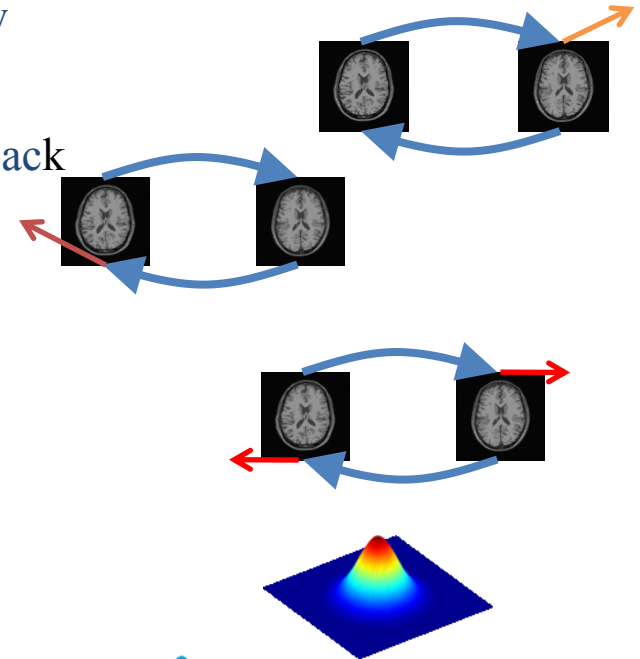
Log-Domain Demons

- Vercauteren et al., MICCAI 08
- Log-domain walking
 - Replace $s \leftarrow s \circ \exp(\mathbf{u})$ by
 - $\exp(\mathbf{v}) \leftarrow \exp(\mathbf{v}) \circ \exp(\mathbf{u})$
 - But this may not exist...
- Approximation with BCH formula
 - $\exp(\mathbf{v}) \circ \exp(\epsilon \mathbf{u}) =$
 $\exp(\mathbf{v} + \epsilon \mathbf{u} + [\mathbf{v}, \epsilon \mathbf{u}]/2 + [\mathbf{v}, [\mathbf{v}, \epsilon \mathbf{u}]]/12 + \dots)$
 - Lie bracket
 $[\mathbf{v}, \mathbf{u}](\mathbf{p}) = \text{Jac}(\mathbf{v})(\mathbf{p}) \cdot \mathbf{u}(\mathbf{p}) - \text{Jac}(\mathbf{u})(\mathbf{p}) \cdot \mathbf{v}(\mathbf{p})$



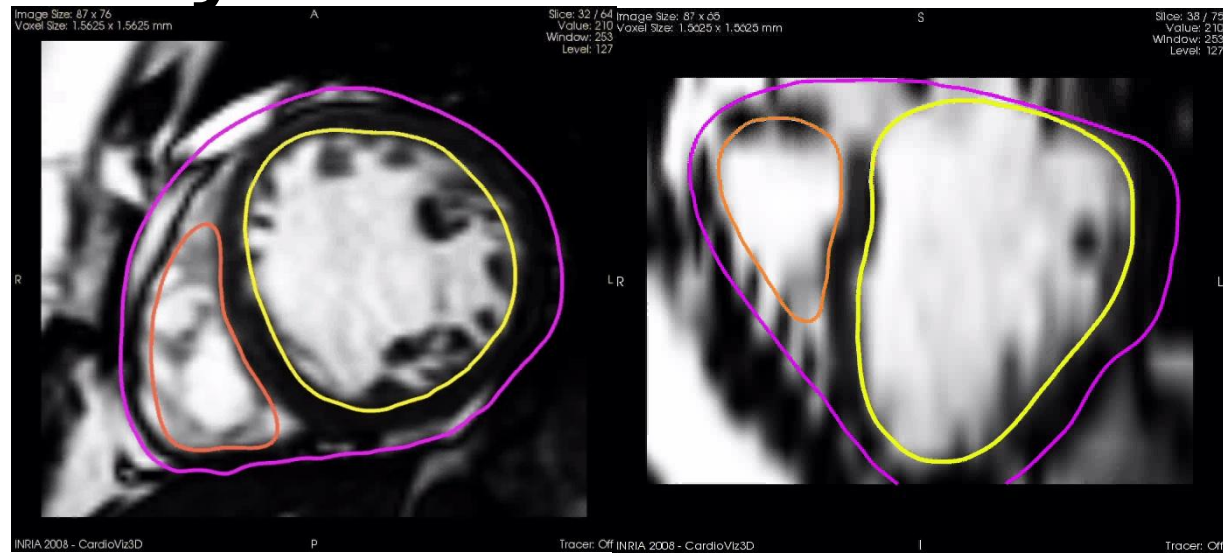
Symmetric Log-Domain Demons

- Use easy inverse: $s^{-1} = \exp(-\mathbf{v})$
- Iteration
 - Images I_0, I_1 and transformation $s = \exp(\mathbf{v})$ given
 - Forward demons forces \mathbf{u}^{forw}
 - Backward demons forces \mathbf{u}^{back}
 - Update
 - $\mathbf{v} \leftarrow \frac{1}{2} (Z(\mathbf{v}, \mathbf{u}^{\text{forw}}) - Z(-\mathbf{v}, \mathbf{u}^{\text{back}}))$
 - Regularize (Gaussian)
 - $\mathbf{v} \leftarrow K_{\text{diff}} * \mathbf{v}$



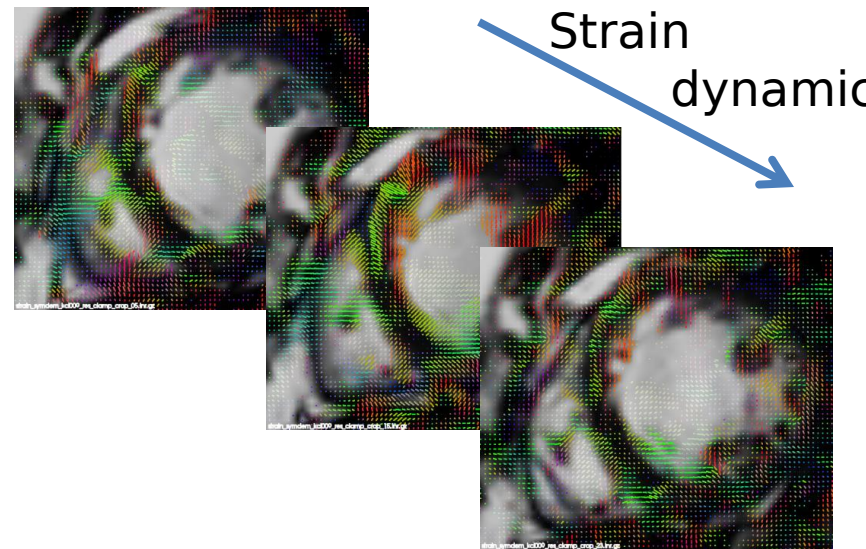
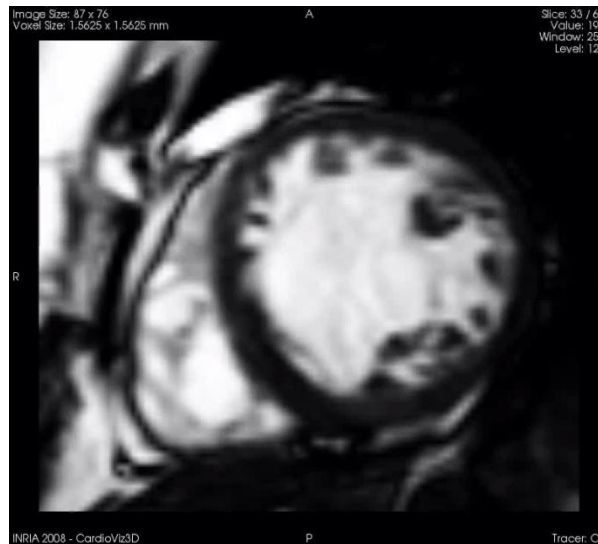
Case Study: 3D+t Heart

- Registration with initial time
 - Forward: Propagation
 - Backward: Strain, Stabilization



Courtesy Div. Imag. Sc., King's College London

Thanks: Tommaso Mansi, Nicolas Toussaint (INRIA)





Dissemination and evaluation is not just a pain

REACHING OUT TO THE WORLD

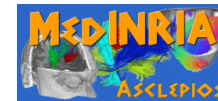


Evaluation with
Arno Klein



Reproducible Research

- Standard in core biomedical fields
- What it means for us?
 - Availability of algorithms
 - Availability of datasets
 - Common rules to compare the results
- Towards open-source
 - Diffeomorphic demons
 - DTI demons
 - Spherical demons
 - Symmetric demons (with F. Dru)

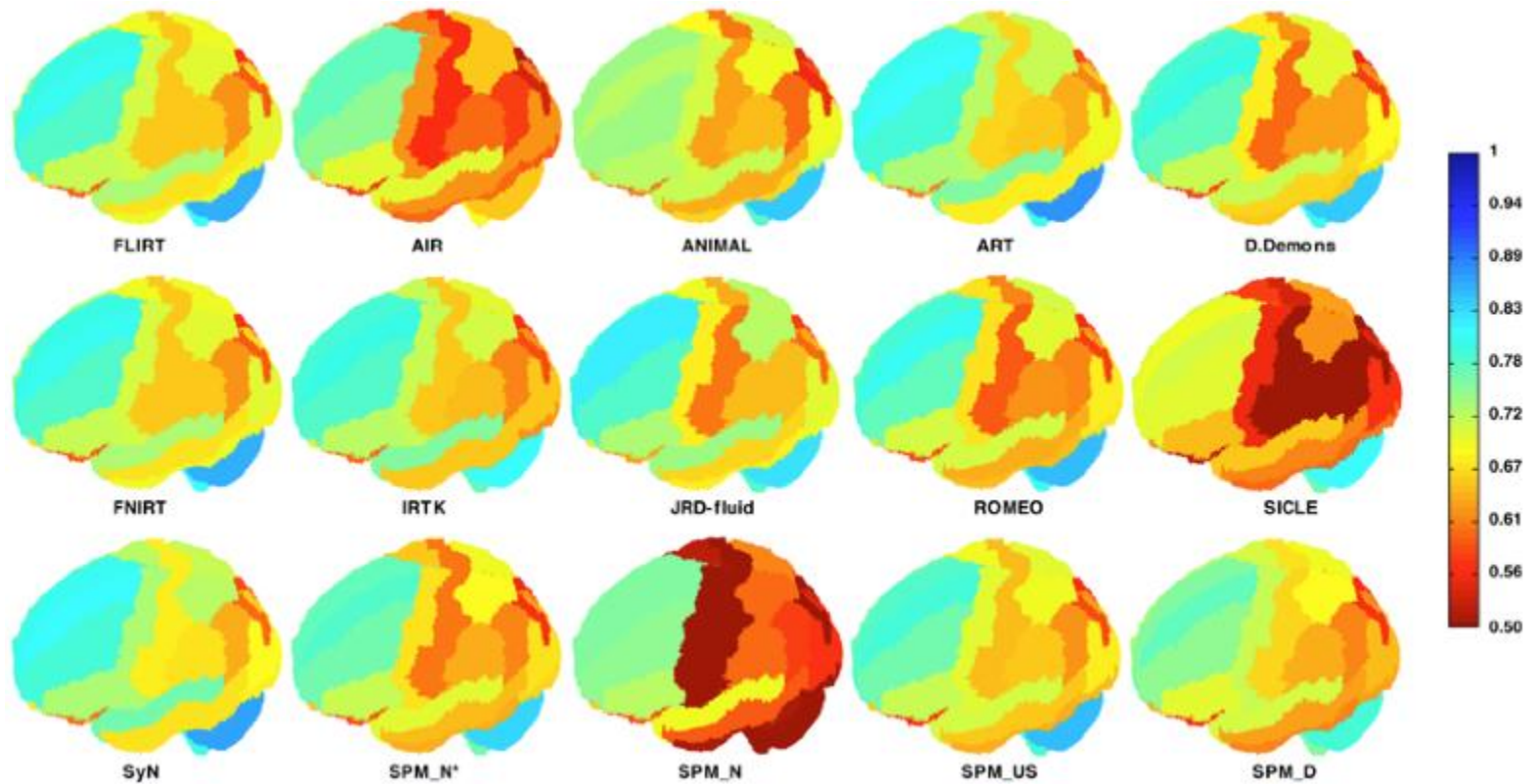


Large Scale Evaluation

- Klein et al., NeuroImage 09
 - 16 groups involved: MKT, INRIA, LONI, Imperial College, UPenn, Ulowa, FMRIB, Wellcome Trust,...
 - 14 registration softwares
 - 80 manually segmented brains
 - Over 45,000 pairwise registrations performed
 - 8 different comparison measures: Dice
 - 3 independent statistical tests
- See also EMPIRE 10 challenge (posters during breaks)



Target Overlap Averaged Across All Registrations



Average Rank

| Algorithm | mean rank | dof | run time: minutes | year |
|---------------------------|-----------|------|----------------------|------|
| SyN | 1.00 | 28M | 77 (15.1) | 2008 |
| ART | 1.00 | 7M | 20.1 (1.6) [Linux] | 2005 |
| IRTK | 1.63 | 1.4M | 120.8 (29.3) | 1999 |
| SPM5 DARTEL Toolbox | 1.88 | 6.4M | 71.8 (6.3) | 2007 |
| JRD-fluid | 2.50 | 2M | 17.1 (1.0) [Solaris] | 2007 |
| Diffeomorphic Demons | 3.00 | 21M | 8.7 (1.2) | 2007 |
| FNIRT | 3.00 | 30K | 29.1 (6.0) | 2008 |
| ROMEO | 3.50 | 2M | 7.5 (0.5) | 2001 |
| ANIMAL | | 69K | 11.2 (0.4) | 1994 |
| SICLE | | 8K | 33.5 (6.6) | 1999 |
| SPM5 Unified Segmentation | | 1K | $\simeq 1$ | 2005 |
| “SPM2-type” Normalize | | 1K | $\simeq 1$ | 1999 |
| SPM5 Normalize | | 1K | $\simeq 1$ | 1999 |
| AIR | | 168 | 6.7 (1.5) | 1998 |

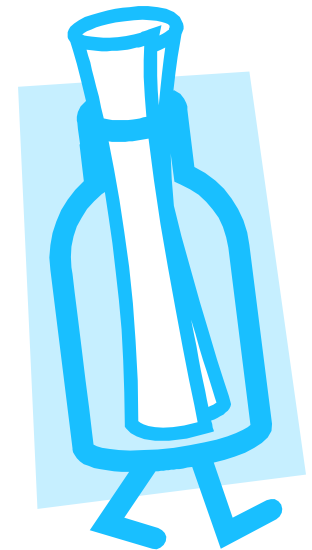
The diagram shows a musical staff in 4/4 time with a treble clef. The notation includes a repeat sign (§) at the beginning, followed by a series of eighth notes, a dotted quarter note, and a half note. A *D.S. al Coda* instruction is placed below the staff, with a diamond symbol (◊) marking the start of the second ending. The second ending consists of a quarter note, a half note, and a whole note. A *Coda* symbol is placed above the staff at the end of the second ending. Annotations include:

- A solid line with arrows: "play to *D.S. al Coda*" from the start to the first diamond symbol.
- A dashed line with arrows: "go back to §" from the first diamond symbol back to the repeat sign.
- A solid line with arrows: "play to ◊" from the start to the second diamond symbol.
- A dashed line with arrows: "skip" from the second diamond symbol to the end of the staff.
- A solid line with arrows: "play coda" from the end of the staff to the right.

CONCLUDING REMARKS AND DISCUSSION

Take Home Messages

- For good image registration performance
 - Study application constraints
 - Pick most suitable tools w.r.t constraints
- Demons algorithms
 - Cover a large variety of constraints
 - Decent accuracy
 - Relatively easy to use
 - Open source implementations
 - Few parameters
 - Computationally efficient
 - Can be thought of as a fine baseline
- Evaluation is more than important
- There is room for research!



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