Technischen Universität München
Winter Semester 2012/2013

TRACKING and DETECTION in COMPUTER VISION

Mean Shift Tracking

Slobodan Ilić

Agenda

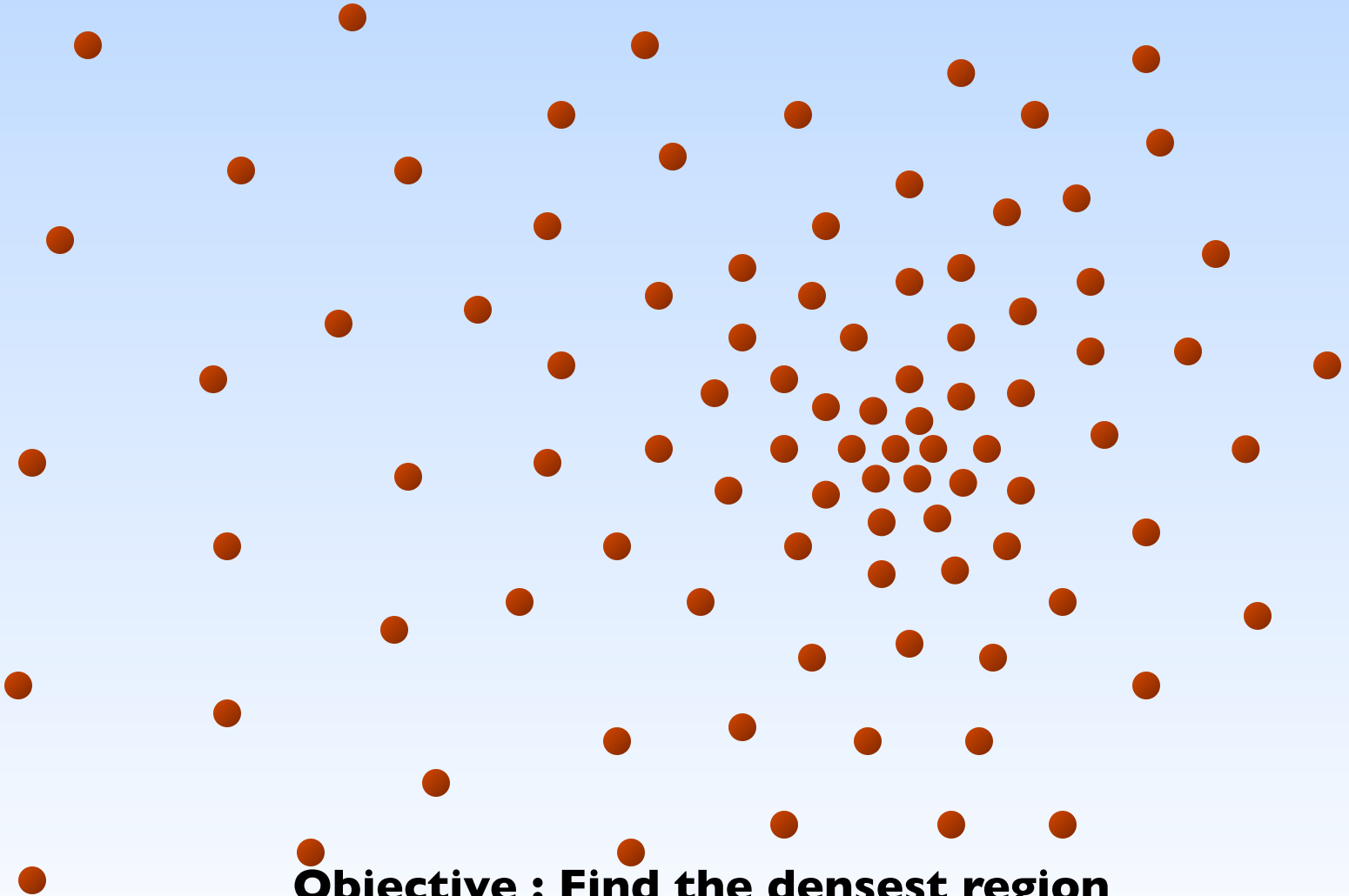
Mean Shift Theory

- What is Mean Shift ?
- Density Estimation Methods
- Deriving the Mean Shift
- Mean shift properties

Applications

- Clustering
- Object Tracking

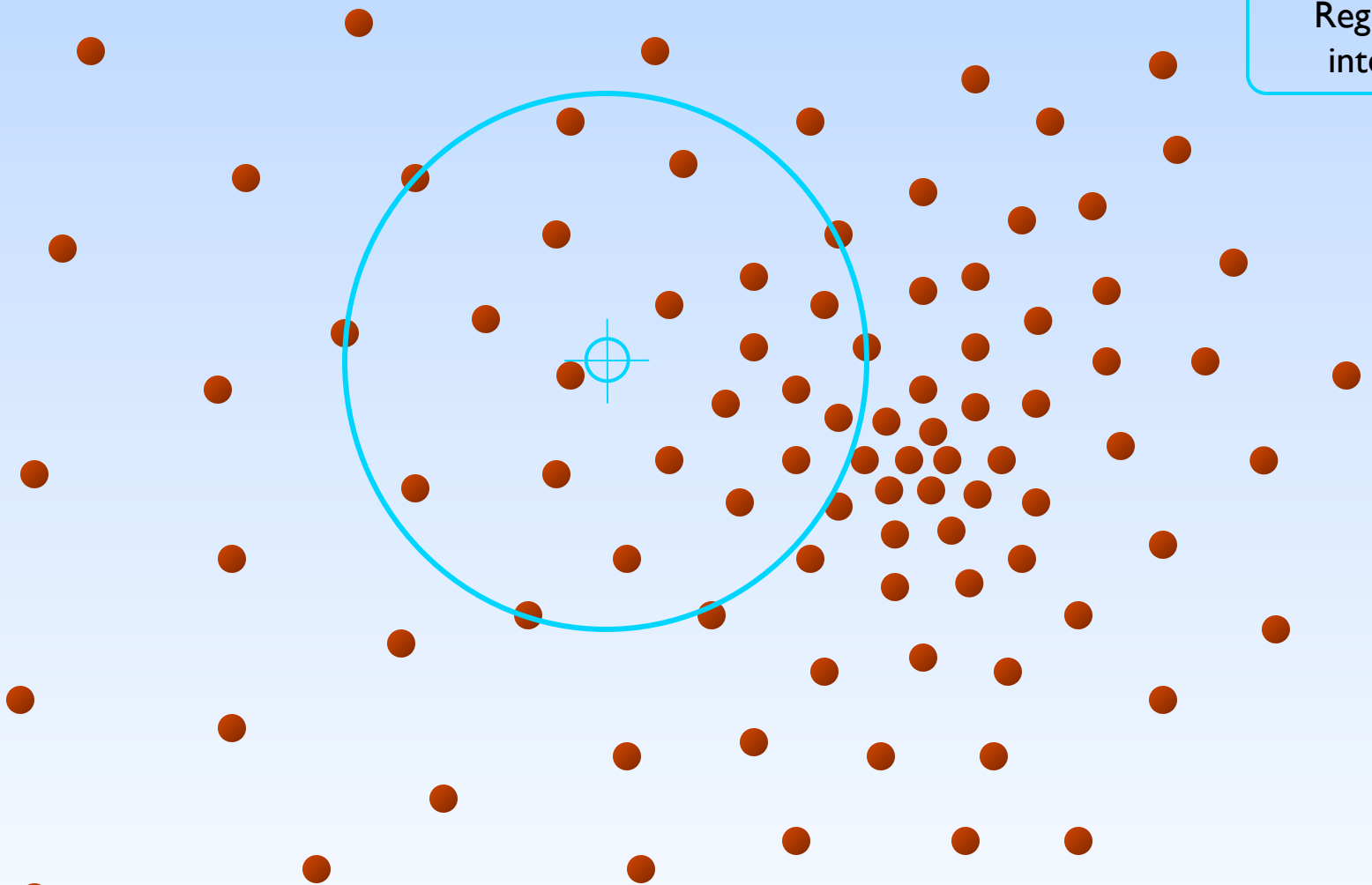
Intuitive Description



Objective : Find the densest region

Distribution of identical billiard balls

Intuitive Description



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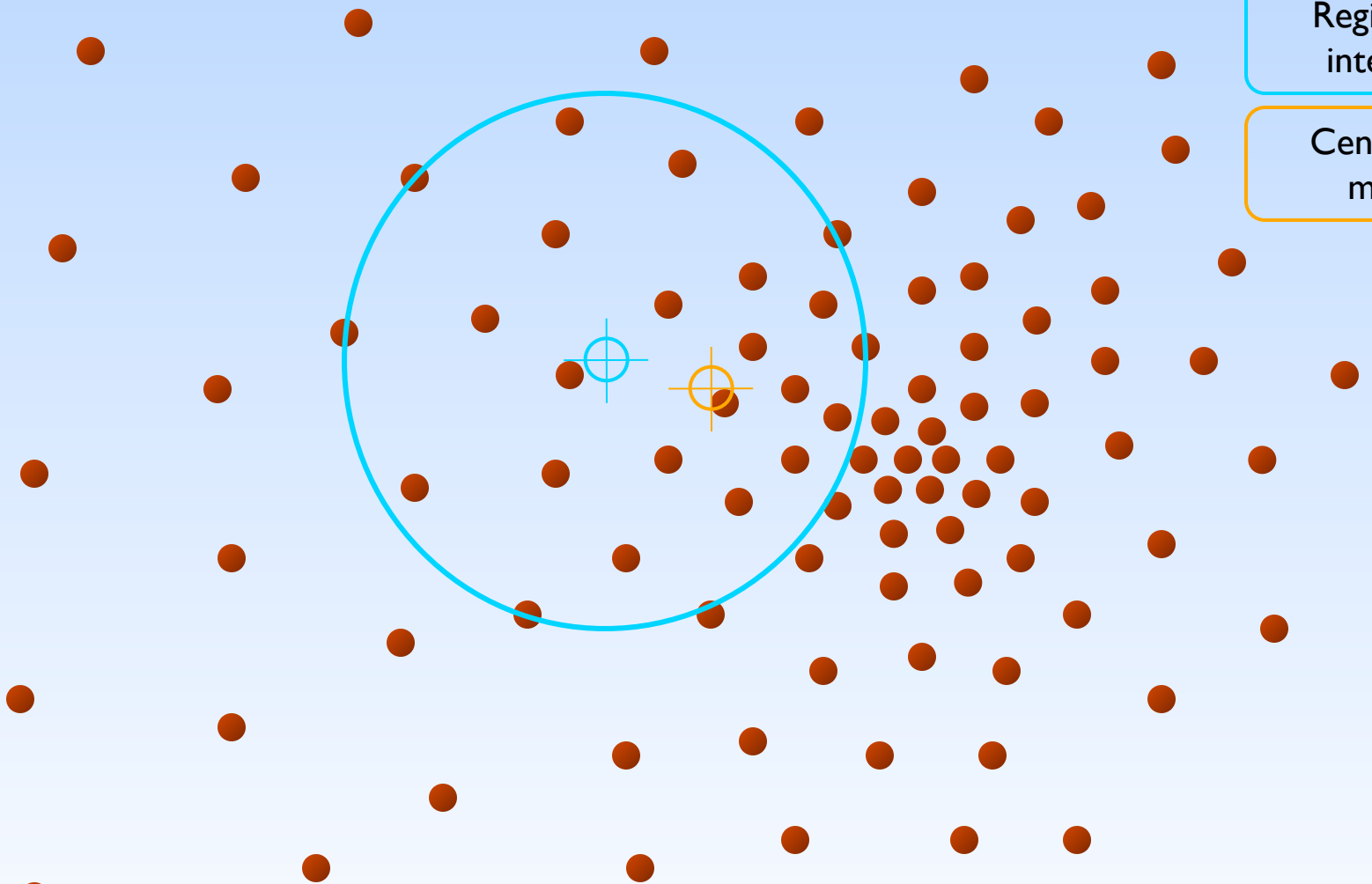

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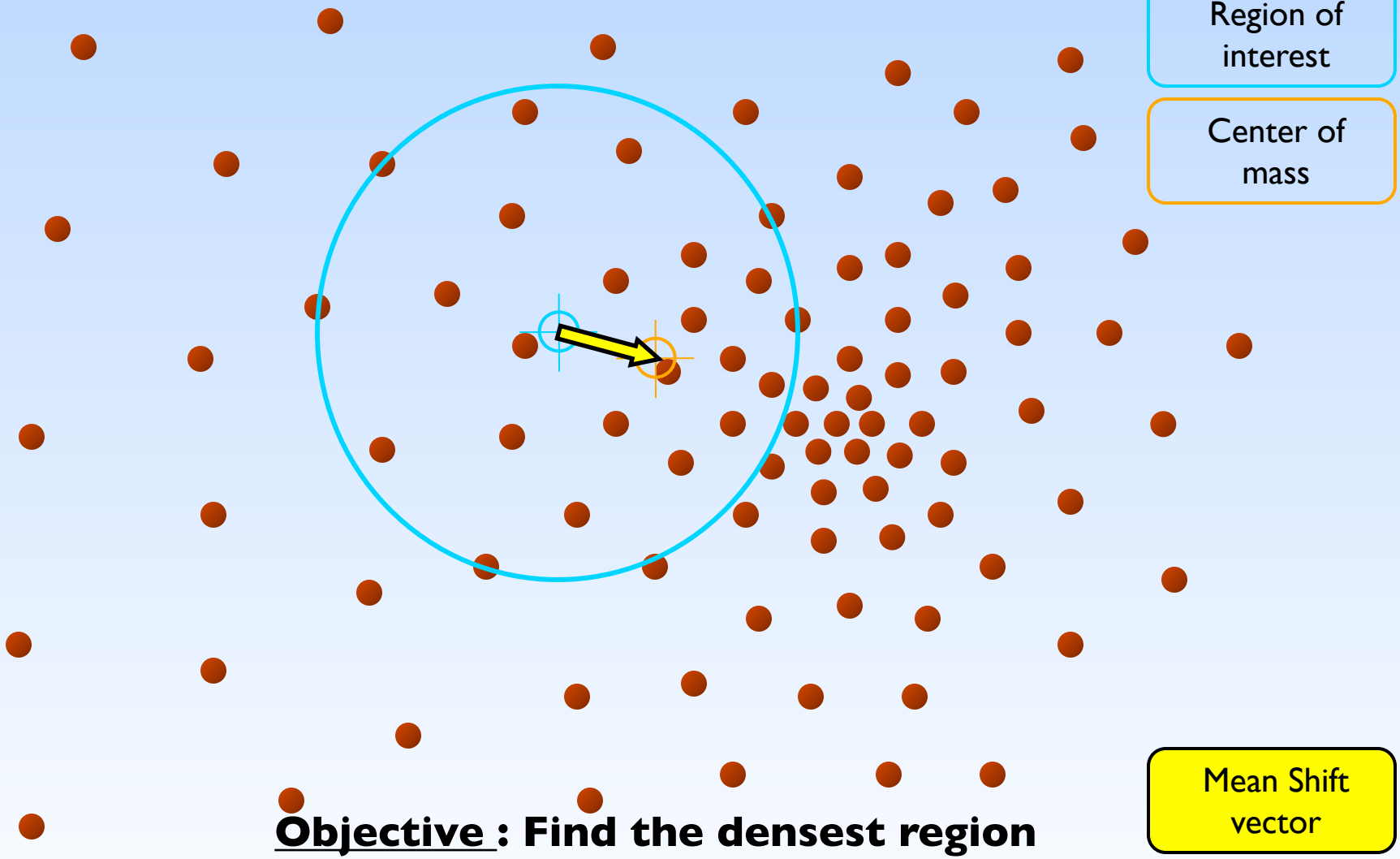

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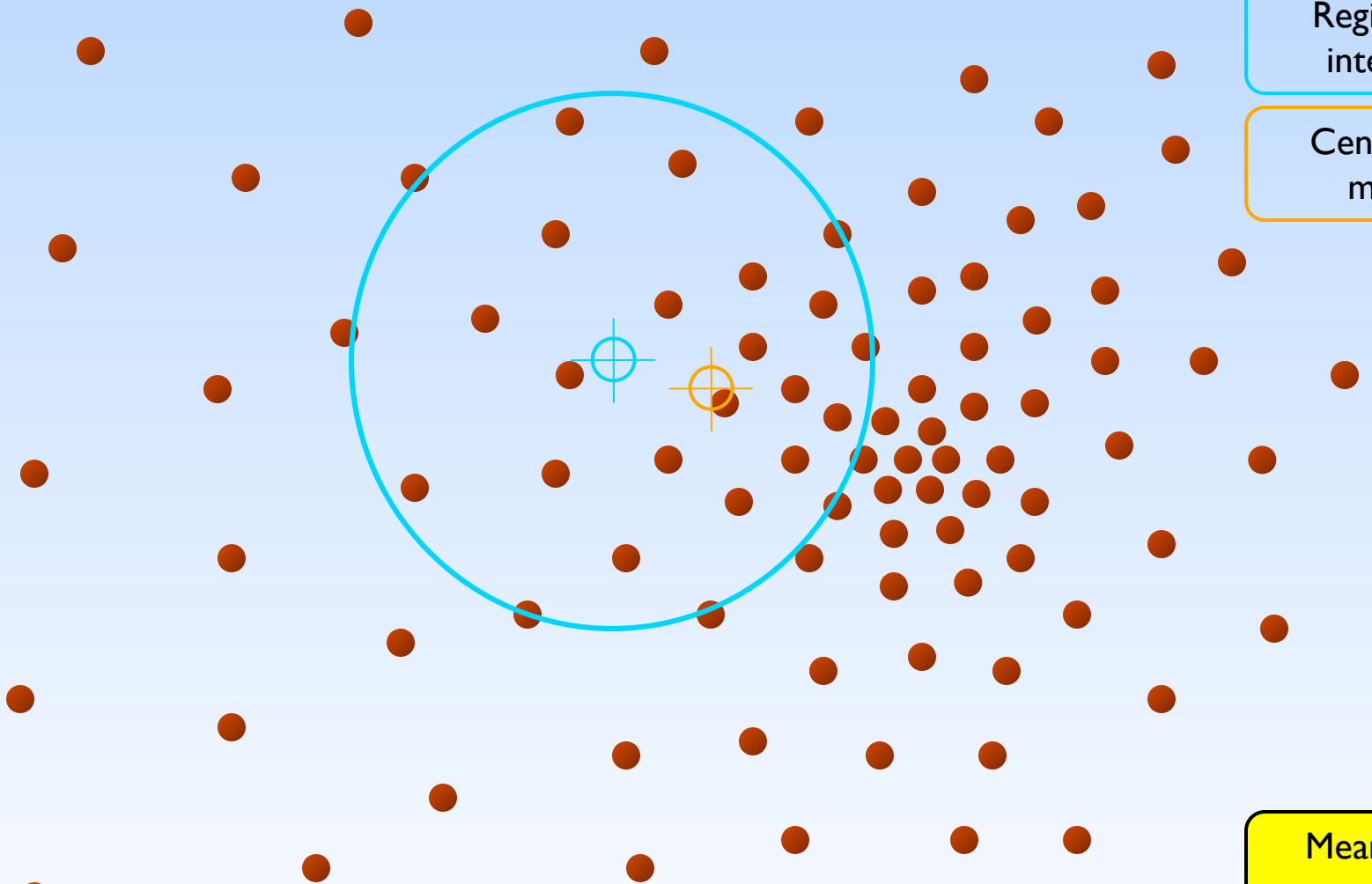

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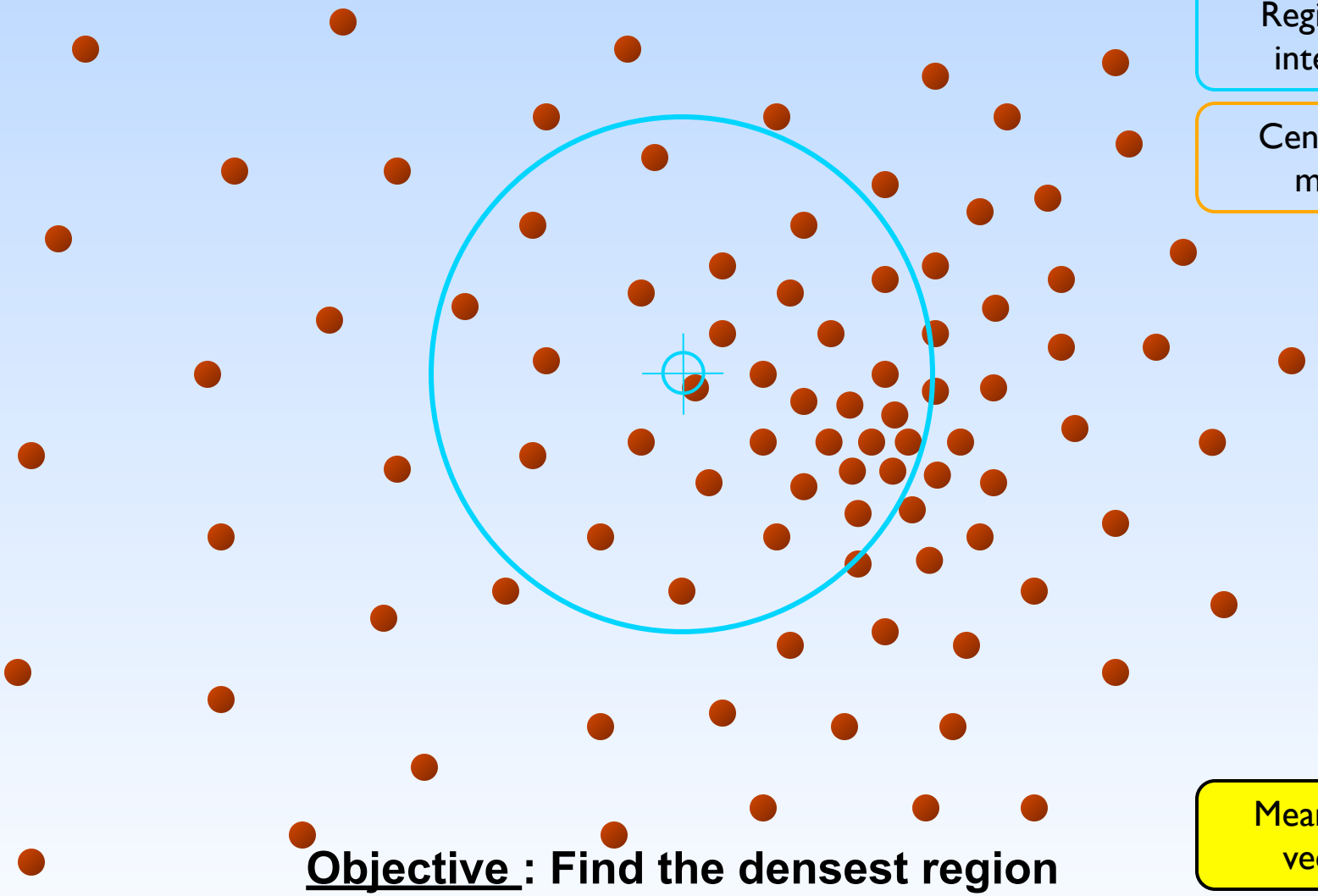

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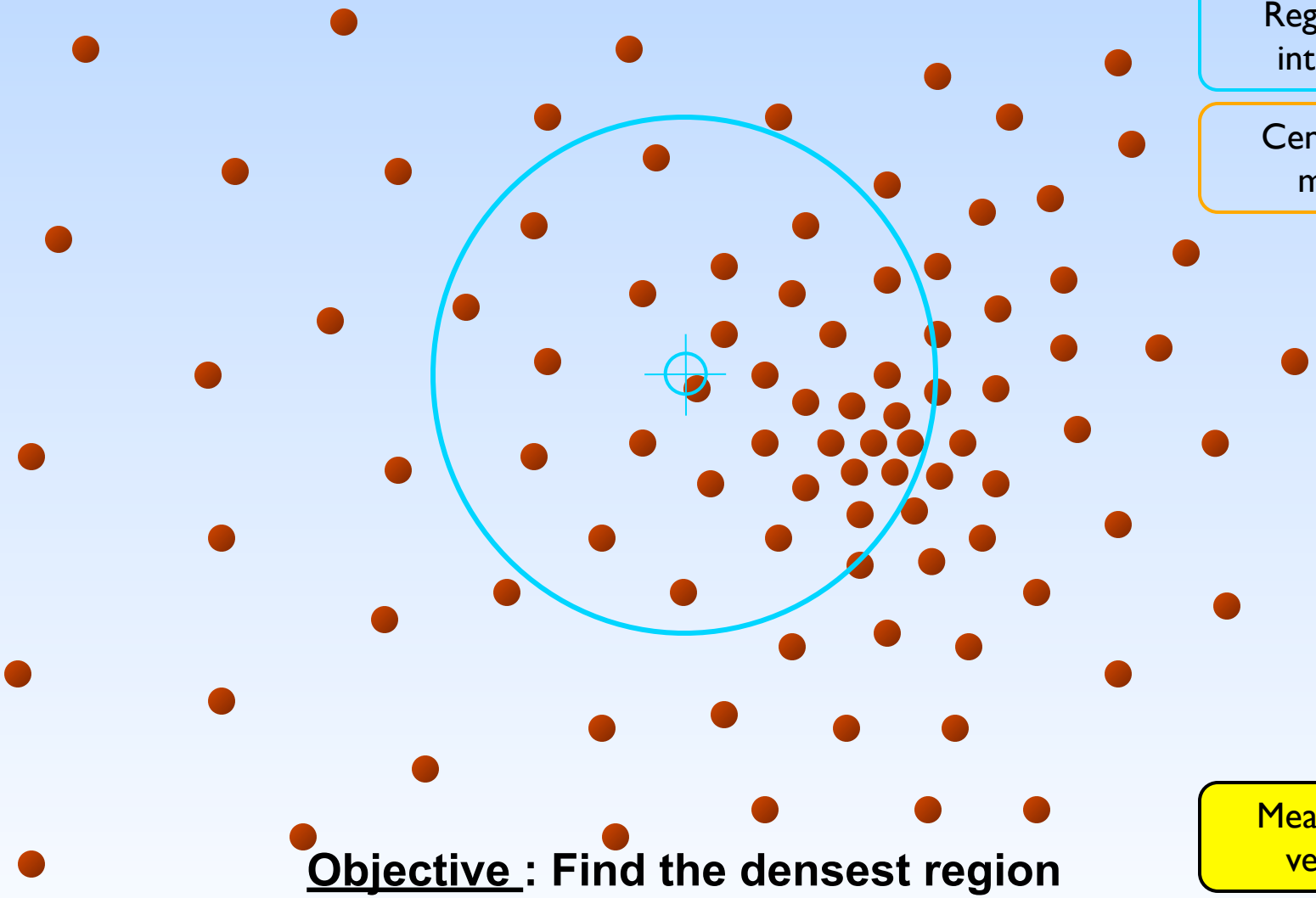

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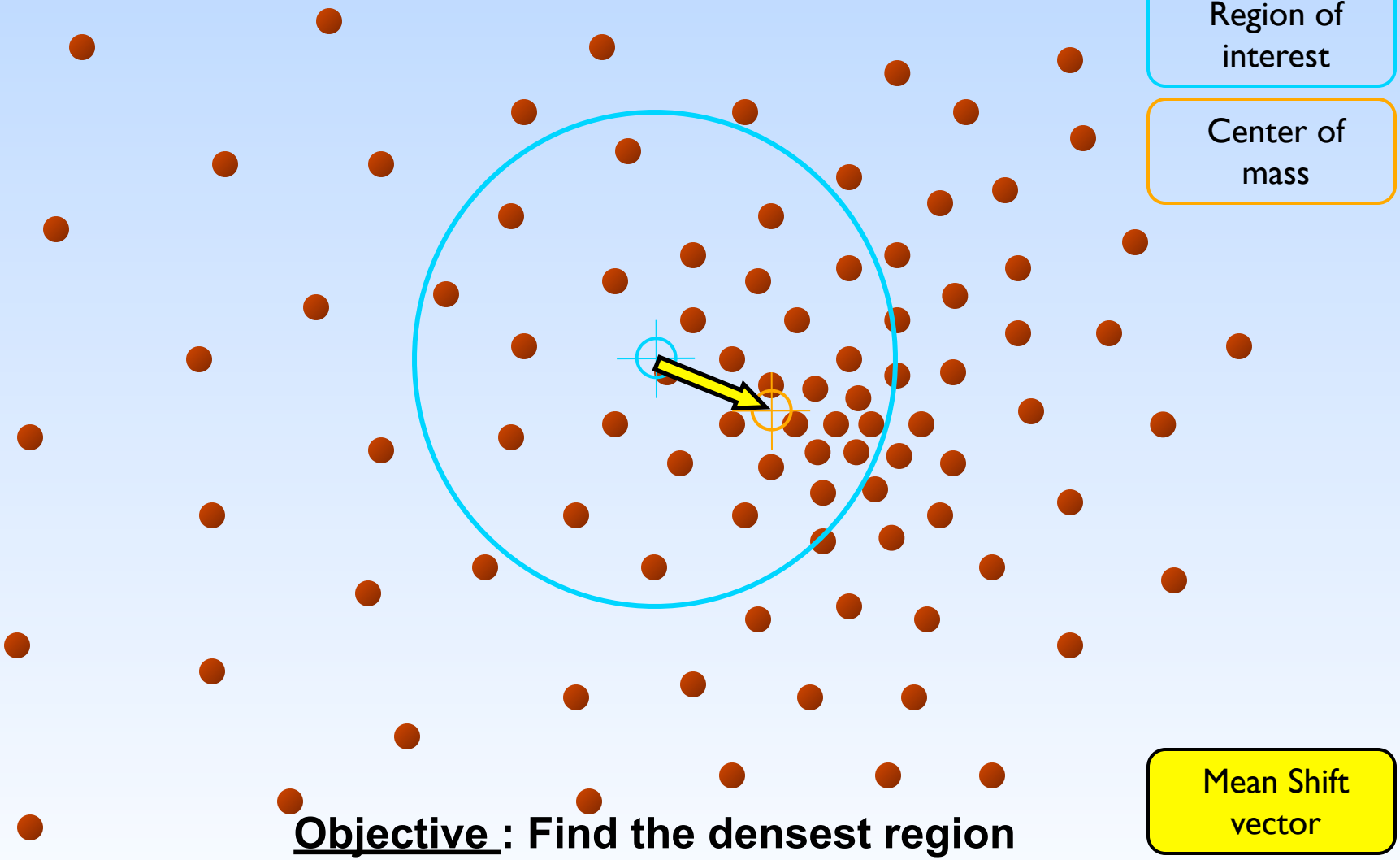

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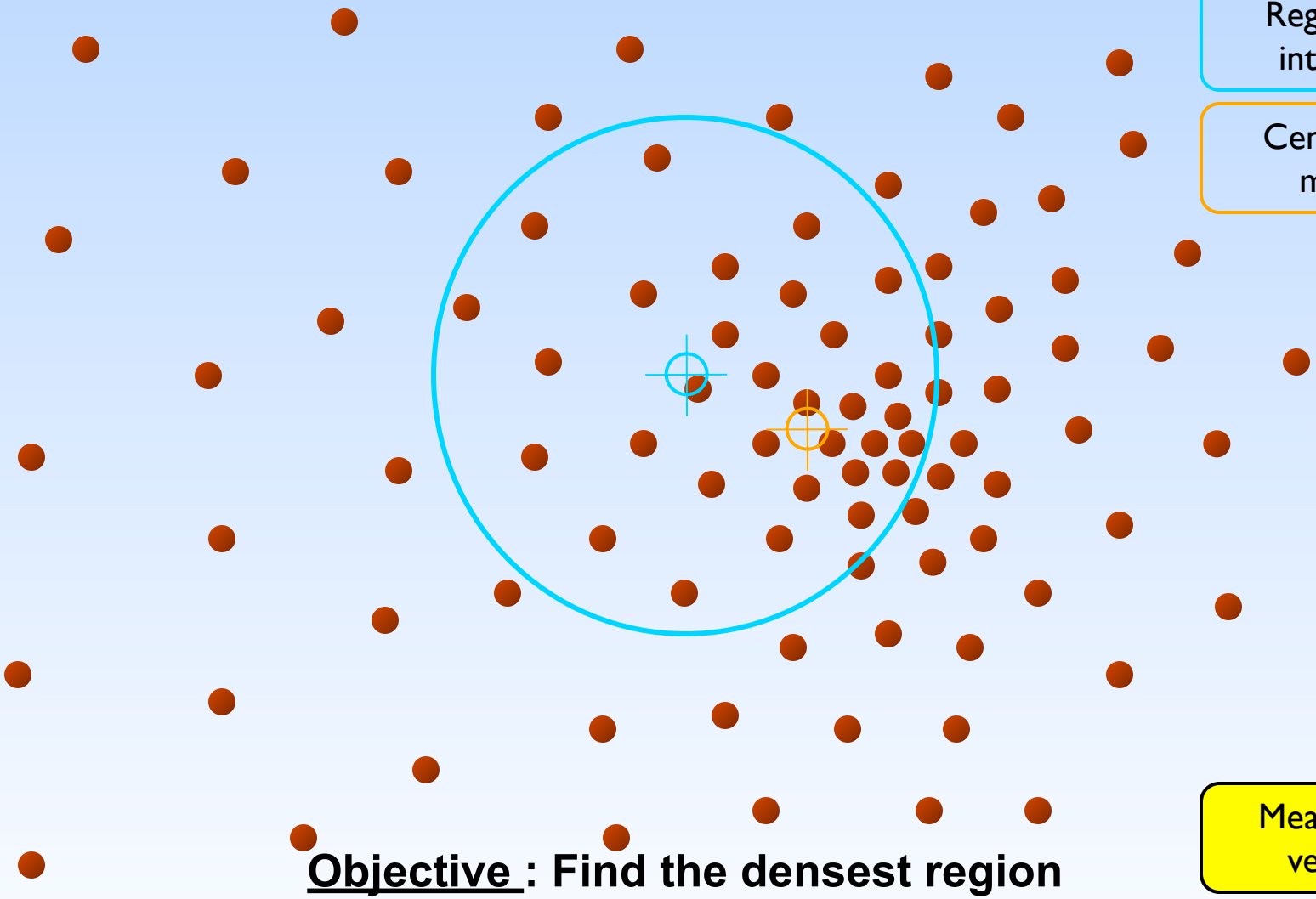

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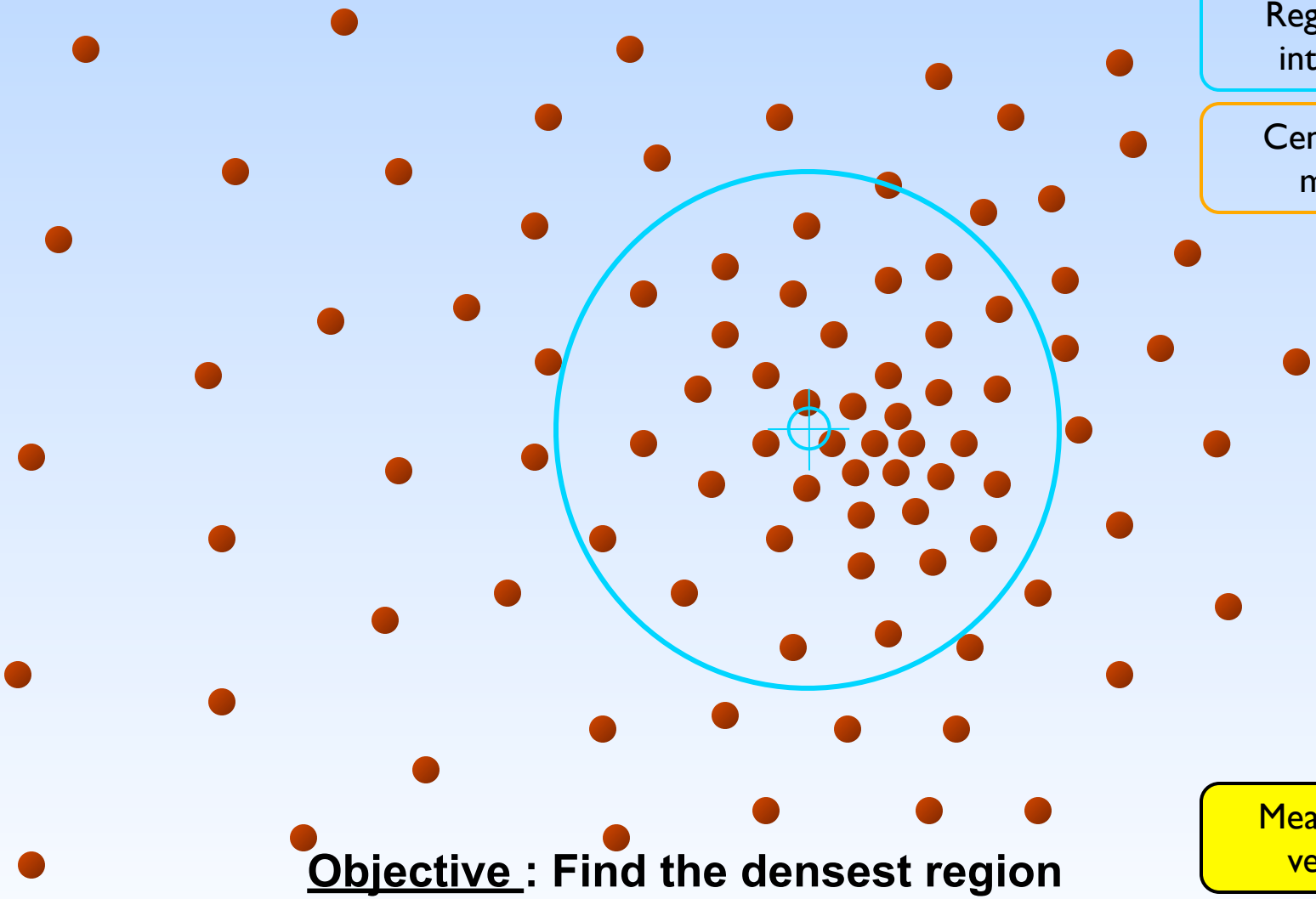

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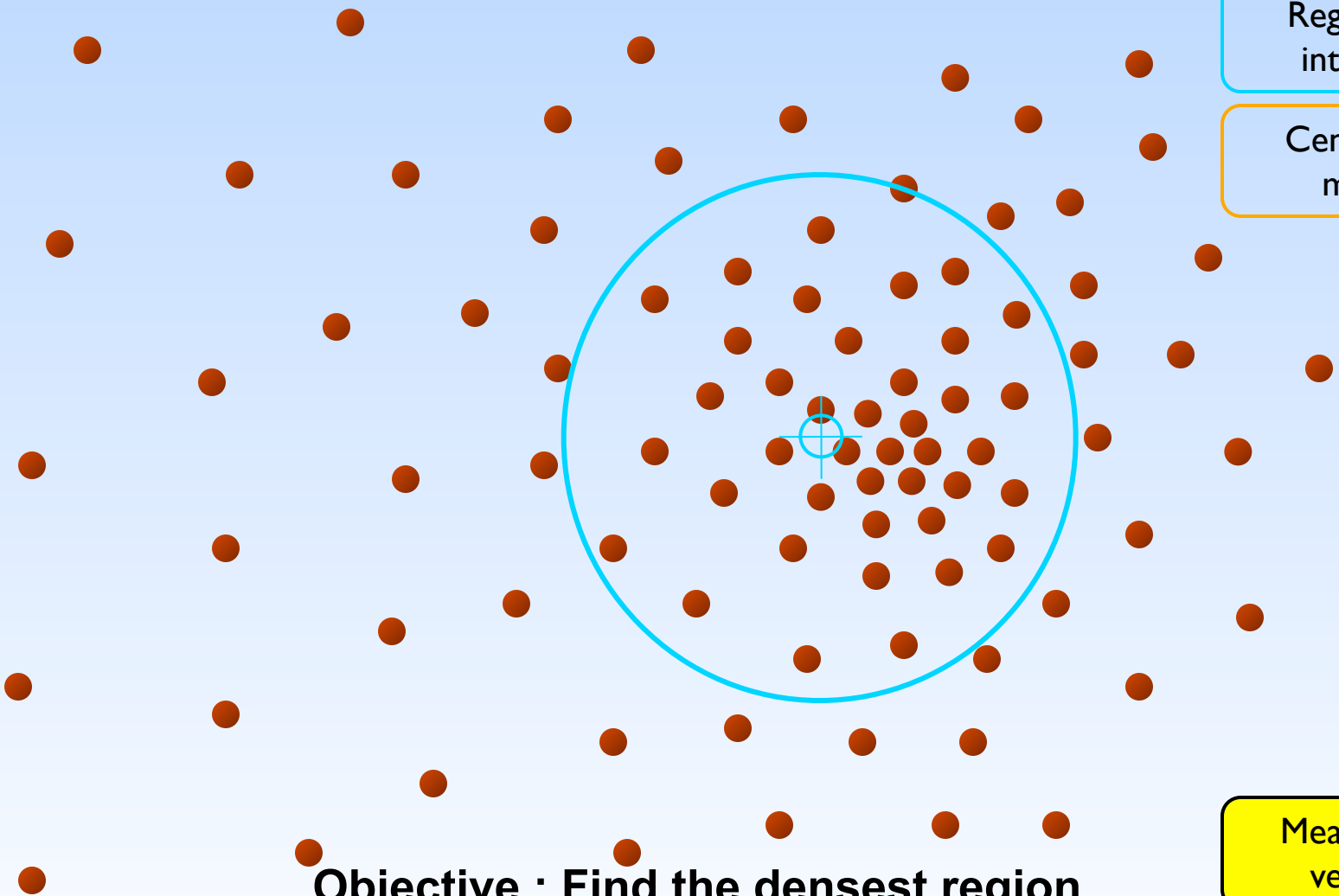

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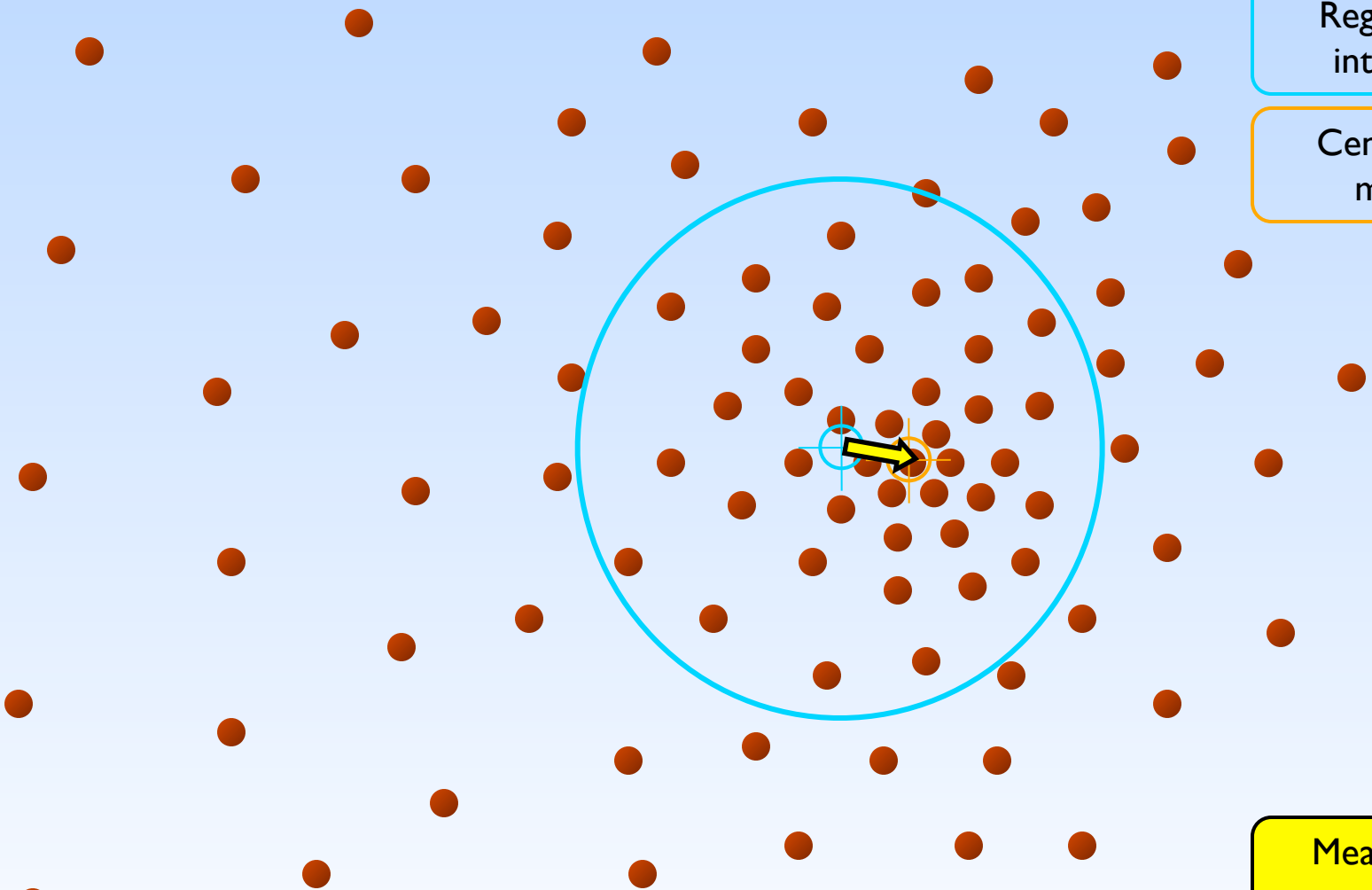

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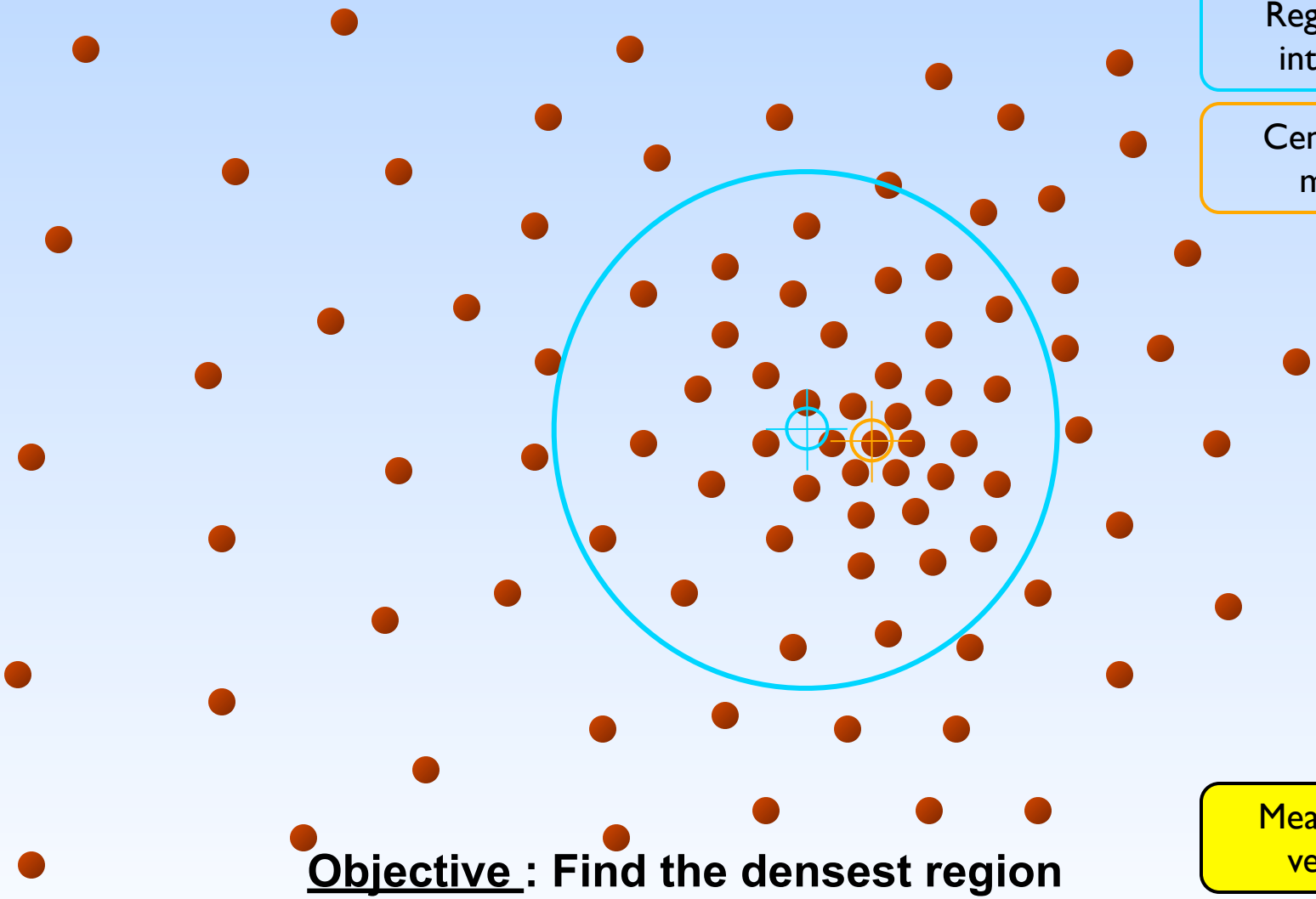

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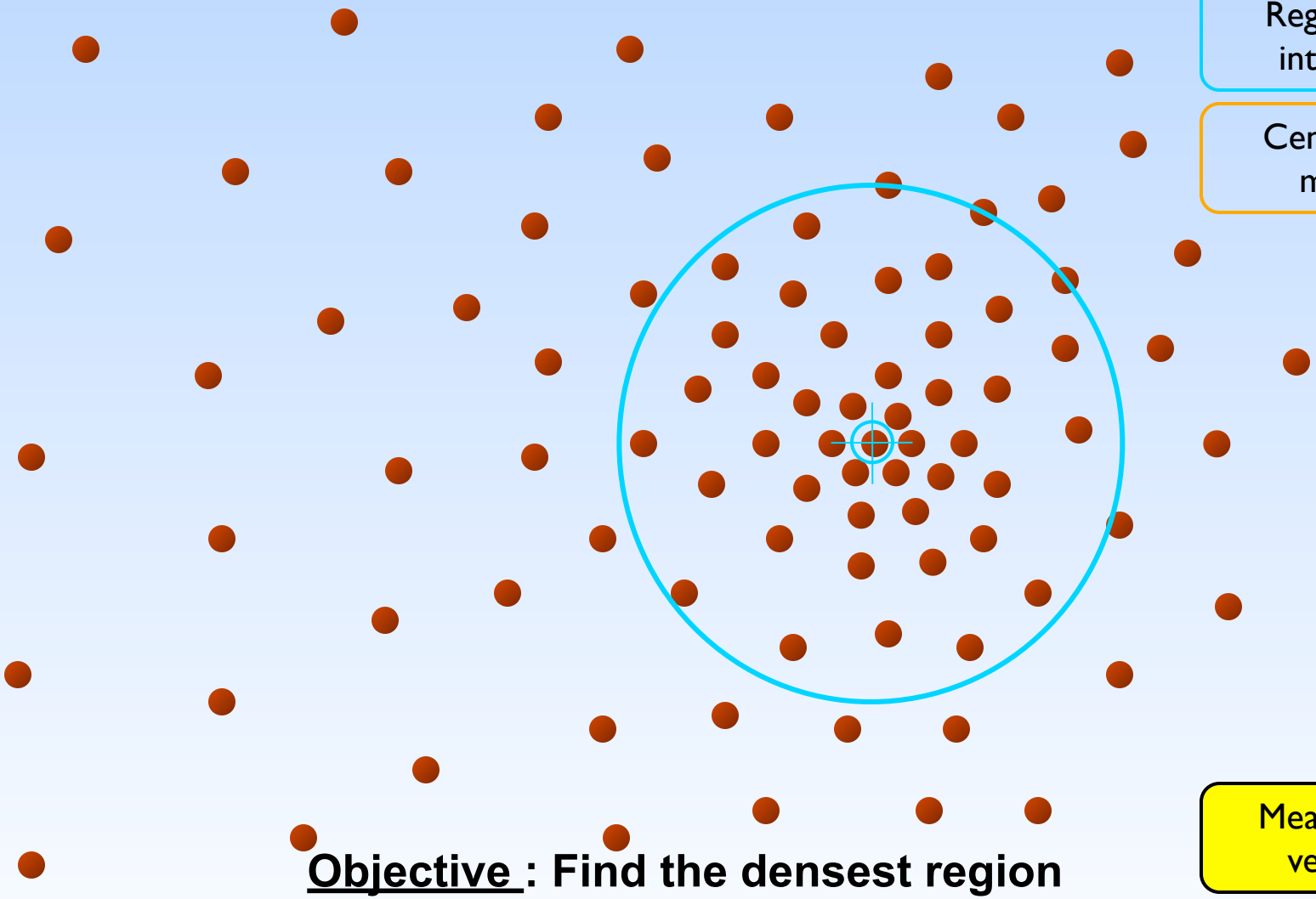

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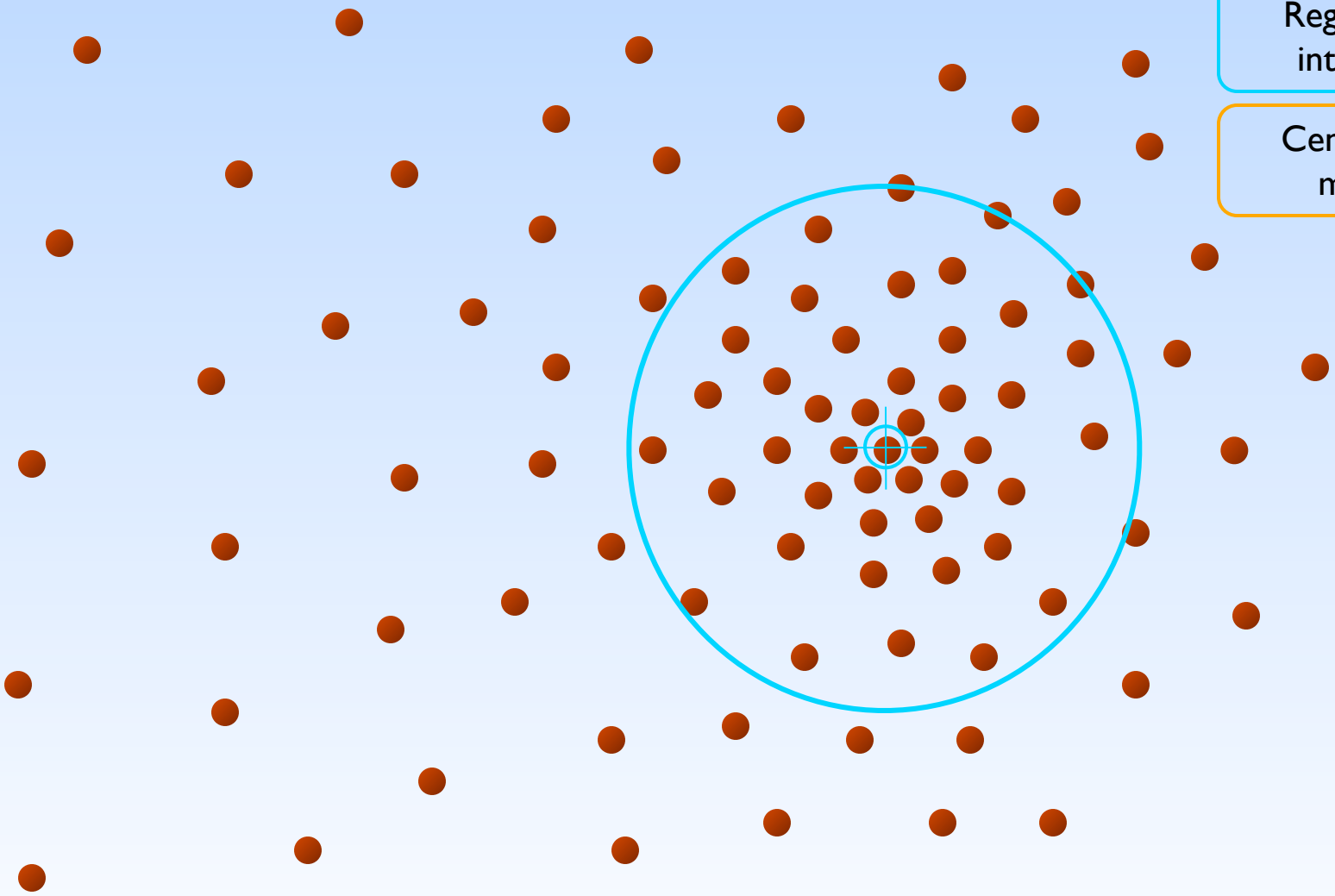

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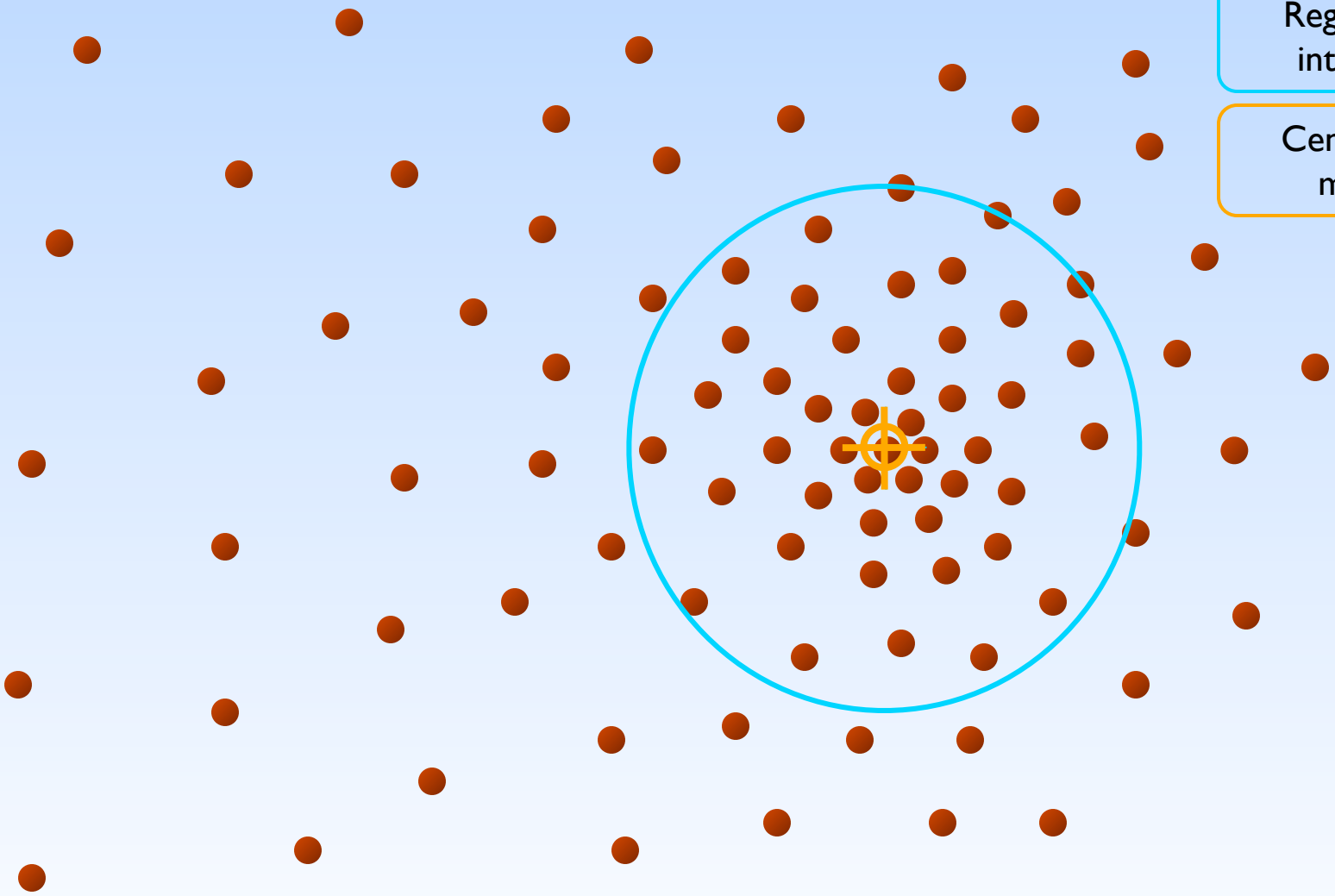

What is Mean Shift ?

A tool for:
Finding modes in a set of data samples representing an underlying probability density function (PDF) in R^N

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PDF in feature space

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PDF in feature space

- **Color space**

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PDF in feature space

- **Color space**
- **Scale space**

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PDF in feature space

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- **Actually any feature space you can conceive**

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PDF in feature space

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- **…**

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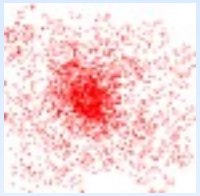

Data

What is Mean Shift ?

A tool for:
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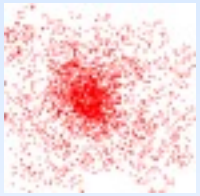
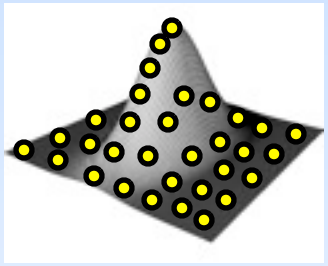


Data

Non-parametric Density Estimation



Discrete PDF Representation

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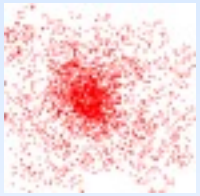
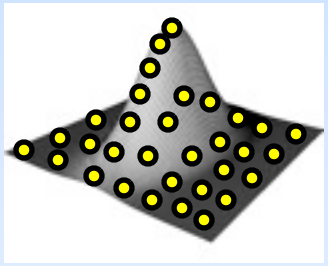


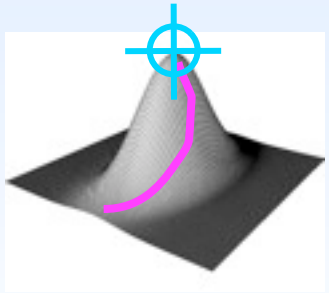
Data

Non-parametric
Density Estimation



Discrete PDF Representation

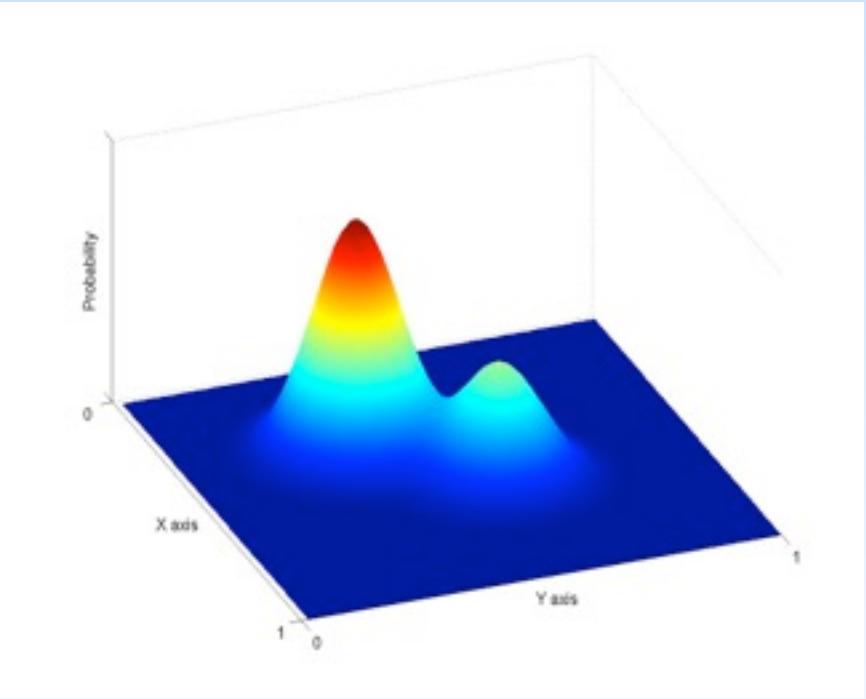
Non-parametric
Density **GRADIENT** Estimation
(Mean Shift)

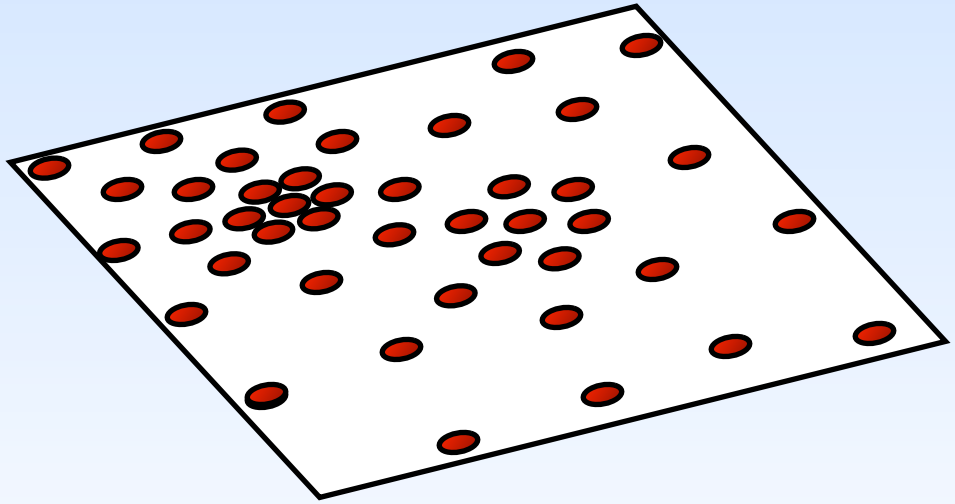


PDF Analysis

Non-Parametric Density Estimation

Assumption : The data points are sampled from an underlying PDF



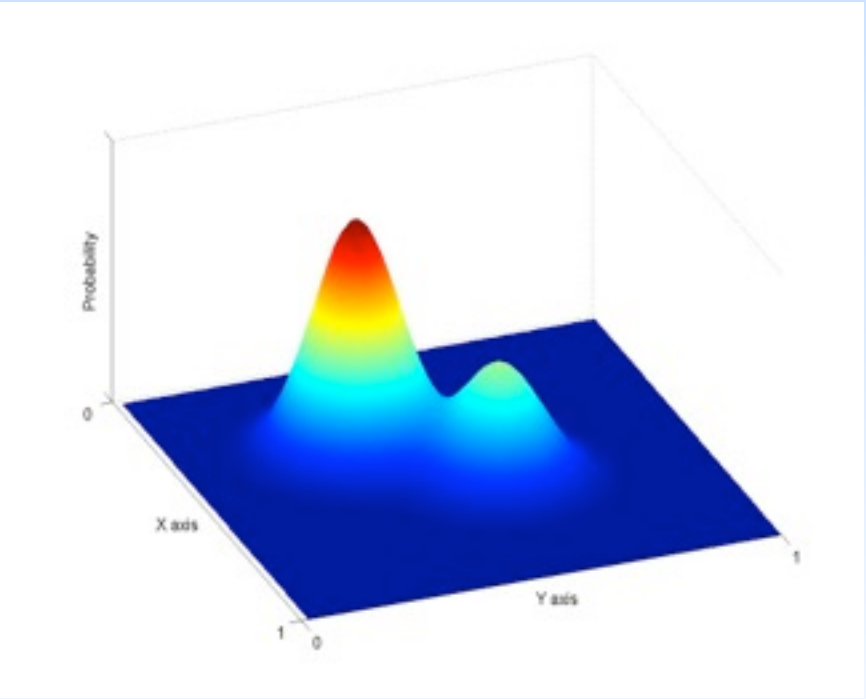


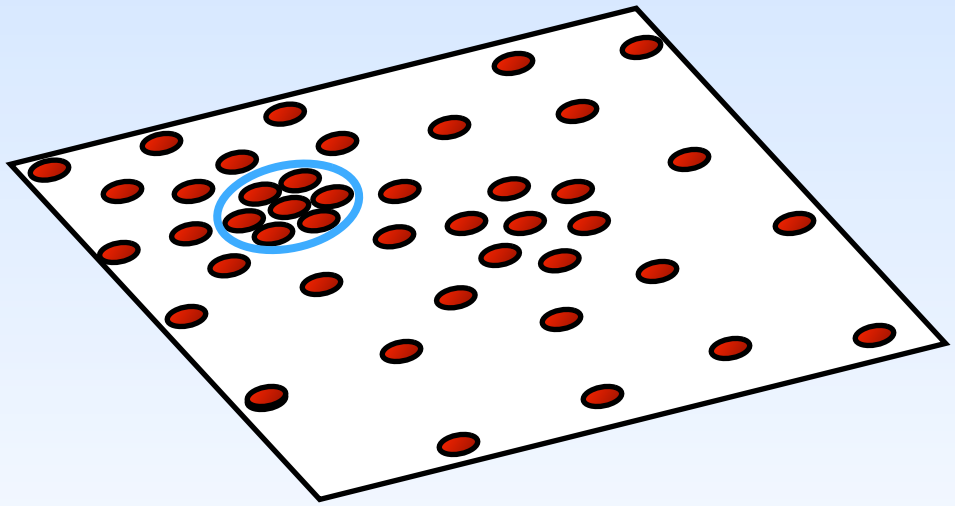
Assumed Underlying PDF

Real Data Samples

Non-Parametric Density Estimation

Assumption : The data points are sampled from an underlying PDF





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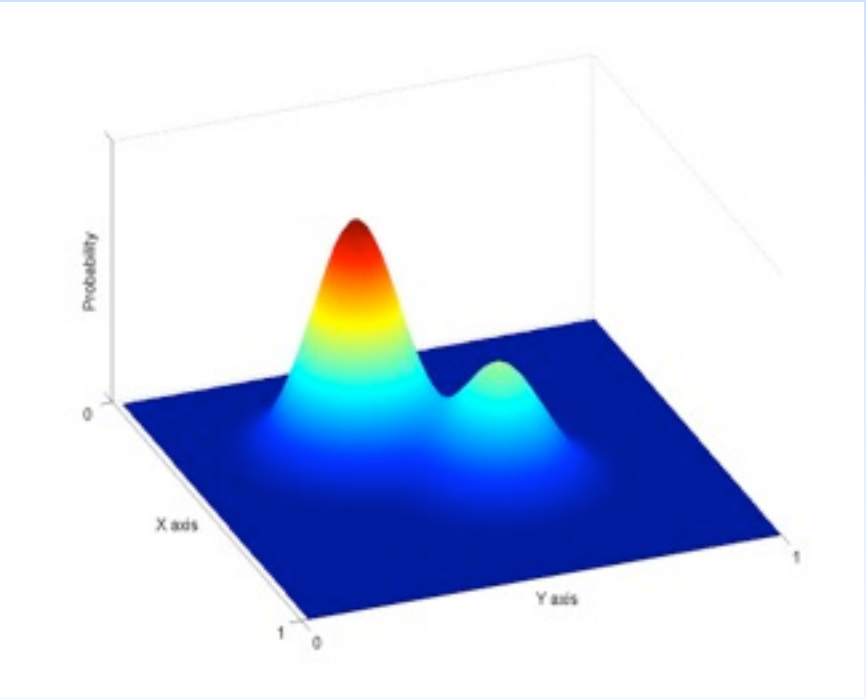

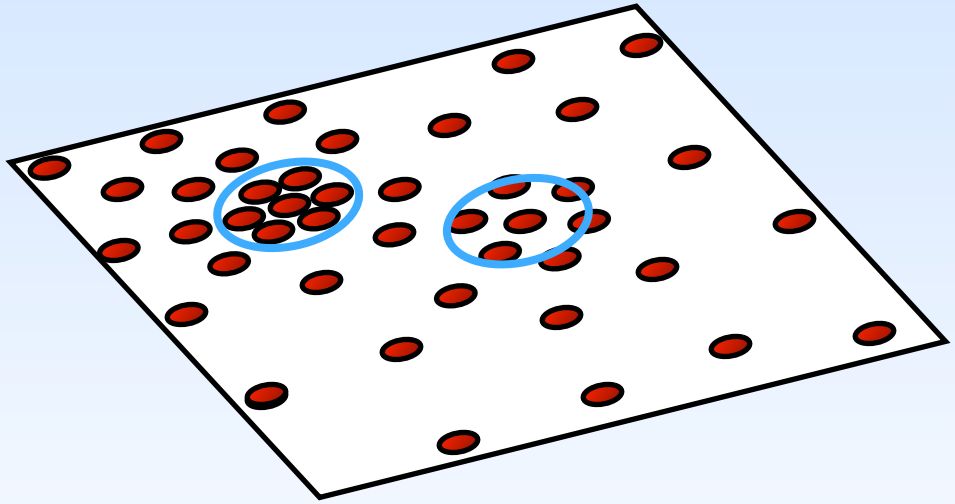

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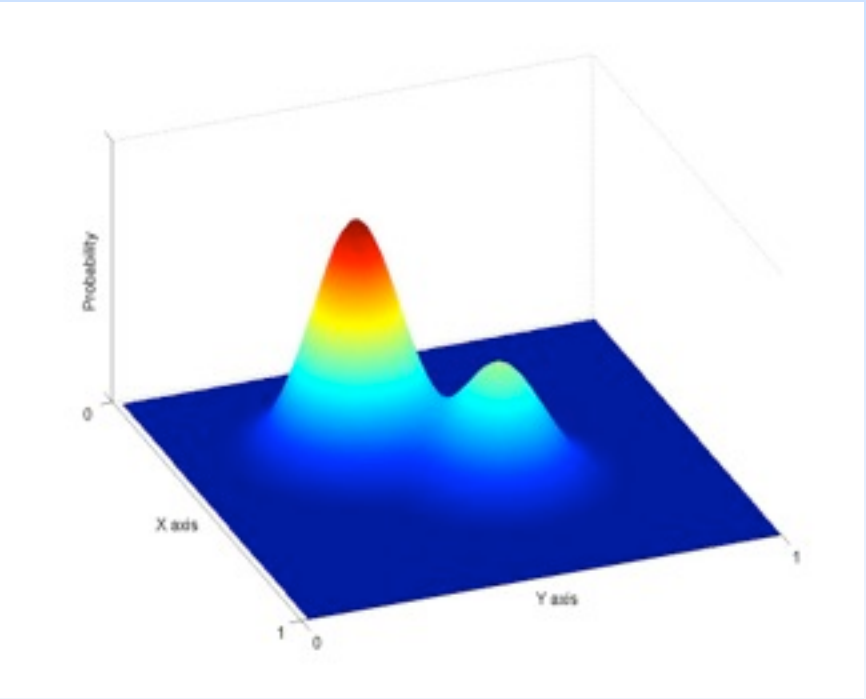

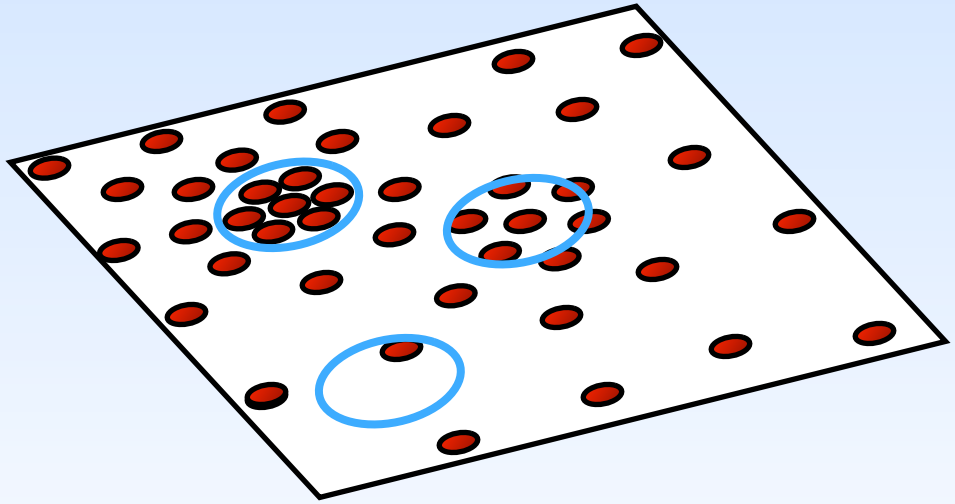

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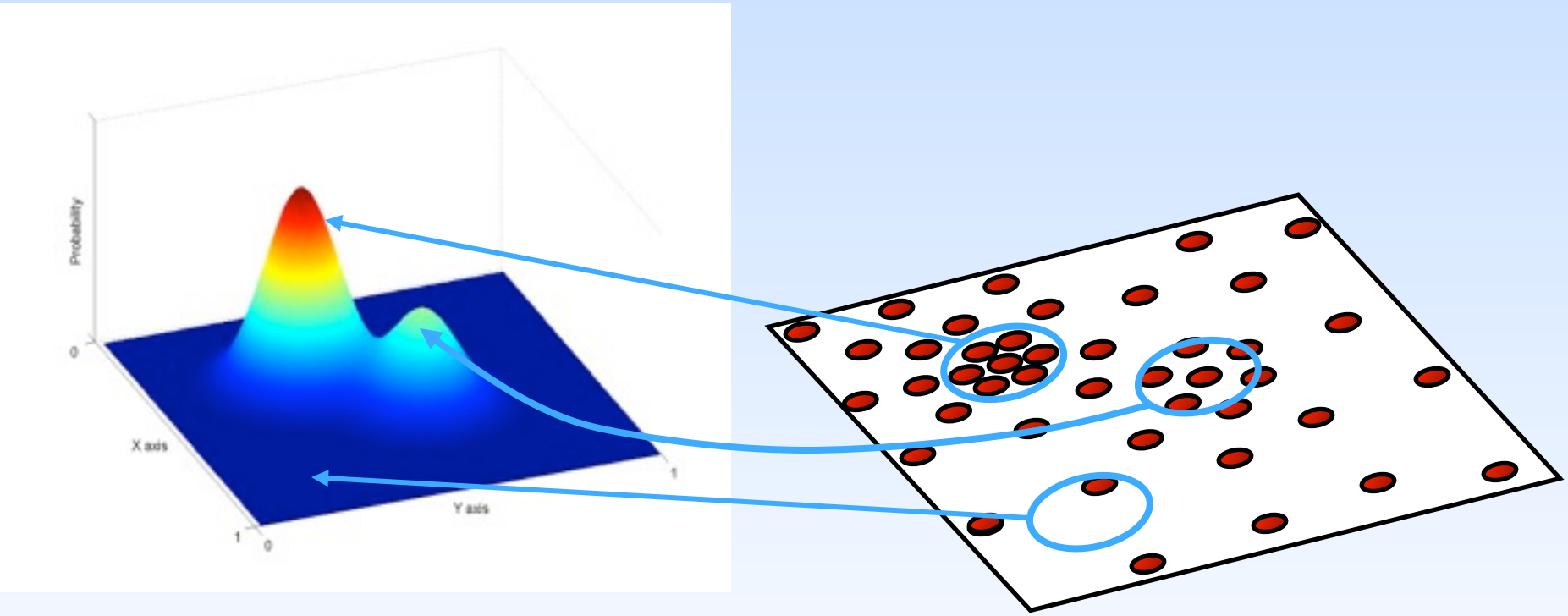

Assumed Underlying PDF **Real Data Samples**

Non-Parametric Density Estimation

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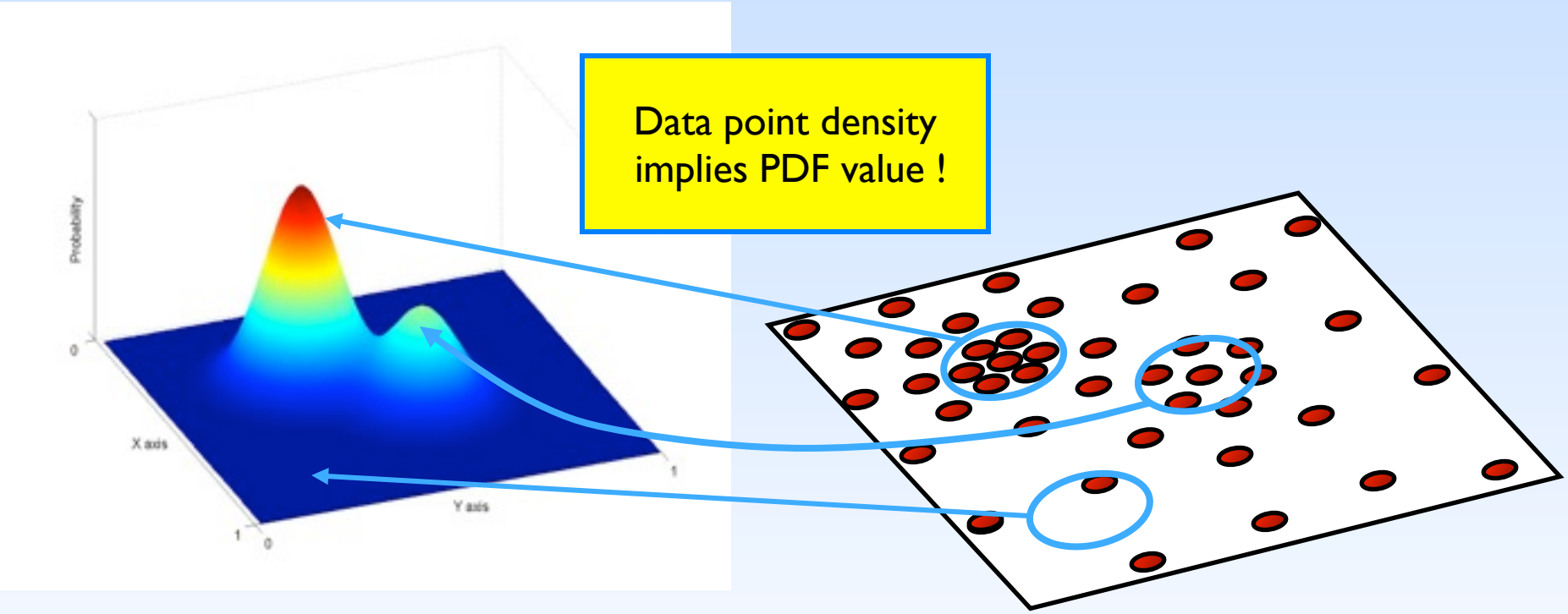

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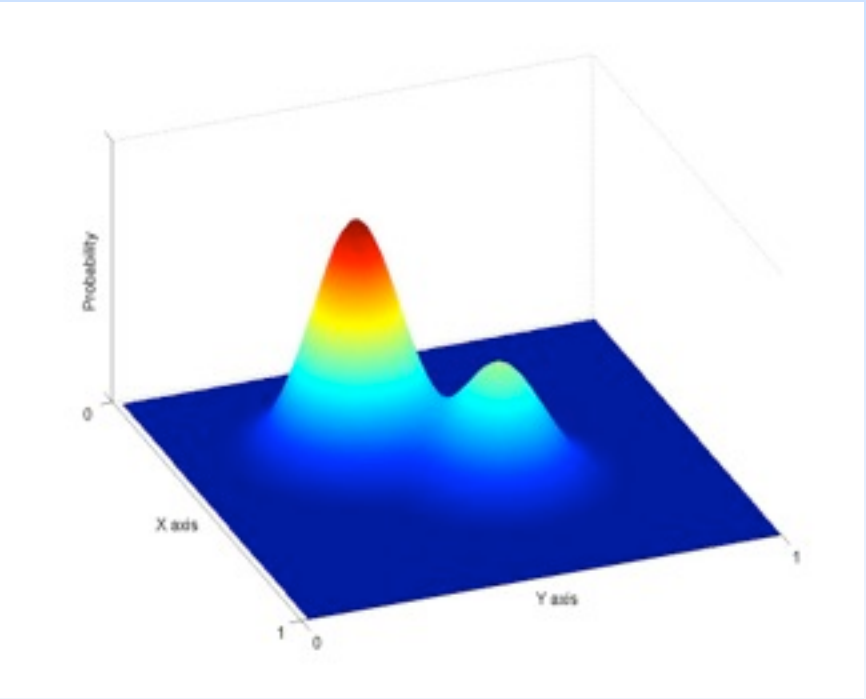

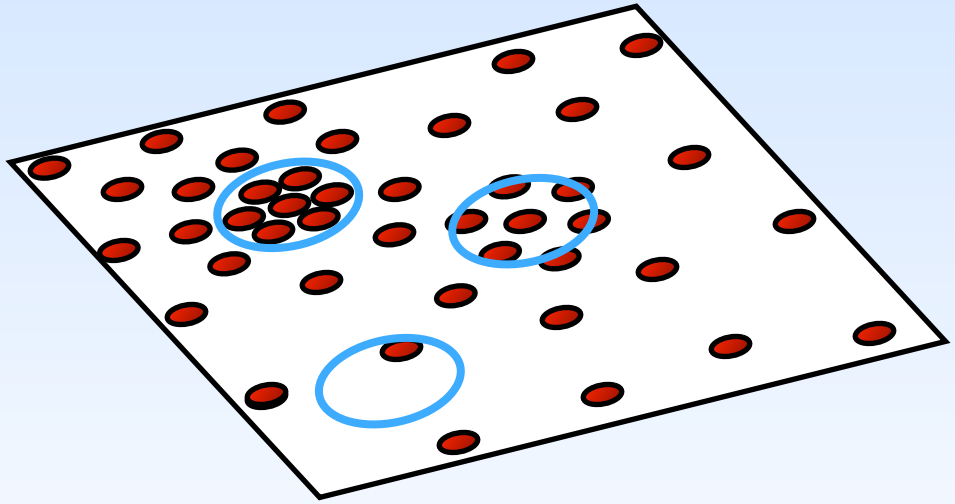

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Real Data Samples

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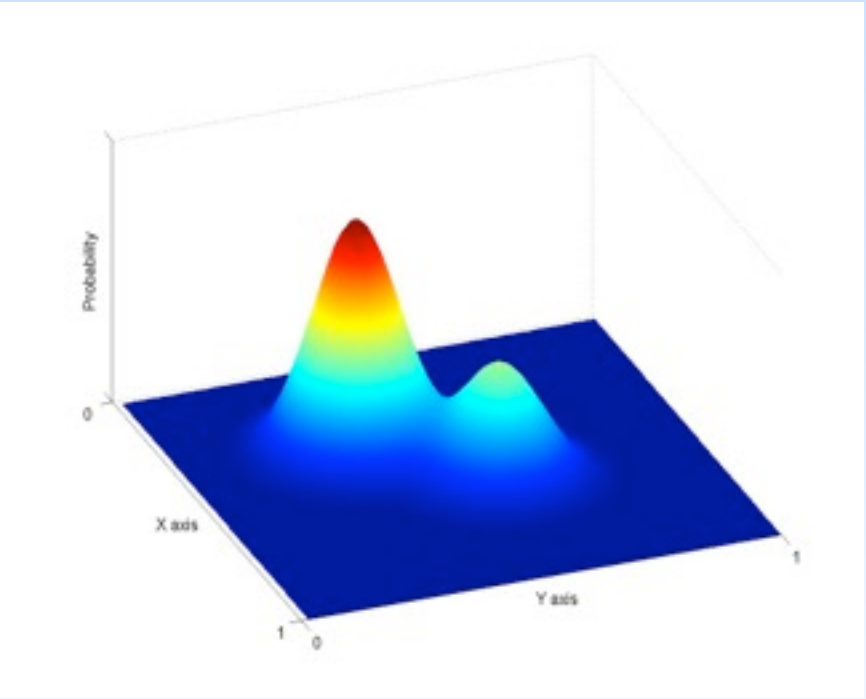

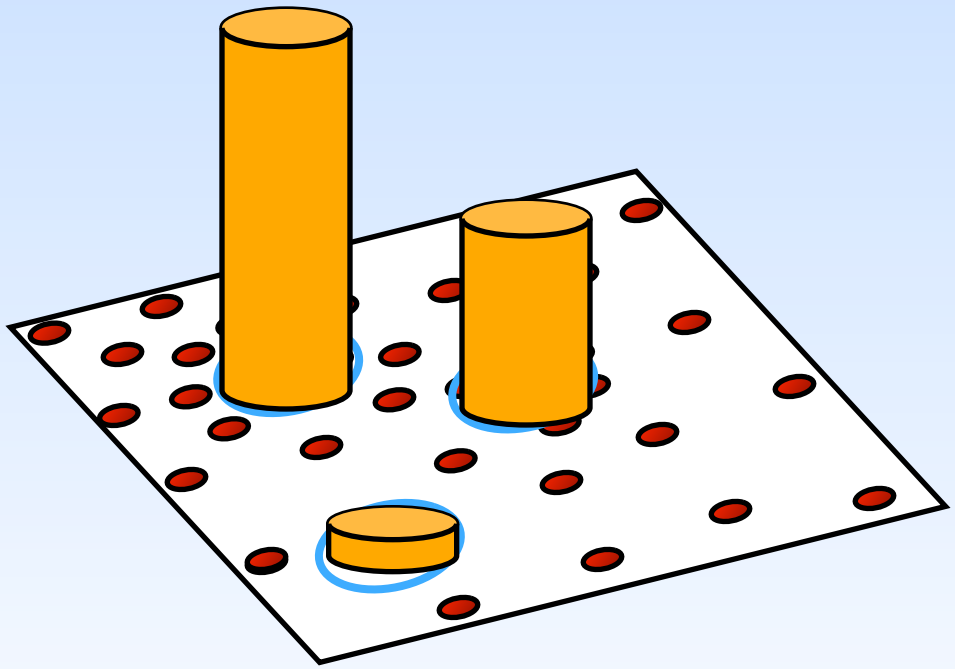

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Real Data Samples

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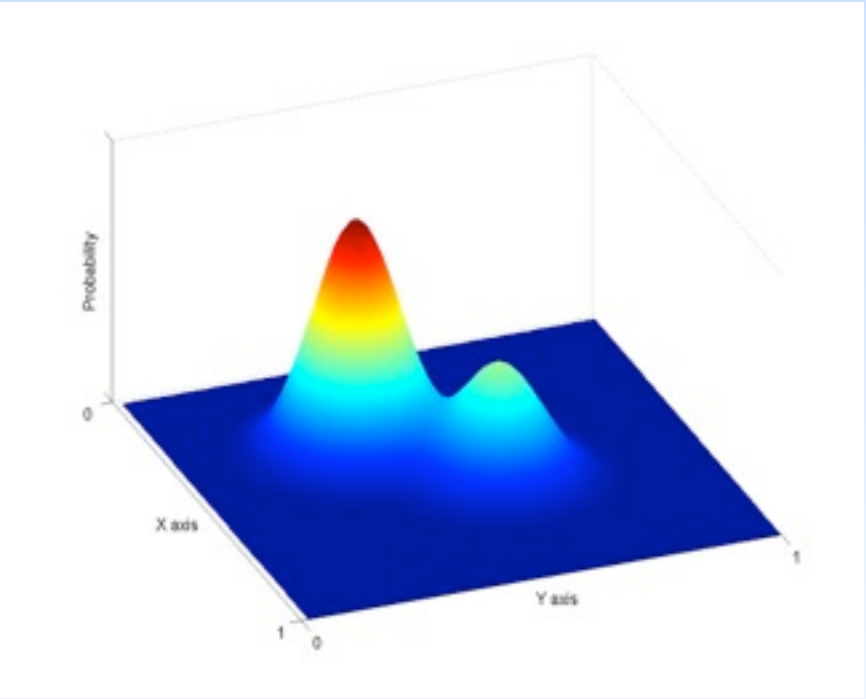

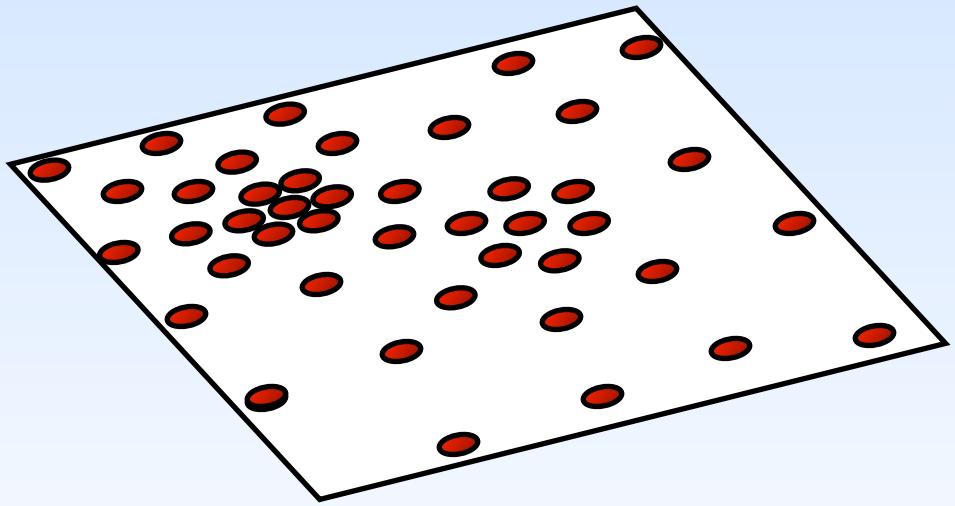

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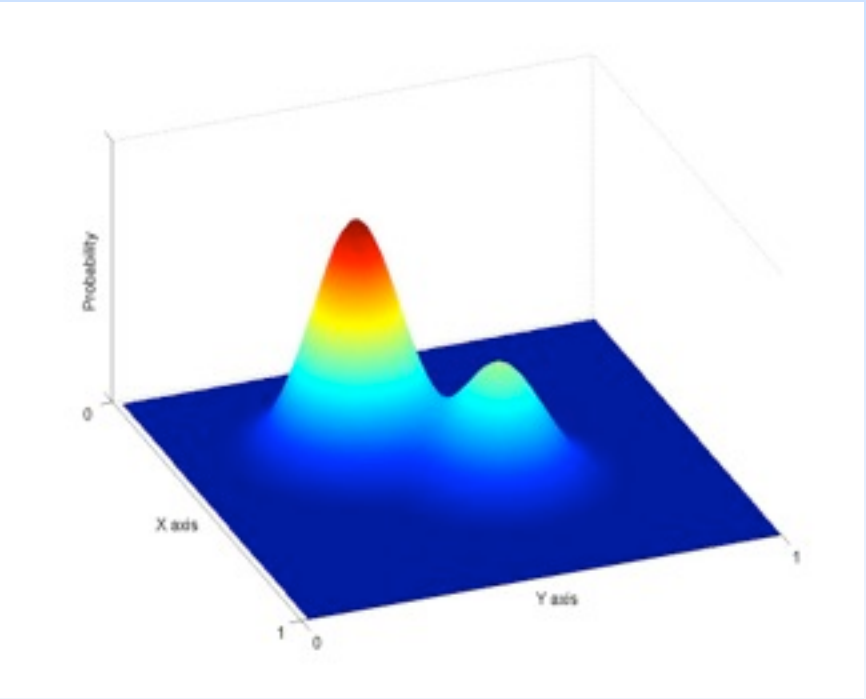

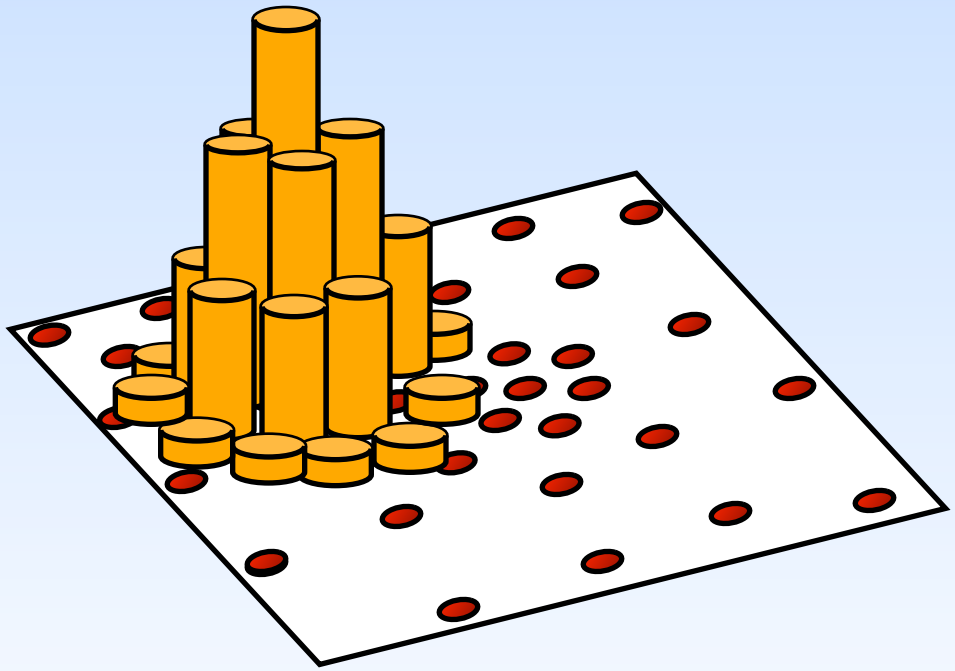

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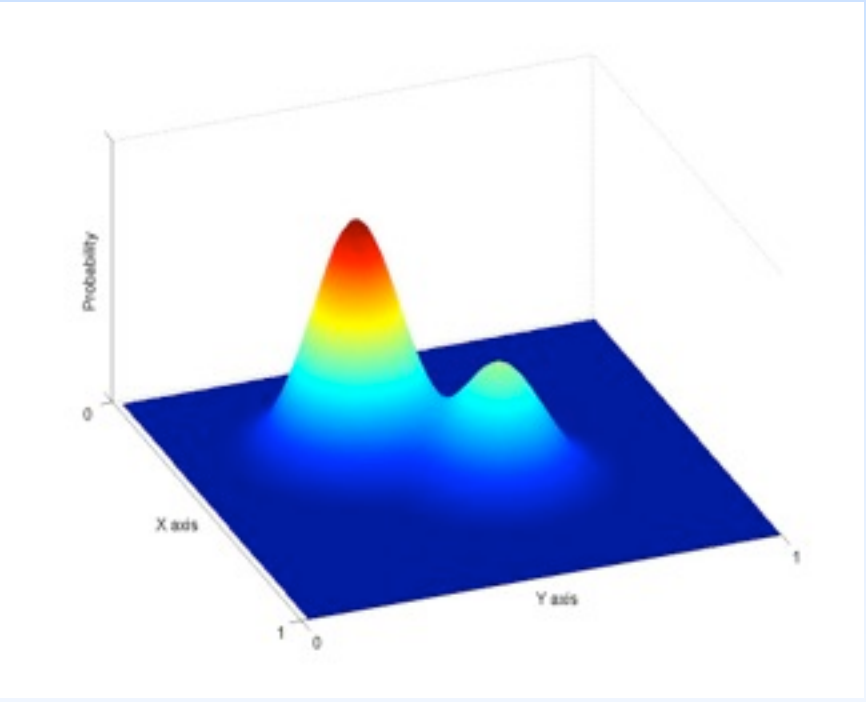
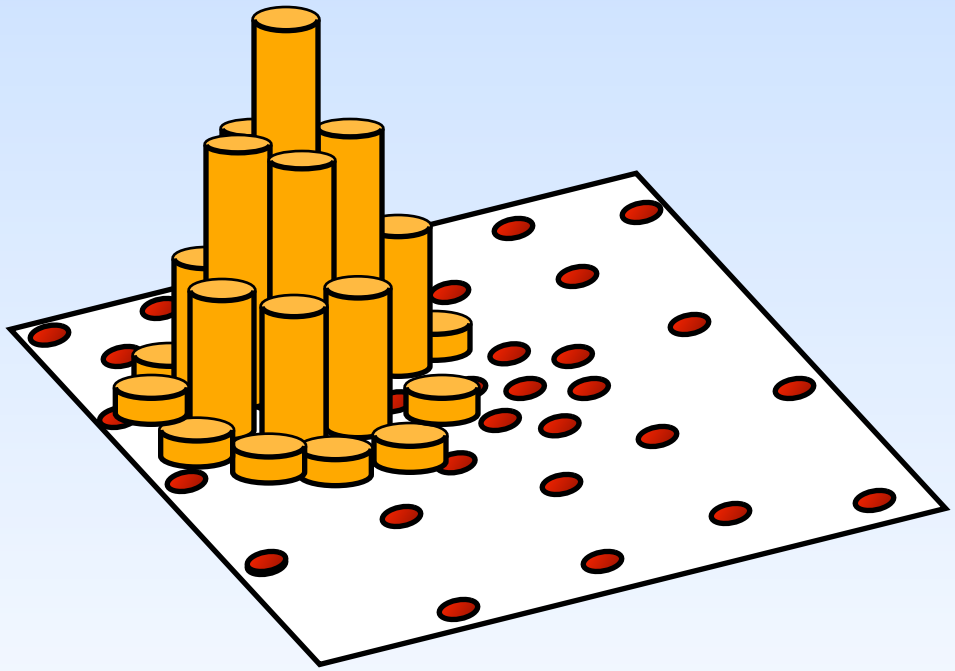


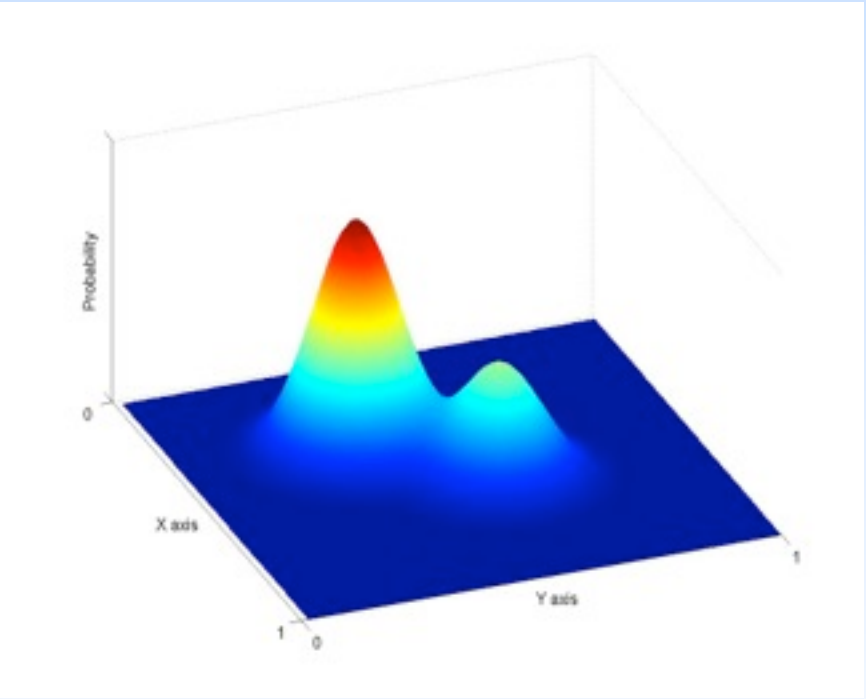


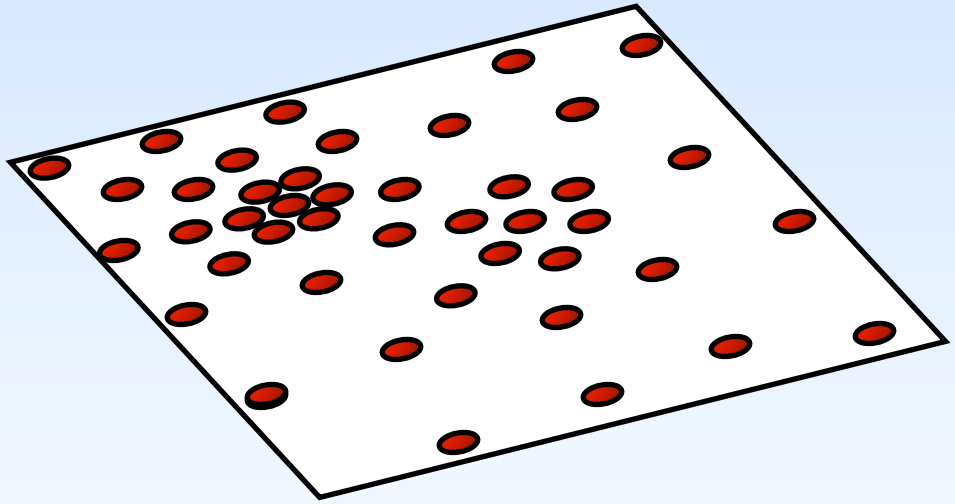
Assumed Underlying PDF

Real Data Samples

Parametric Density

Assumption : The data points are sampled from an underlying PDF





Assumed Underlying PDF

Real Data Samples

Parametric Density

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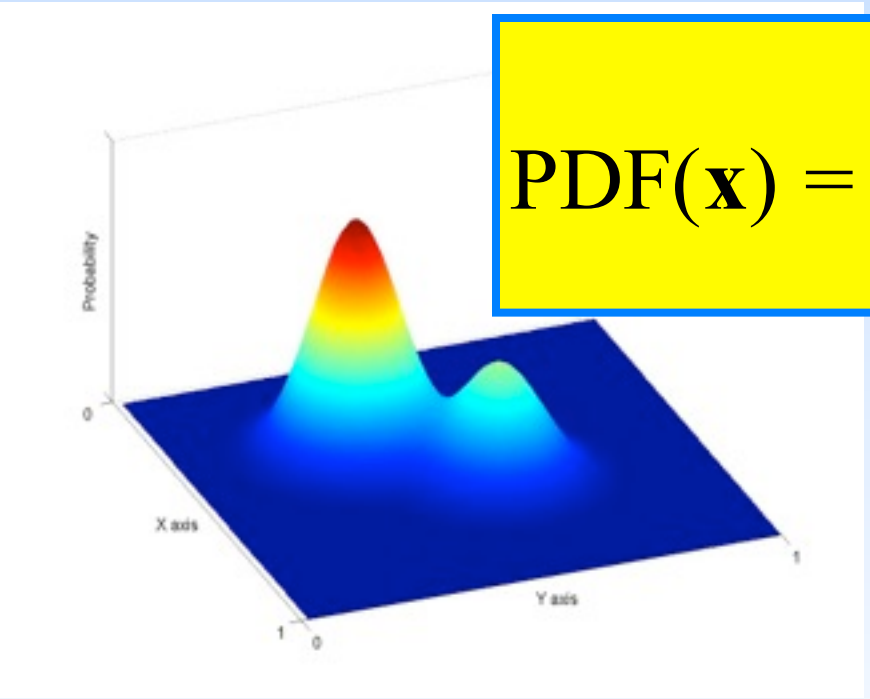
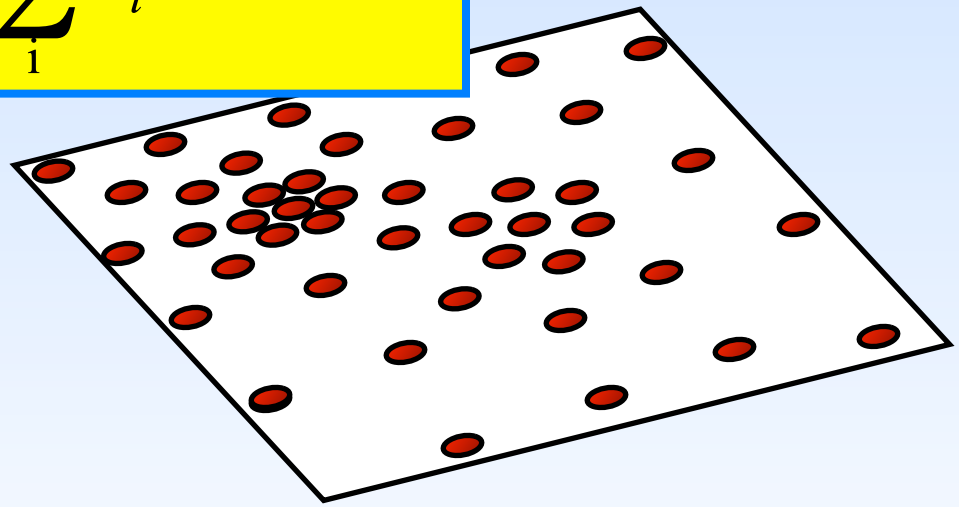
$$\mathrm{PDF}(\mathbf{x}) = \sum_{1} c_i \times e^{-\frac{(\mathbf{x}-\mu_i)^2}{2\sigma_i^2}}$$





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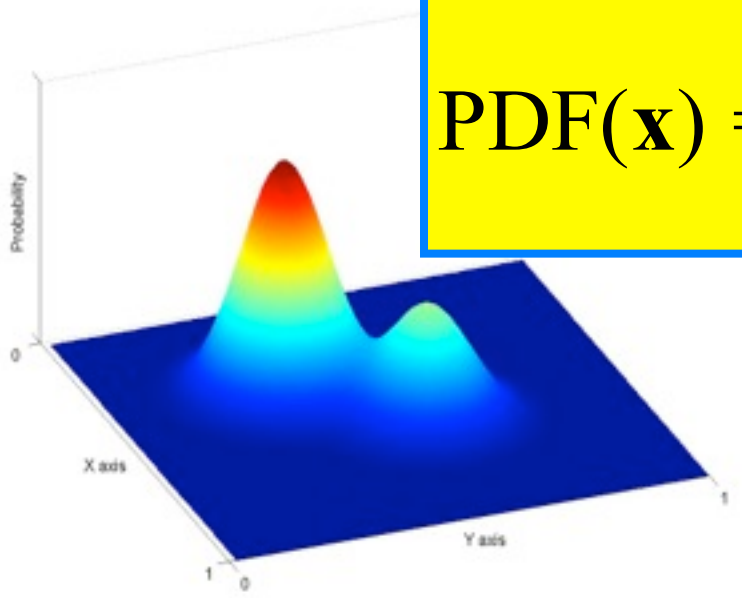
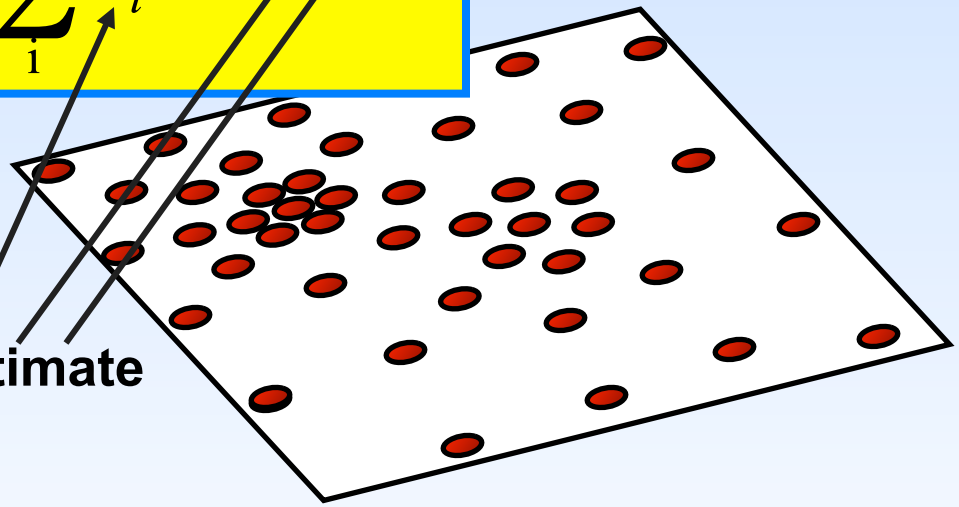


Estimate



Assumed Underlying PDF **Real Data Samples**

Kernel Density Estimation

Parzen Windows - Function Forms

$$P(\mathbf{x}) = \frac{1}{n}\sum_{i=1}^{n} K(\mathbf{x} - \mathbf{x}_i)$$

A function of some finite number of data points $x_1 \ldots x_n$

Data

Kernel Density Estimation

Parzen Windows - Function Forms

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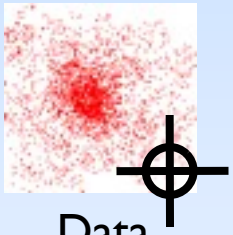

Data

Kernel Density Estimation

Parzen Windows - Function Forms

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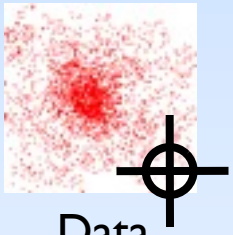


Data

In practice one uses the forms:

$$K(\mathbf{x}) = c\prod_{i=1}^{d} k(x_i)$$

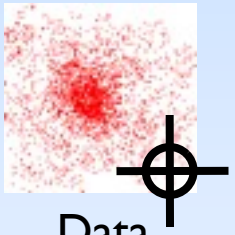
Same function on each dimension

Kernel Density Estimation

Parzen Windows - Function Forms

$$P(\mathbf{x}) = \frac{1}{n}\sum_{i=1}^{n} K(\mathbf{x} - \mathbf{x}_i)$$

A function of some finite number of data points $x_1 \ldots x_n$



Data

In practice one uses the forms:

$$K(\mathbf{x}) = c\prod_{i=1}^{d} k(x_i)$$ or $$K(\mathbf{x}) = ck\left(\|\mathbf{x}\|\right)$$

Same function on each dimension

Function of vector length only

Kernel Density Estimation

Various Kernels

$$P(\mathbf{x}) = \frac{1}{n}\sum_{i=1}^{n} K(\mathbf{x} - \mathbf{x}_i)$$

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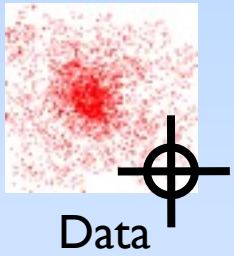

Data

Kernel Density Estimation

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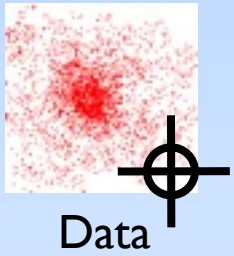
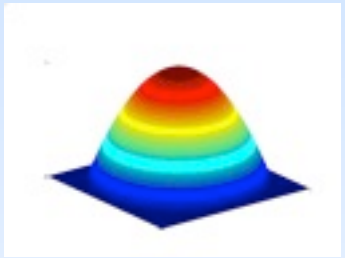


Data

Examples:

- Epanechnikov Kernel

$$K_E(\mathbf{x}) = \begin{cases} c\left(1 - \|\mathbf{x}\|^2\right) & \|\mathbf{x}\| \le 1 \\ 0 & \text{otherwise} \end{cases}$$



Kernel Density Estimation

Various Kernels

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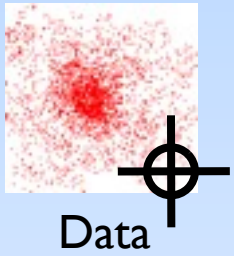

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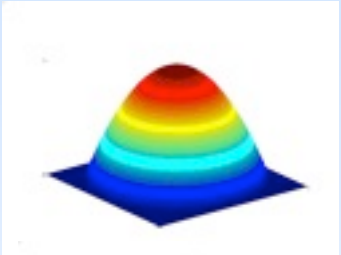
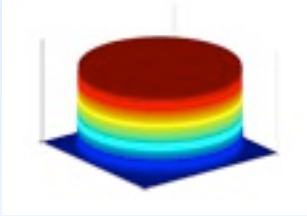


- Uniform Kernel

$$K_U(\mathbf{x}) = \begin{cases} c & \|\mathbf{x}\| \le 1 \\ 0 & \text{otherwise} \end{cases}$$



Kernel Density Estimation

Various Kernels

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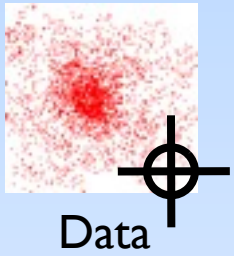
A function of some finite number of data points $x_1 \ldots x_n$



Data

Examples:

- Epanechnikov Kernel

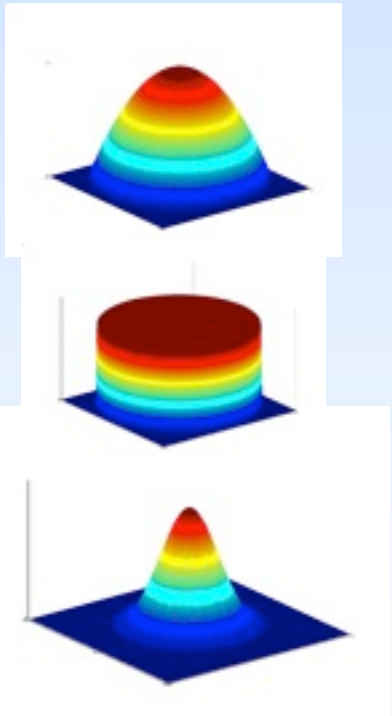
$$K_E(\mathbf{x}) = \begin{cases} c\left(1 - \|\mathbf{x}\|^2\right) & \|\mathbf{x}\| \le 1 \\ 0 & \text{otherwise} \end{cases}$$

- Uniform Kernel

$$K_U(\mathbf{x}) = \begin{cases} c & \|\mathbf{x}\| \le 1 \\ 0 & \text{otherwise} \end{cases}$$

- Normal Kernel

$$K_N(\mathbf{x}) = c \cdot \exp\left(-\frac{1}{2}\|\mathbf{x}\|^2\right)$$



Kernel Density Estimation

$$P(\mathbf{x}) = \frac{1}{n}\sum_{i=1}^{n} K(\mathbf{x} - \mathbf{x}_i)$$

Kernel Density Estimation

Gradient

$$P(\mathbf{x}) = \frac{1}{n}\sum_{i=1}^{n} K(\mathbf{x} - \mathbf{x}_i)$$

Kernel Density Estimation

Gradient

$$\nabla P(\mathbf{x}) = \frac{1}{n}\sum_{i=1}^{n} \nabla K(\mathbf{x} - \mathbf{x}_i)$$

Kernel Density Estimation

Gradient

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Give up estimating the PDF !
Estimate **ONLY** the gradient

Kernel Density Estimation

Gradient

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Using the Kernel form:

$$K(\mathbf{x} - \mathbf{x}_i) = ck\left(\left\|\frac{\mathbf{x} - \mathbf{x}_i}{h}\right\|^2\right)$$

Kernel Density Estimation

Gradient

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Size of window

Kernel Density Estimation

Gradient

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$$K(\mathbf{x} - \mathbf{x}_i) = ck\left(\left\|\frac{\mathbf{x} - \mathbf{x}_i}{h}\right\|^2\right)$$

We get :

$$\nabla P(\mathbf{x}) = \frac{c}{n}\sum_{i=1}^{n} \nabla k_i = \frac{c}{n}\left[\sum_{i=1}^{n} g_i\right] \cdot \left[\frac{\sum_{i=1}^{n} \mathbf{x}_i g_i}{\sum_{i=1}^{n} g_i} - \mathbf{x}\right]$$

$$g(\mathbf{x}) = -k'(\mathbf{x})$$

Kernel Density Estimation

Gradient

$$\nabla P(\mathbf{x}) = \frac{c}{n}\sum_{i=1}^{n}\nabla k_i = \frac{c}{n}\left[\sum_{i=1}^{n} g_i\right]\cdot\left[\frac{\sum_{i=1}^{n}\mathbf{x}_i g_i}{\sum_{i=1}^{n} g_i} - \mathbf{x}\right]$$

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Kernel Density Estimation

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Computing The Mean Shift

$$\nabla P(\mathbf{x}) = \frac{c}{n}\sum_{i=1}^{n}\nabla k_i = \frac{c}{n}\left[\sum_{i=1}^{n} g_i\right]\cdot\left[\frac{\sum_{i=1}^{n}\mathbf{x}_i g_i}{\sum_{i=1}^{n} g_i} - \mathbf{x}\right]$$

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Yet another Kernel density estimation !

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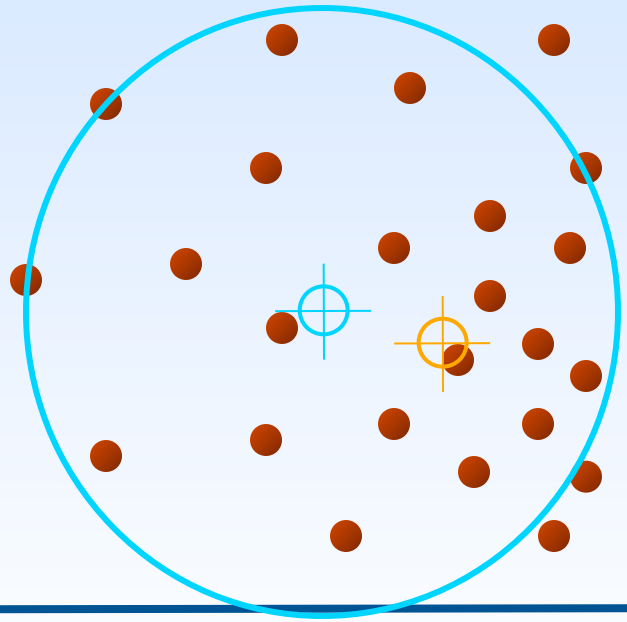
Yet another Kernel density estimation !



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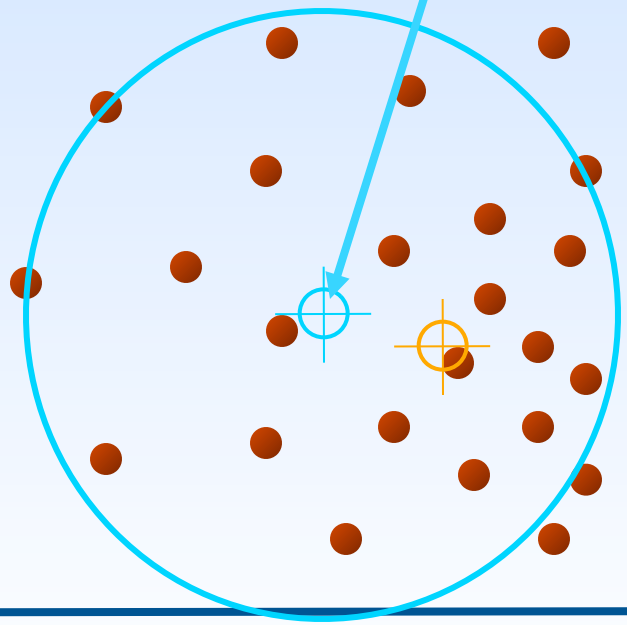

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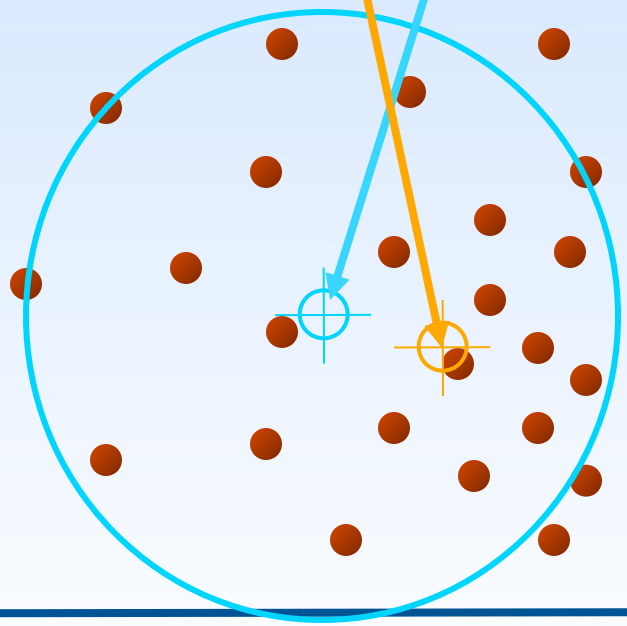

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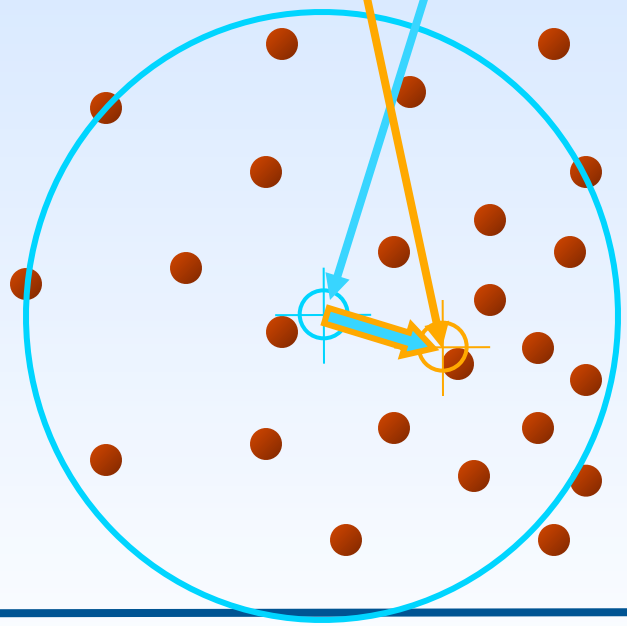

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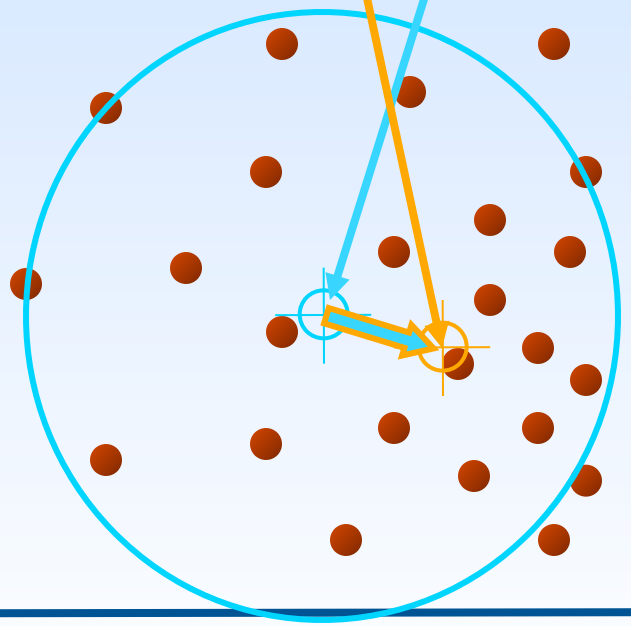
Simple Mean Shift procedure:

- Compute mean shift vector

$$\mathbf{m}(\mathbf{x}) = \left[\frac{\sum_{i=1}^{n}\mathbf{x}_i g\left(\left\|\frac{\mathbf{x}-\mathbf{x}_i}{h}\right\|^2\right)}{\sum_{i=1}^{n} g\left(\left\|\frac{\mathbf{x}-\mathbf{x}_i}{h}\right\|^2\right)} - \mathbf{x}\right]$$



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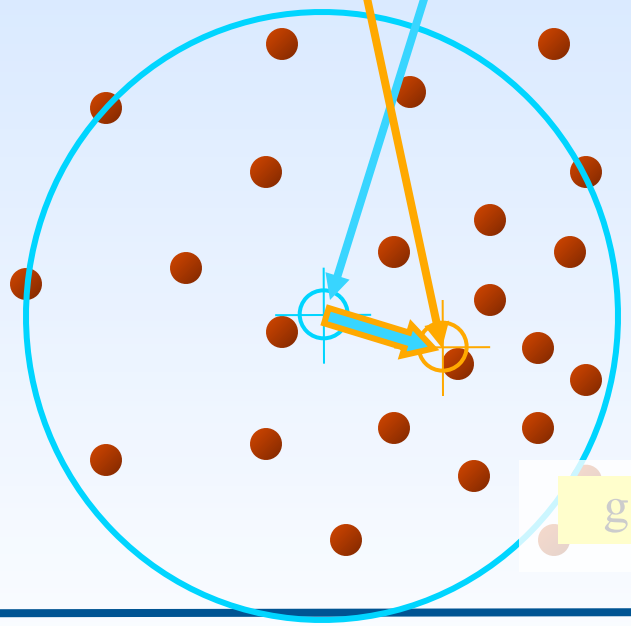
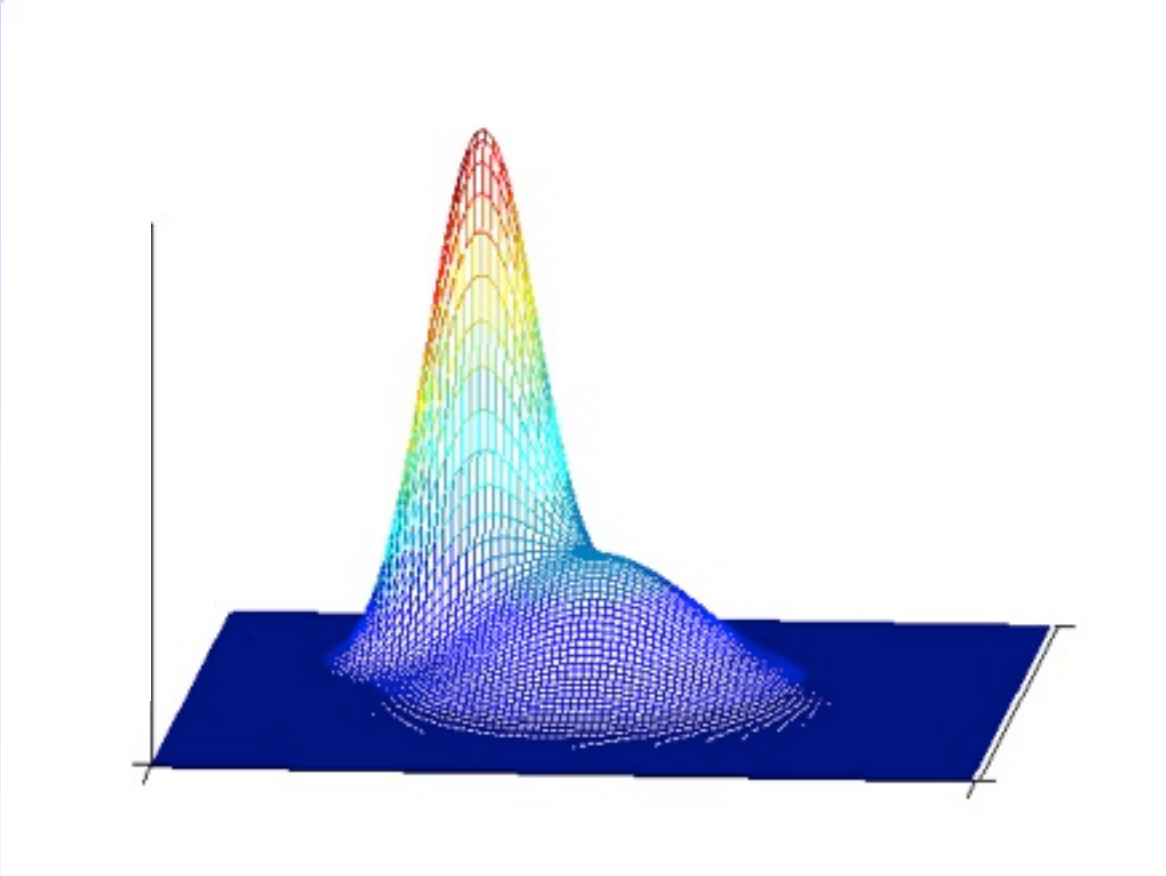
- Translate the Kernel window by **m(x)**



$$g(\mathbf{x}) = -k'(\mathbf{x})$$

Mean Shift Mode Detection



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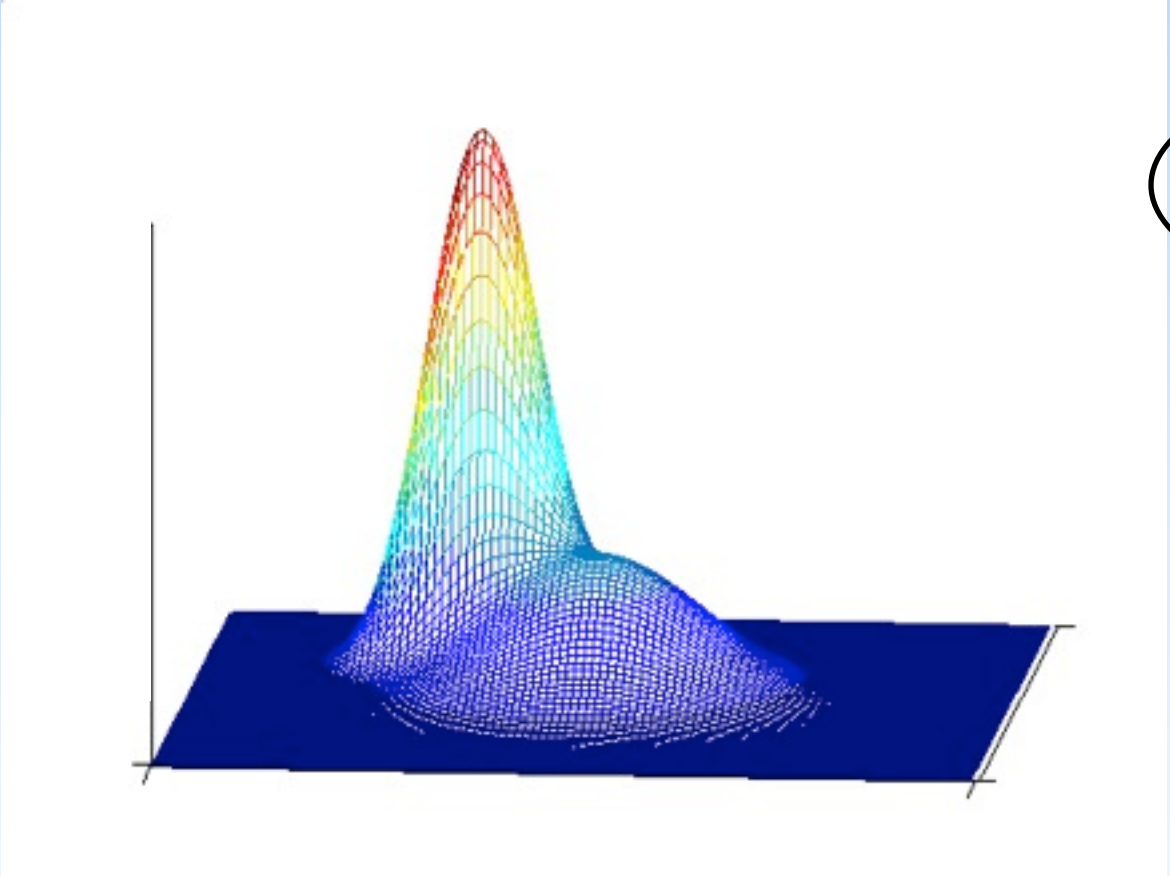
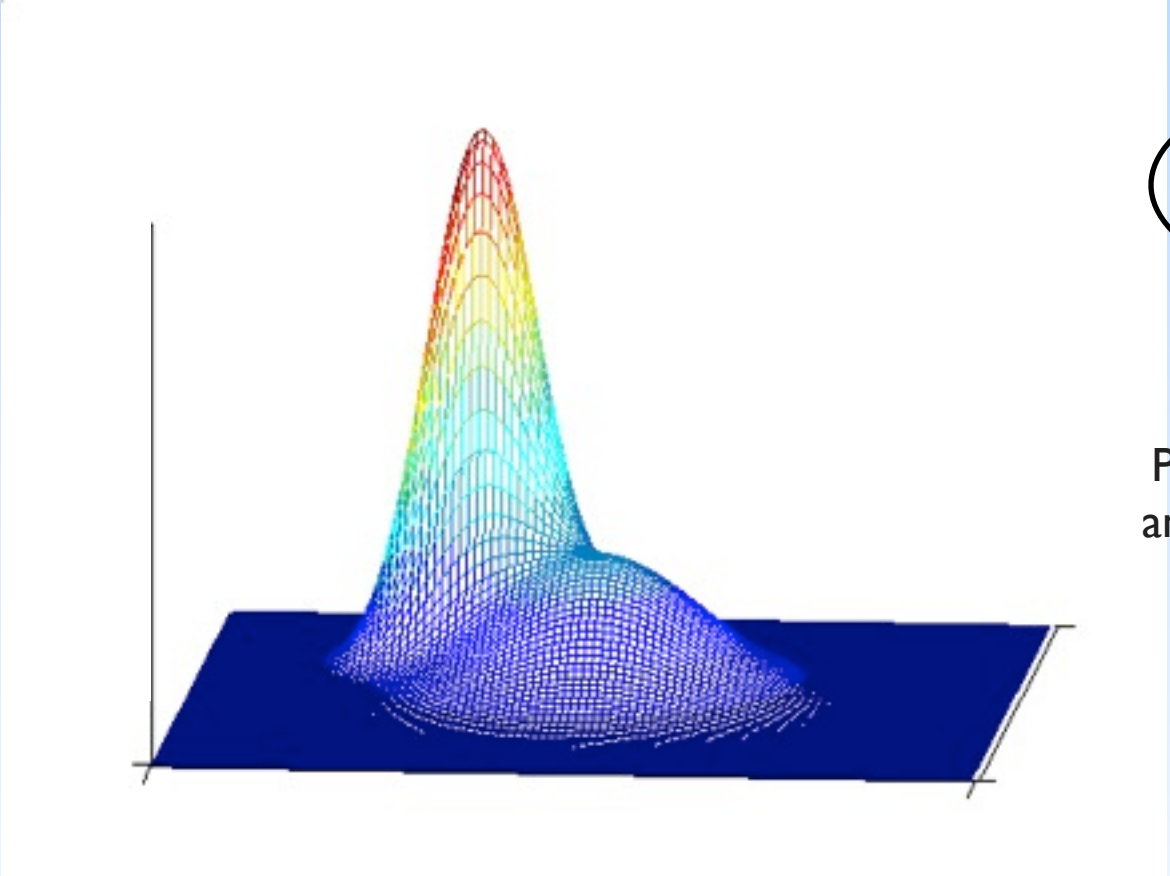


What happens if we reach a saddle point ?

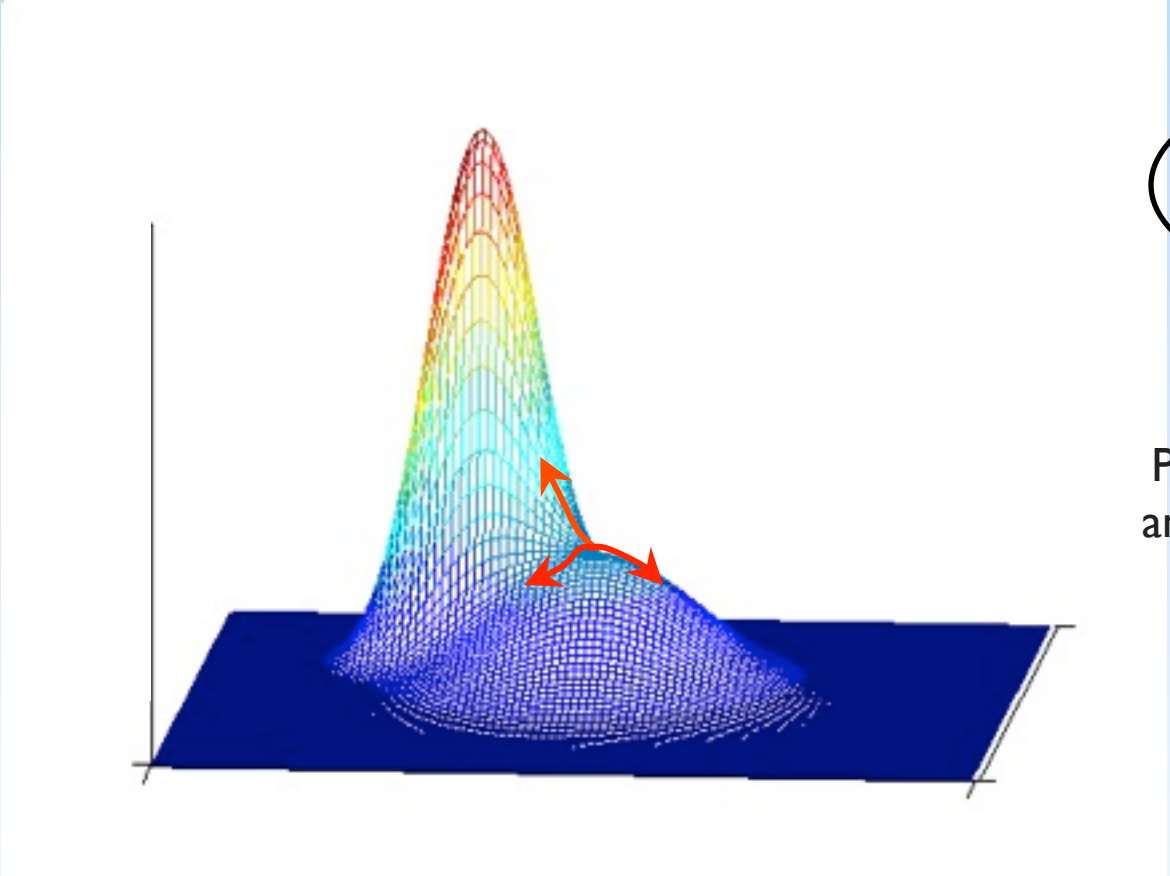
Mean Shift Mode Detection



What happens if we reach a saddle point ?

Perturb the mode position and check if we return back

Mean Shift Mode Detection



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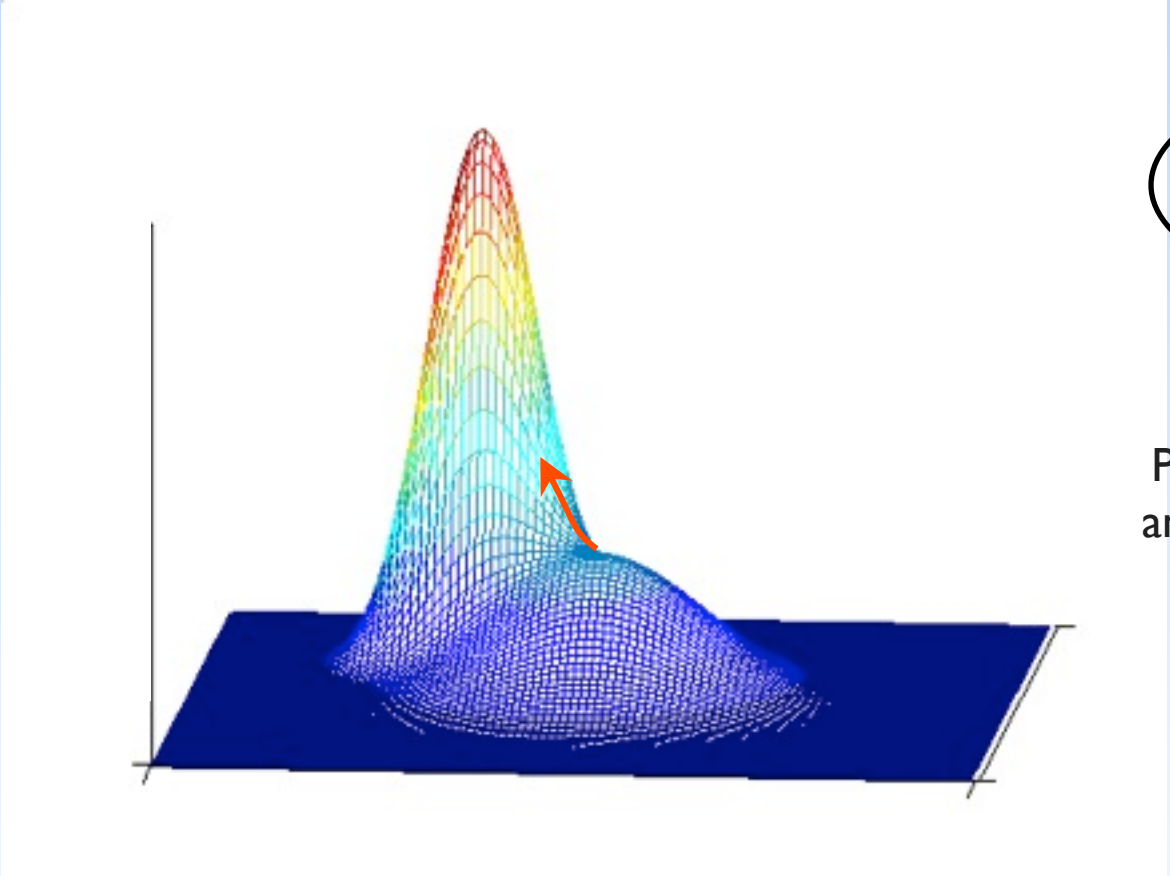

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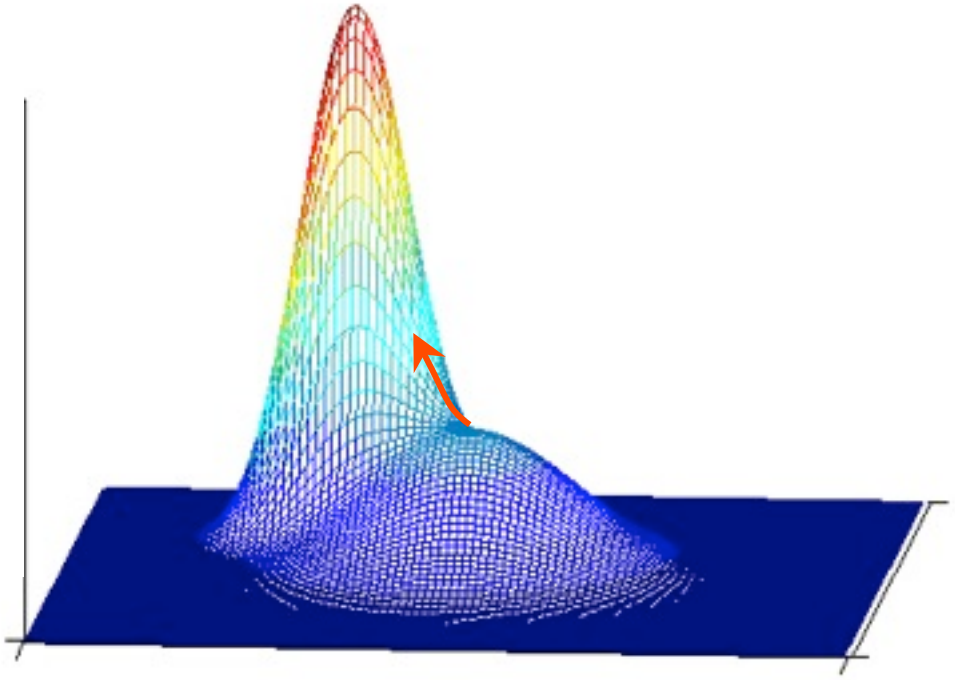

What happens if we reach a saddle point ?

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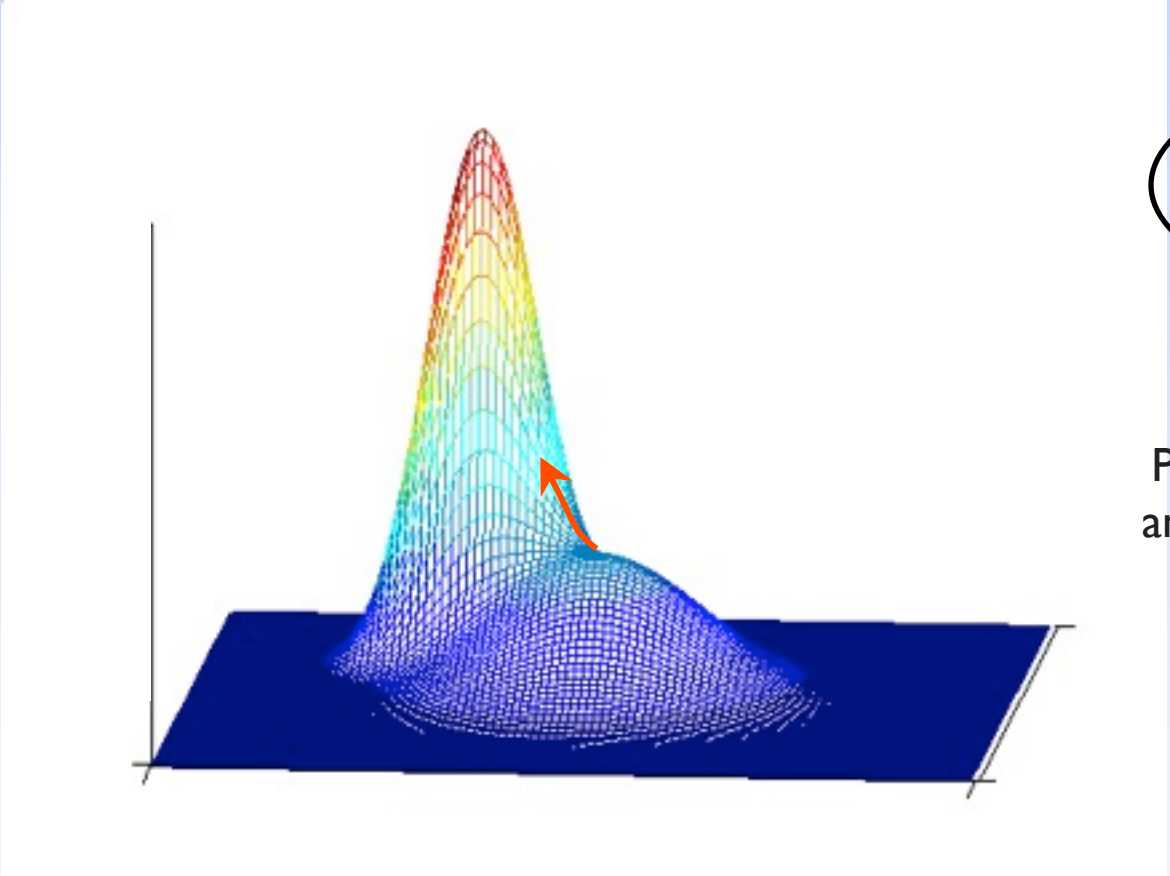
Updated Mean Shift Procedure:

Mean Shift Mode Detection



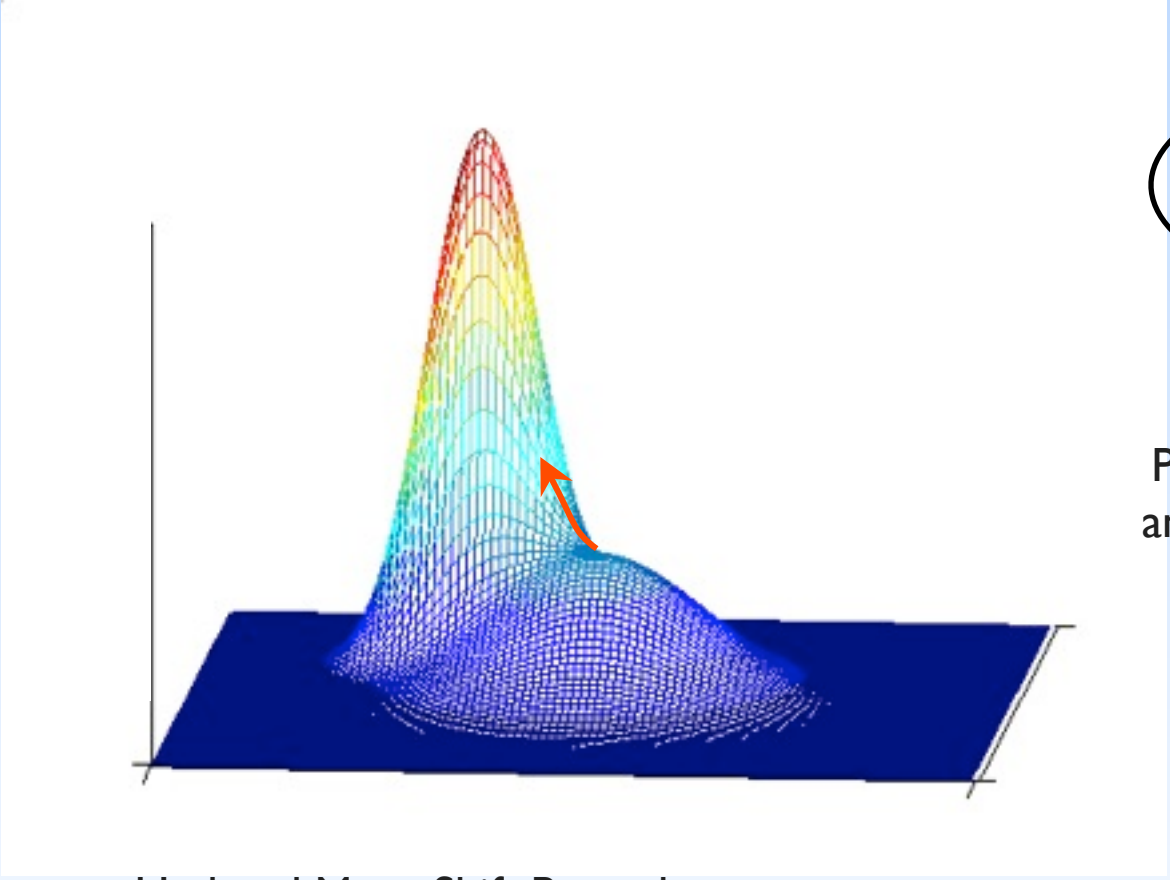
What happens if we reach a saddle point ?

Perturb the mode position and check if we return back

Updated Mean Shift Procedure:

- Find all modes using the Simple Mean Shift Procedure

Mean Shift Mode Detection



What happens if we reach a saddle point ?

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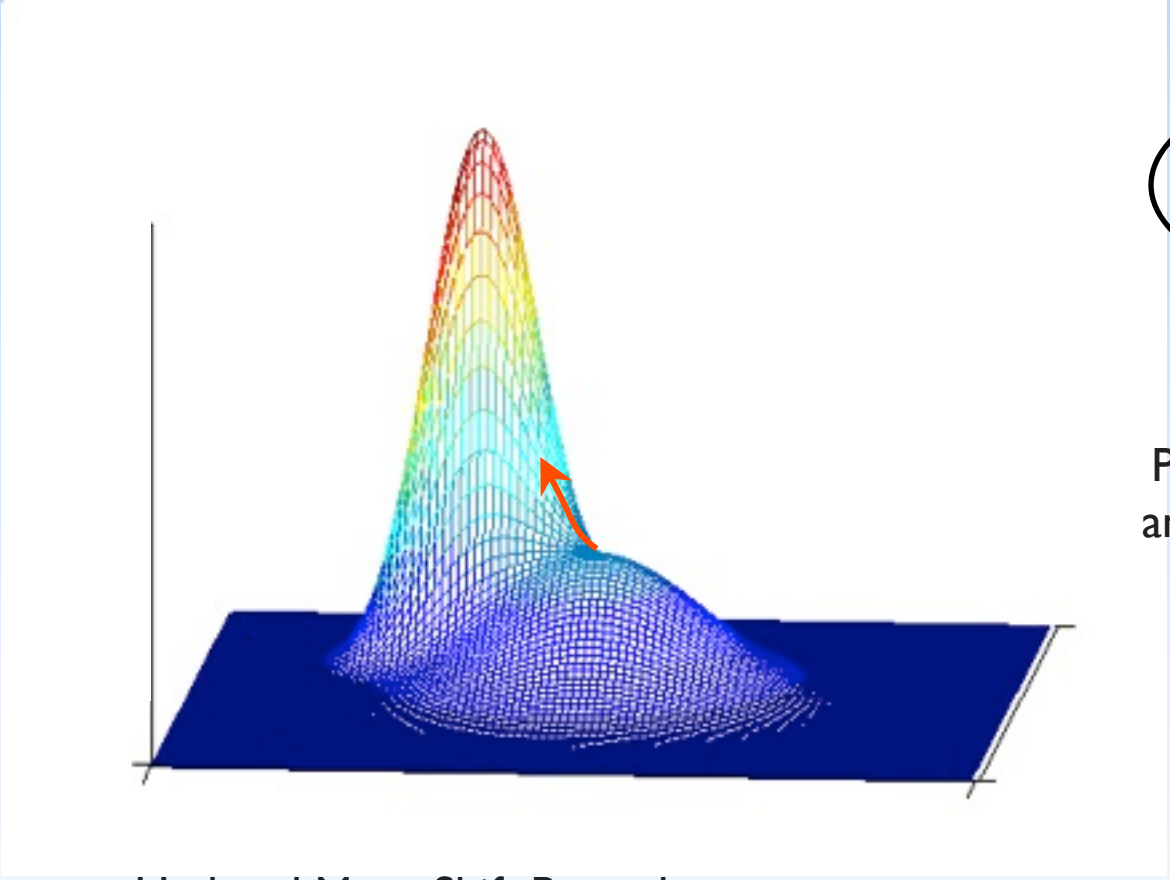
Updated Mean Shift Procedure:

- Find all modes using the Simple Mean Shift Procedure
- Prune modes by perturbing them (find saddle points and plateaus)

Mean Shift Mode Detection



What happens if we reach a saddle point ?

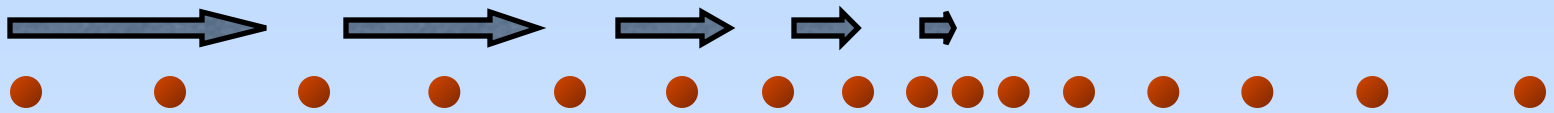
Perturb the mode position and check if we return back

Updated Mean Shift Procedure:

- Find all modes using the Simple Mean Shift Procedure
- Prune modes by perturbing them (find saddle points and plateaus)
- Prune nearby – take highest mode in the window

Mean Shift Properties

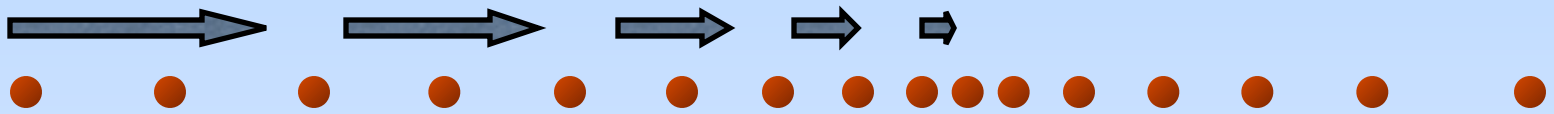
Mean Shift Properties



- Automatic convergence speed – the mean shift vector size depends on the gradient itself.

- Near maxima, the steps are small and refined

Mean Shift Properties



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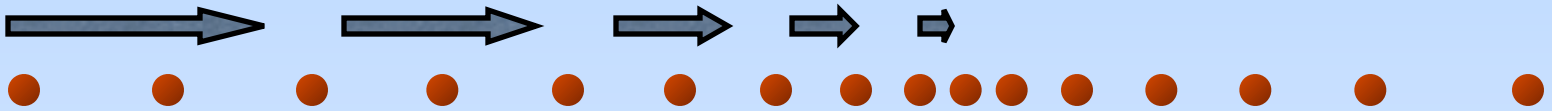
Adaptive Gradient Ascent

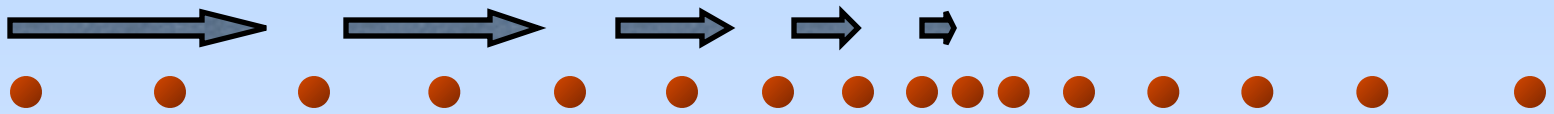
Mean Shift Properties



- Automatic convergence speed – the mean shift vector size depends on the gradient itself.
- Near maxima, the steps are small and refined
- Convergence is guaranteed for infinitesimal steps only → infinitely convergent, (therefore set a lower bound)

Adaptive Gradient Ascent

Mean Shift Properties

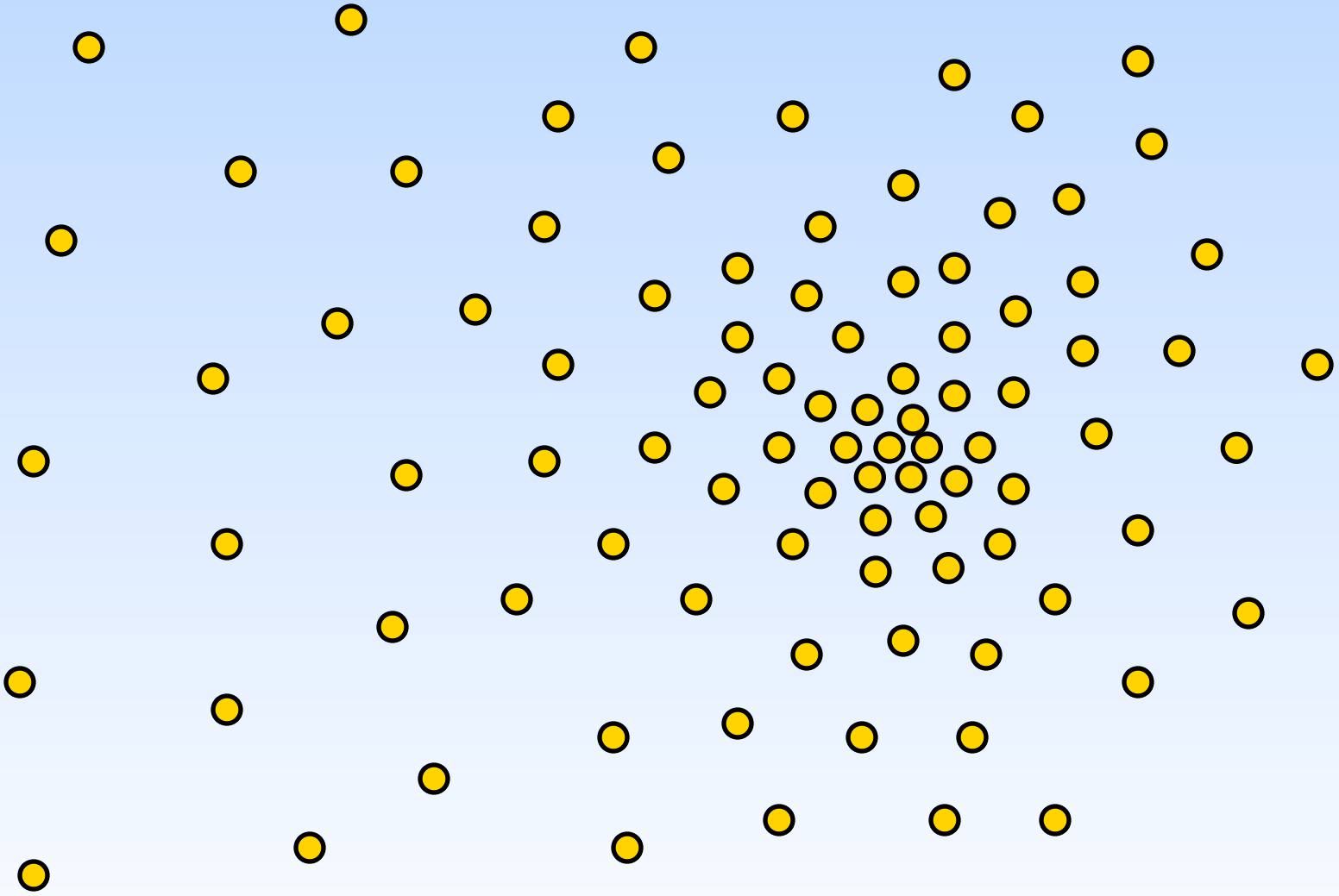


- Automatic convergence speed – the mean shift vector size depends on the gradient itself.
- Near maxima, the steps are small and refined

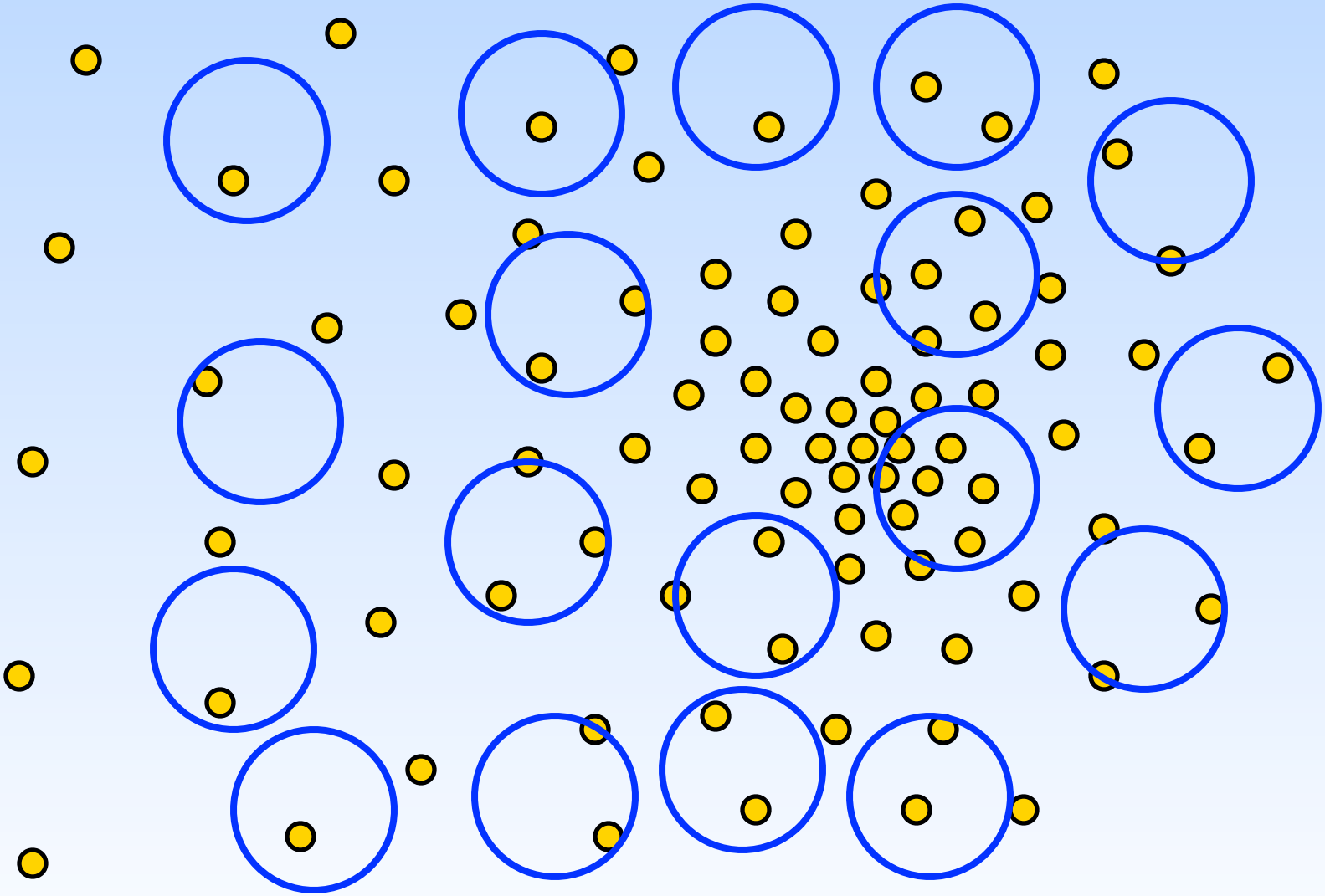
Adaptive Gradient Ascent

- Convergence is guaranteed for infinitesimal steps only → infinitely convergent, (therefore set a lower bound)
- For Uniform Kernel (), convergence is achieved in a finite number of steps
- Normal Kernel () exhibits a smooth trajectory, but is slower than Uniform Kernel ().

Real Modality Analysis

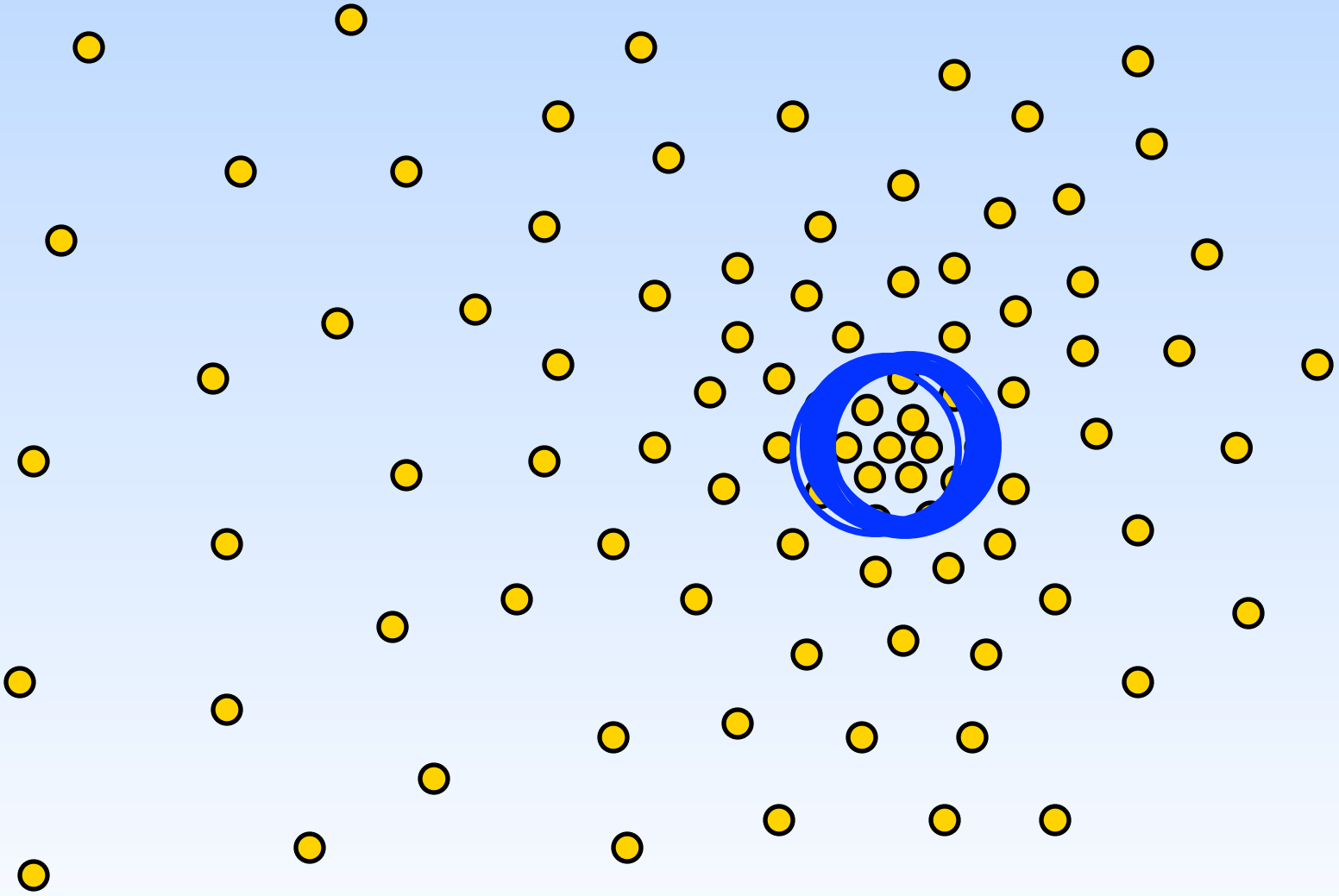


Real Modality Analysis



Run the procedure in parallel

Real Modality Analysis



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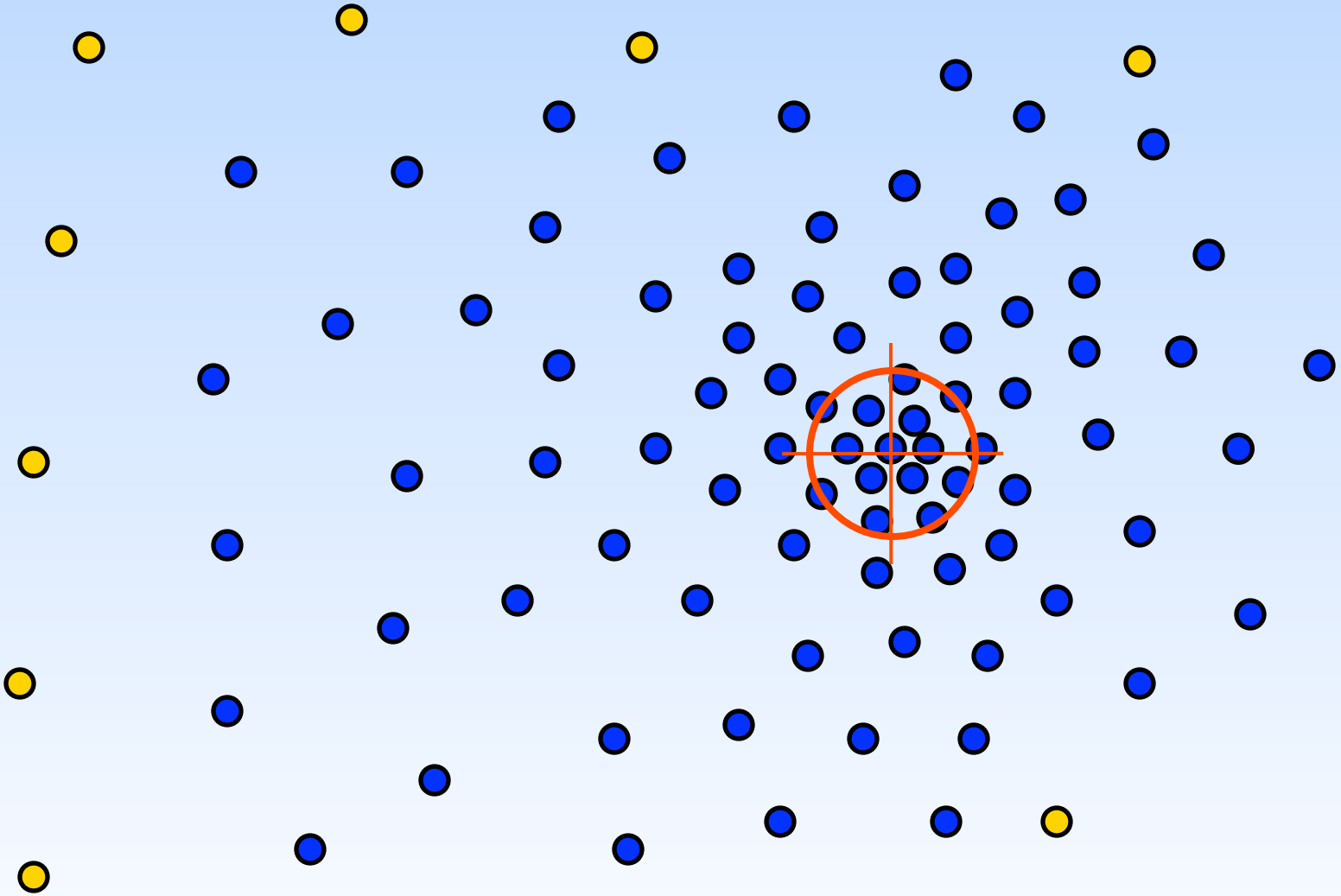
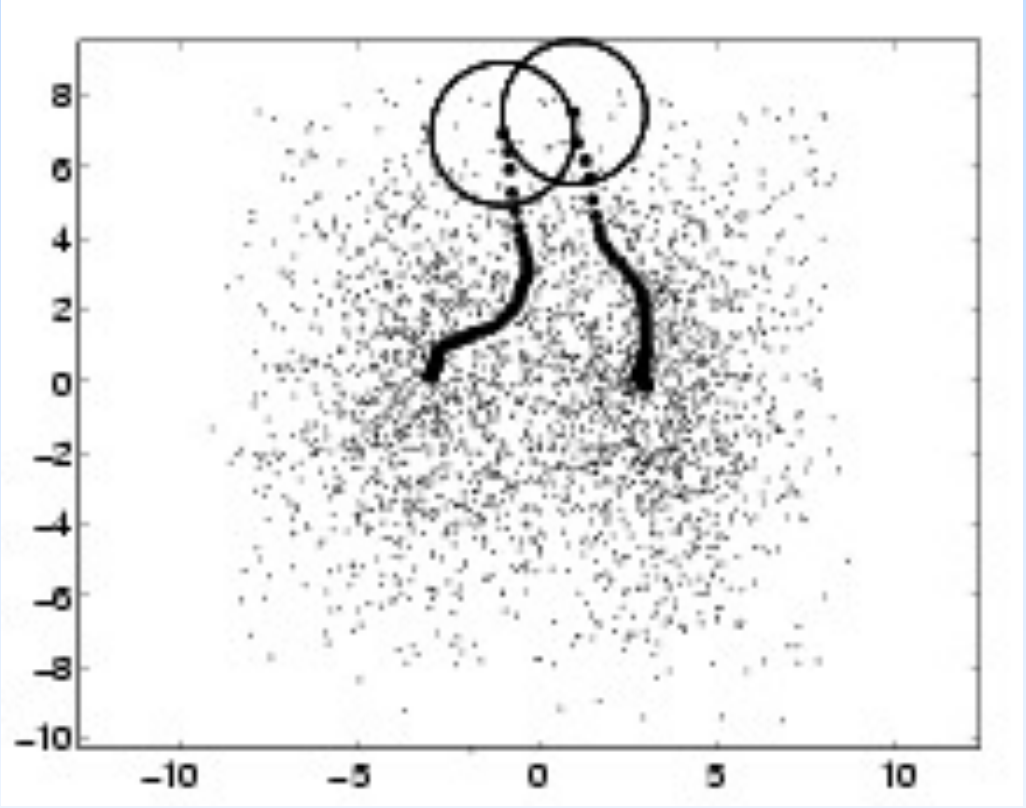


The blue data points were traversed by the windows towards the mode

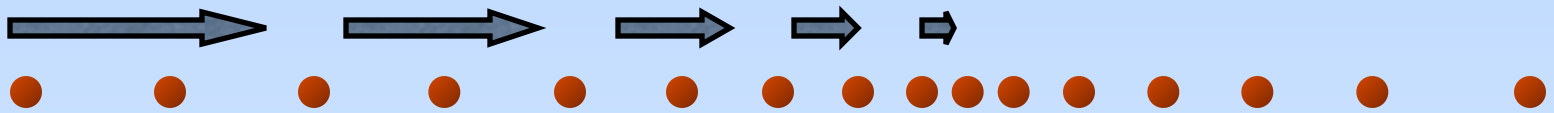
Real Modality Analysis

An example

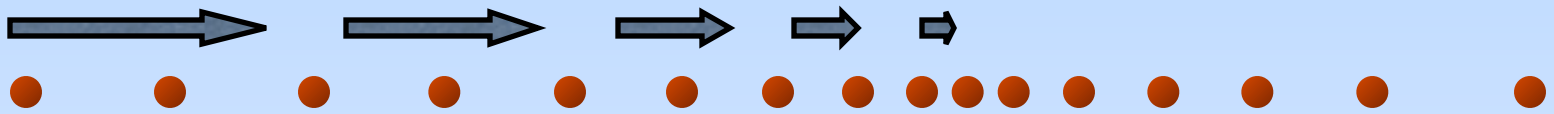


Window tracks signify the steepest ascent directions

Mean Shift Strengths & Weaknesses

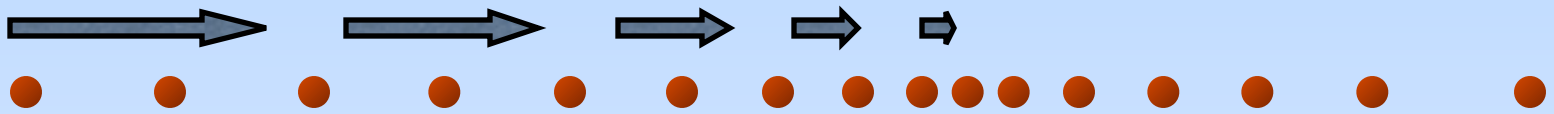


Mean Shift Strengths & Weaknesses



Strengths :

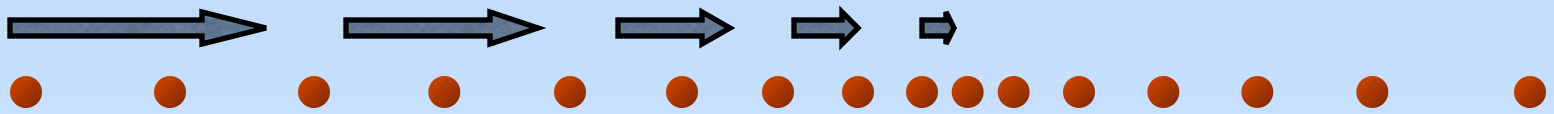
Mean Shift Strengths & Weaknesses



Strengths :

- Application independent tool

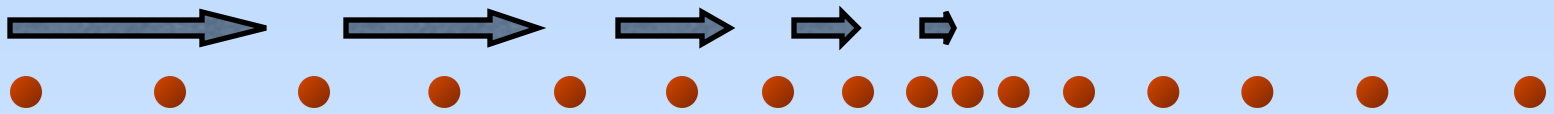
Mean Shift Strengths & Weaknesses



Strengths :

- Application independent tool
- Suitable for real data analysis

Mean Shift Strengths & Weaknesses



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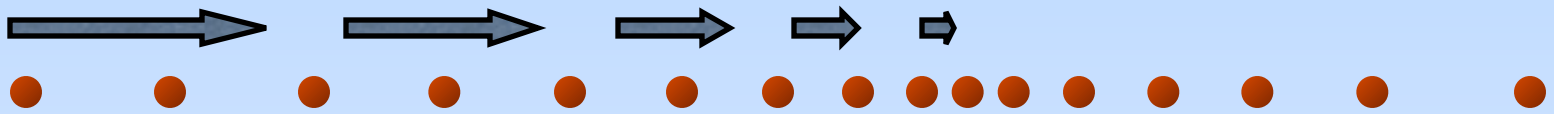

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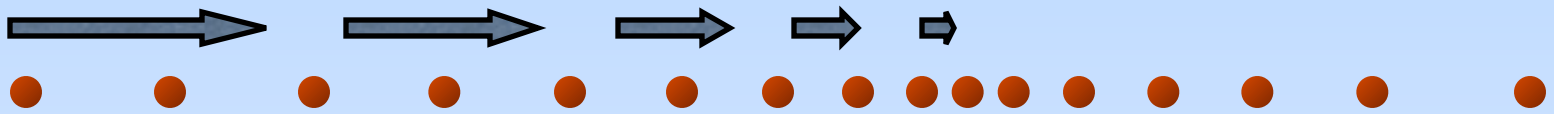

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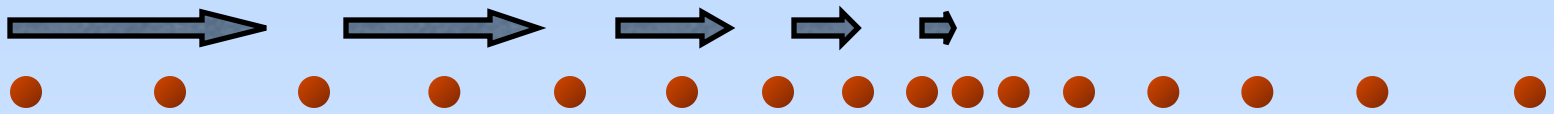
Mean Shift Strengths & Weaknesses



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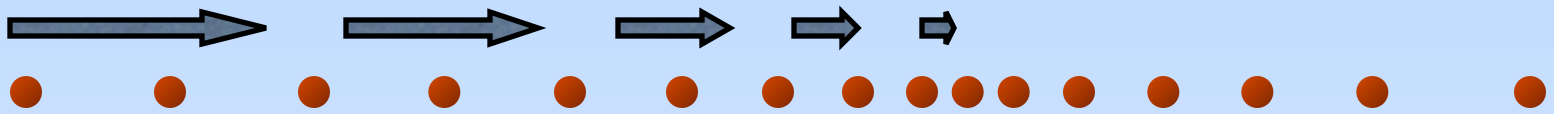
Mean Shift Strengths & Weaknesses



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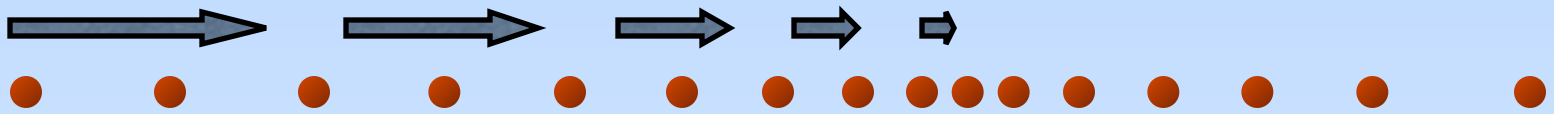
Mean Shift Strengths & Weaknesses



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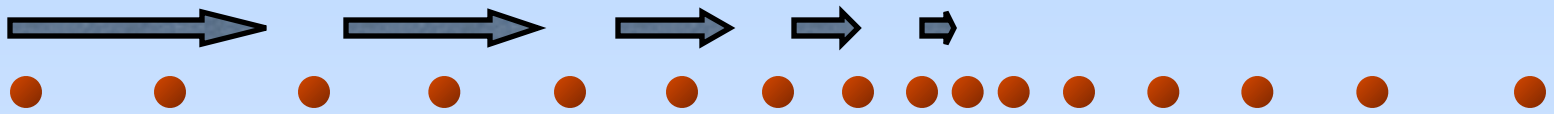
Mean Shift Strengths & Weaknesses



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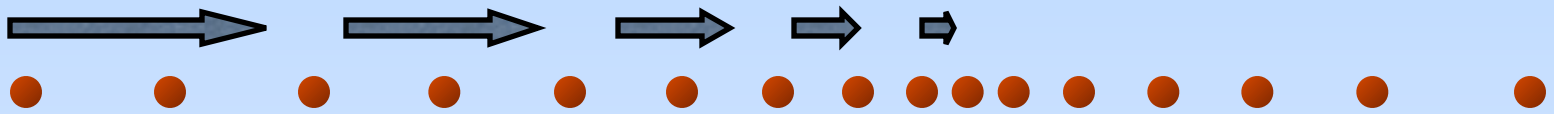
- Inappropriate window size can cause modes to be merged, or generate additional "shallow" modes

Mean Shift Strengths & Weaknesses



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- Only ONE parameter to choose
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Weaknesses :

- The window size (bandwidth selection) is not trivial
- Inappropriate window size can cause modes to be merged, or generate additional "shallow" modes
- Use adaptive window size

Mean Shift Applications

Clustering

Cluster : All data points in the ***attraction basin*** of a mode

Mean Shift : A robust Approach Toward Feature Space Analysis, by Comaniciu, Meer

Clustering

Cluster : All data points in the ***attraction basin*** of a mode

Attraction basin : the region for which all trajectories lead to the same mode

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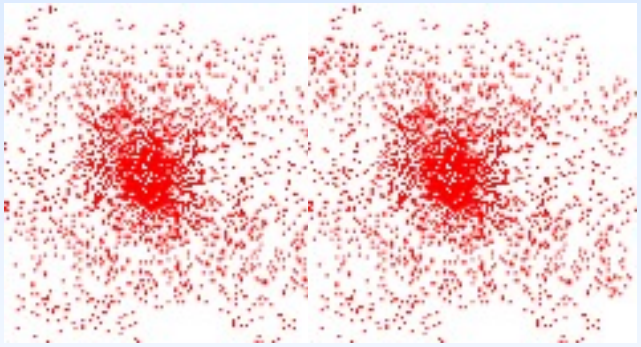

Mean Shift : A robust Approach Toward Feature Space Analysis, by Comaniciu, Meer

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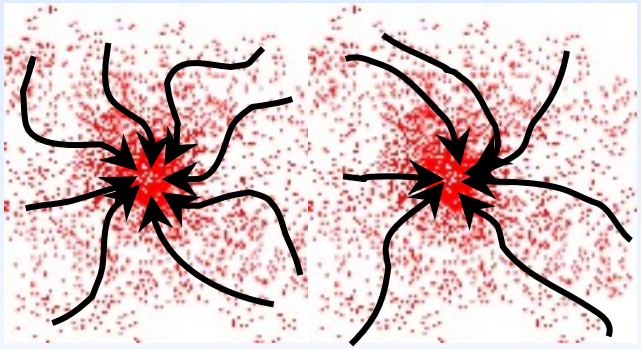

Mean Shift : A robust Approach Toward Feature Space Analysis, by Comaniciu, Meer

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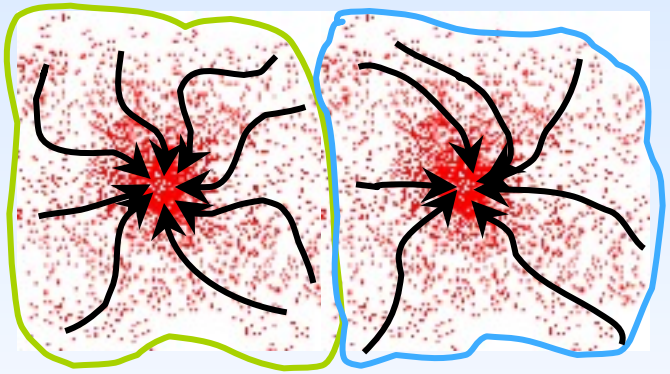
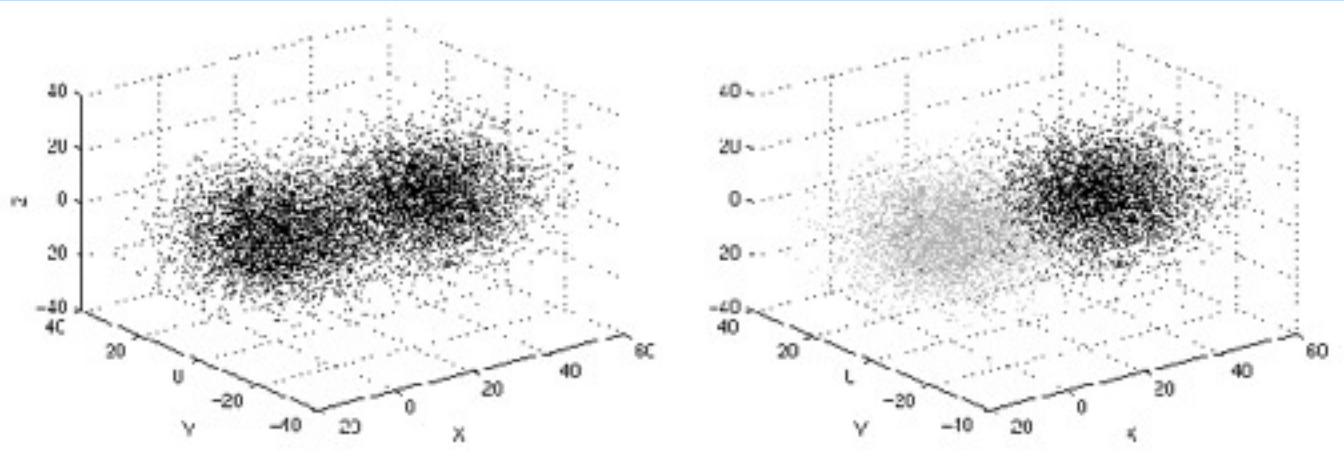


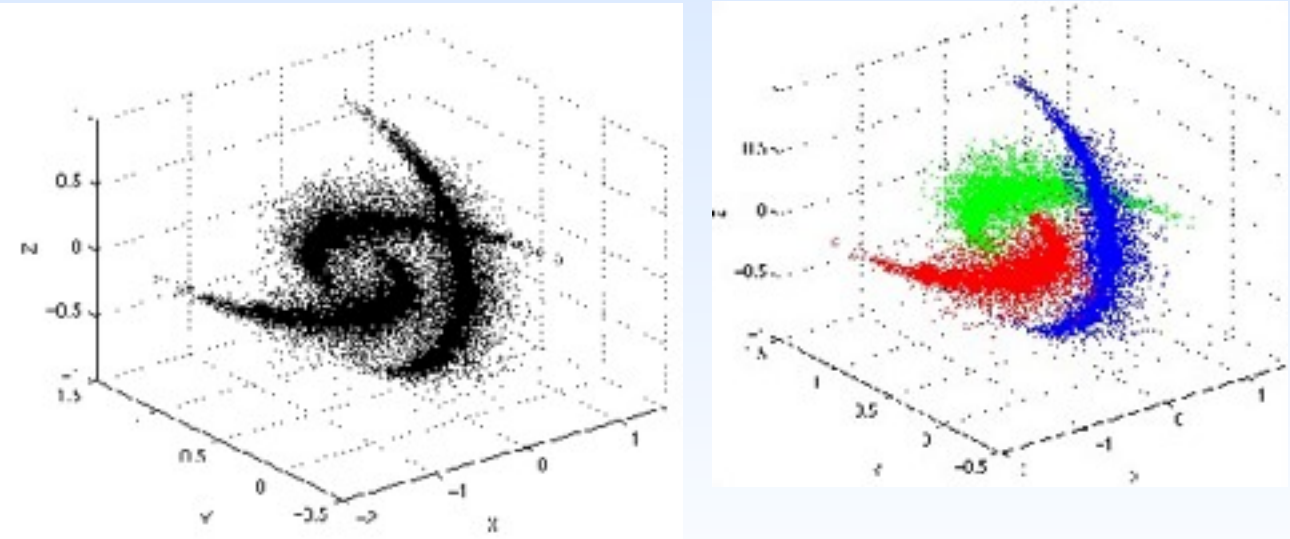
Mean Shift : A robust Approach Toward Feature Space Analysis, by Comaniciu, Meer

Clustering

Synthetic Examples

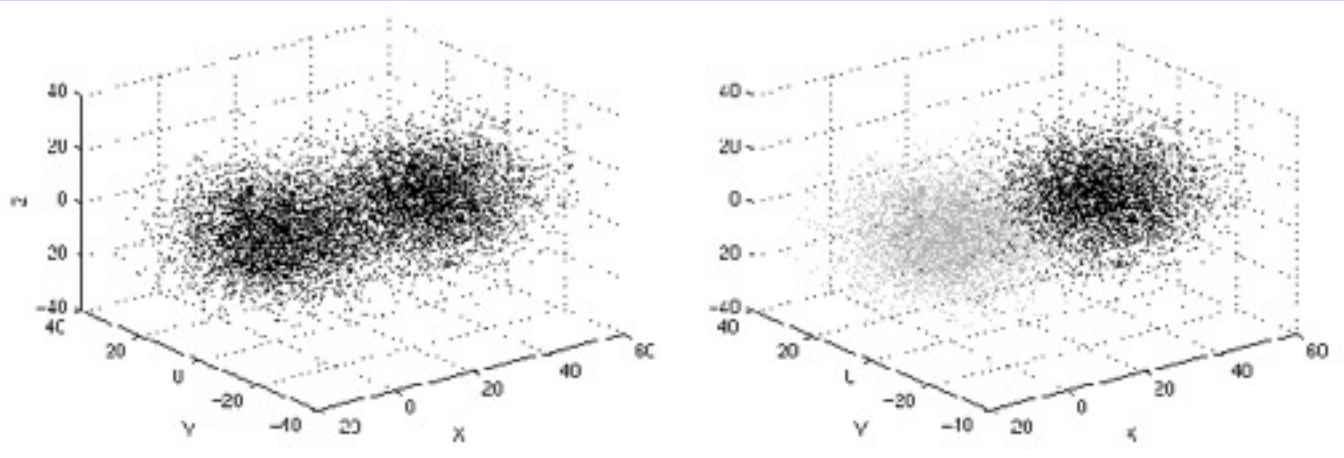


Simple Modal Structures



Clustering

Synthetic Examples



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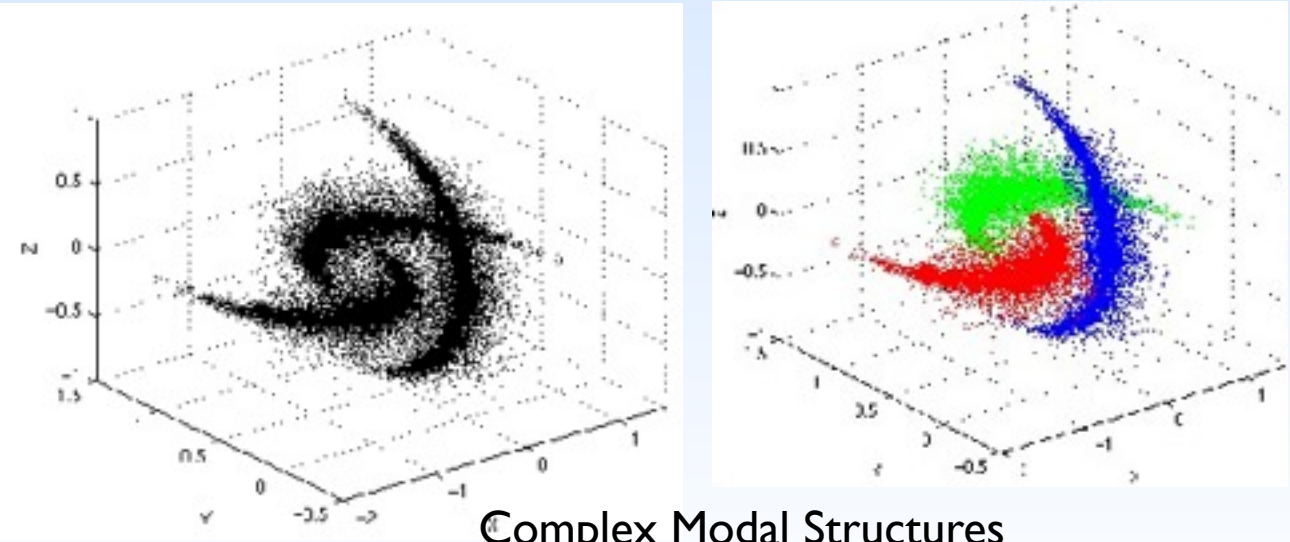
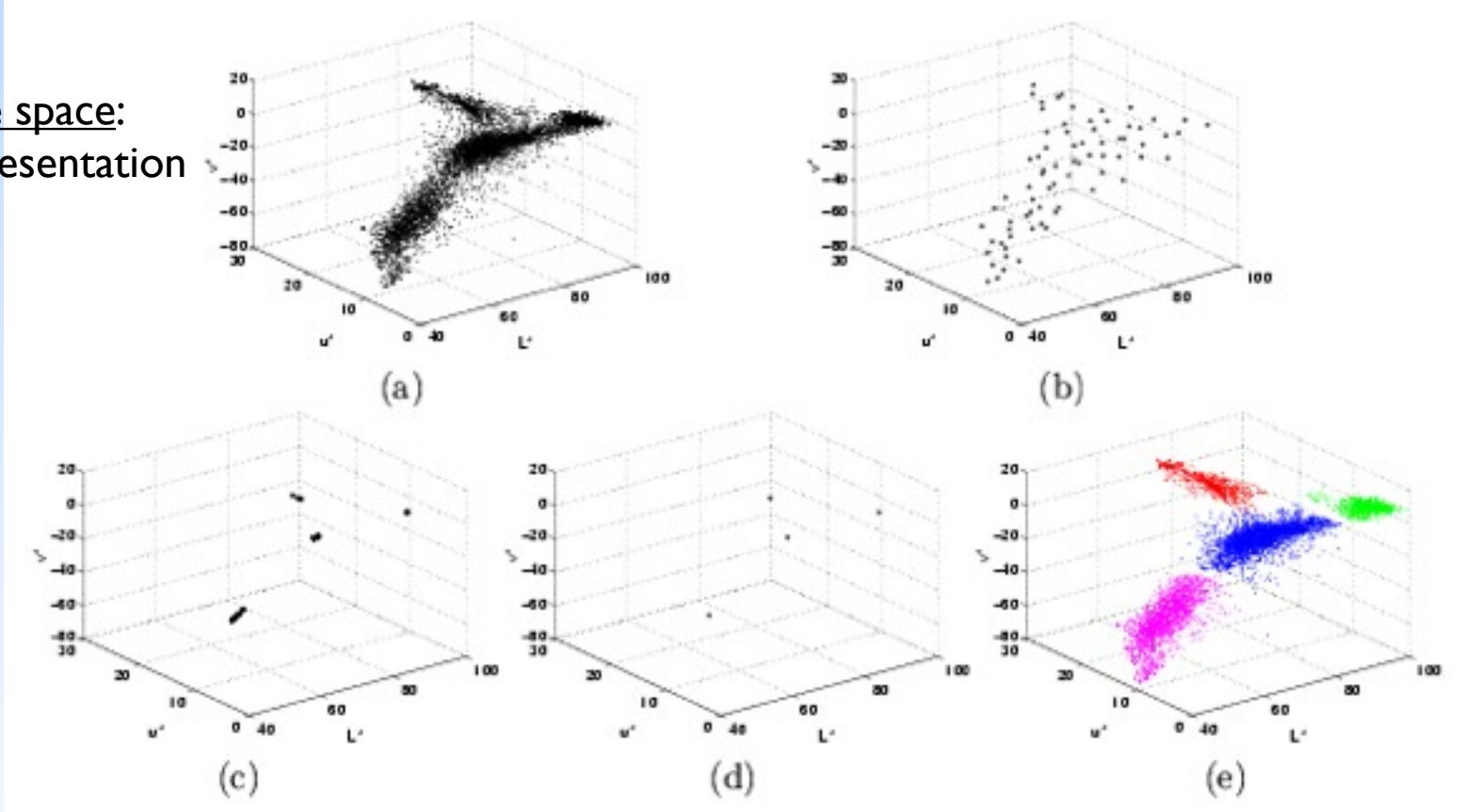


Complex Modal Structures

Clustering

Real Example

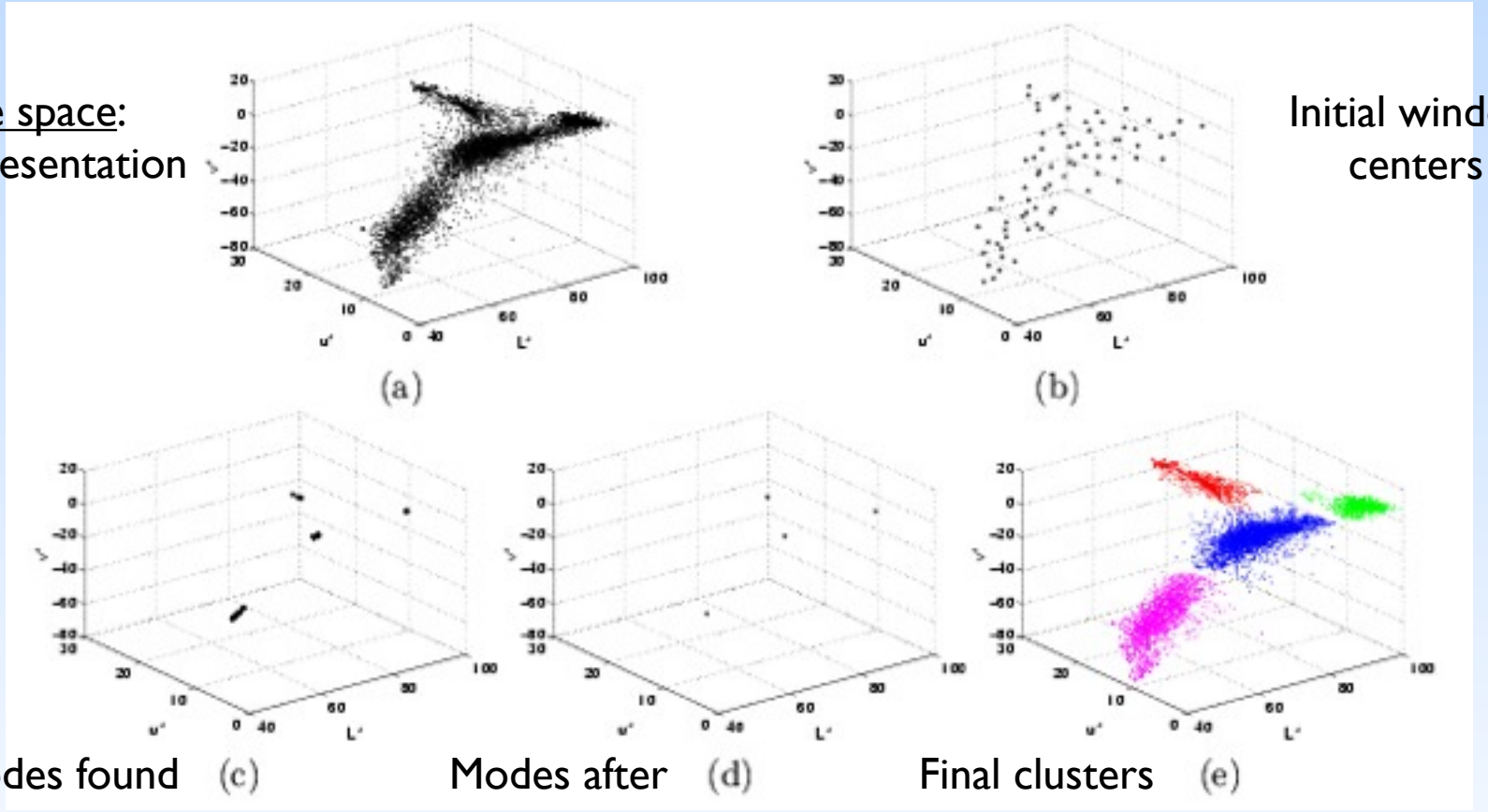
Feature space:
L*u*v representation



Clustering

Real Example

Feature space:
L*u*v representation



Initial window centers

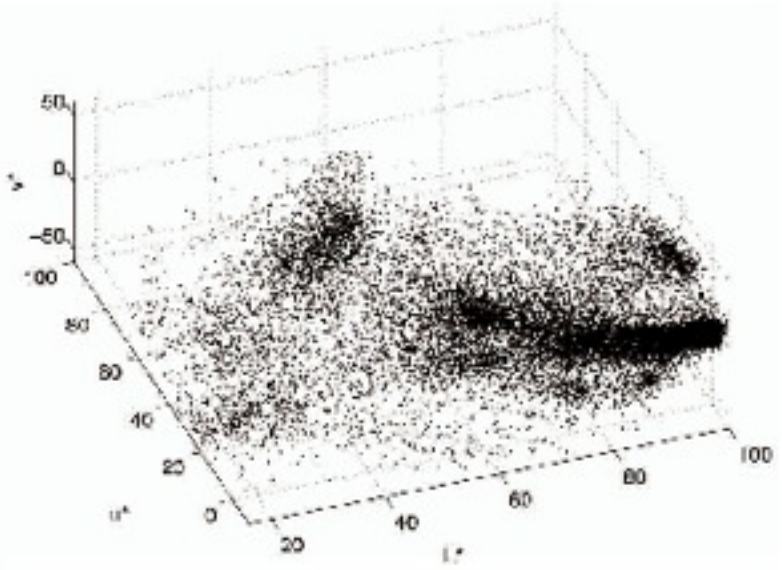
Modes found

Modes after pruning

Final clusters

Clustering

Real Example

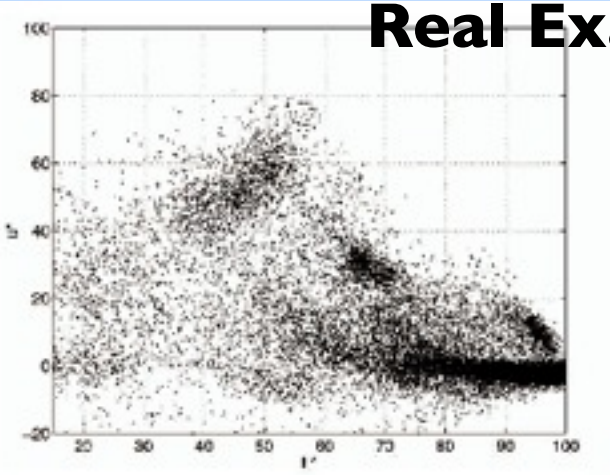


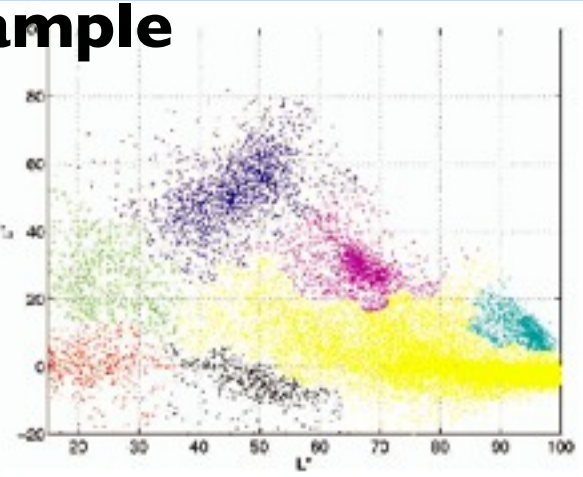
L*u*v space representation

Clustering

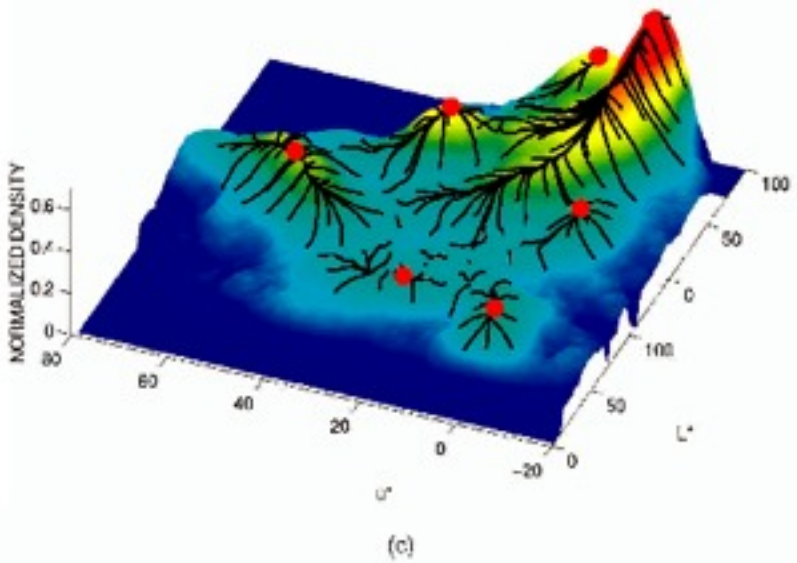
Real Example

2D (L*u) space representation





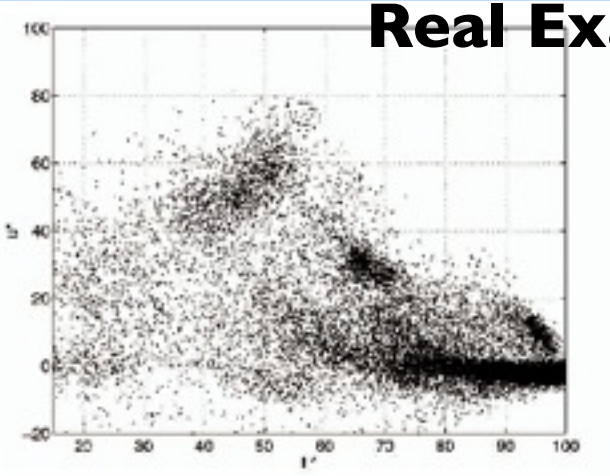
Final clusters

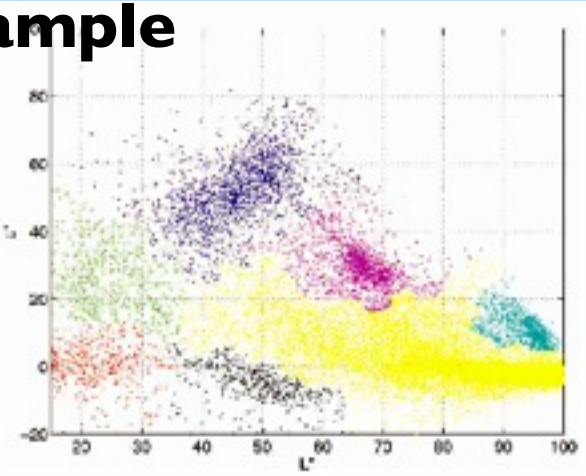


Clustering

Real Example

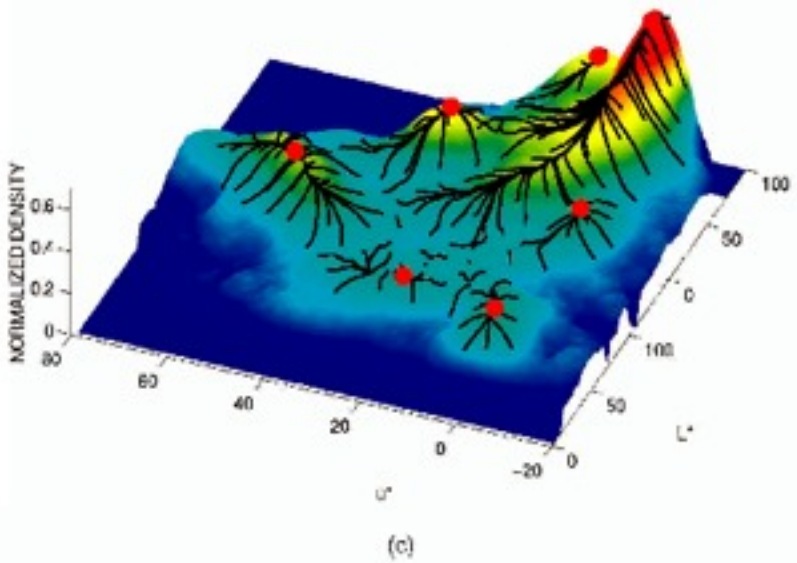
2D (L*u) space representation



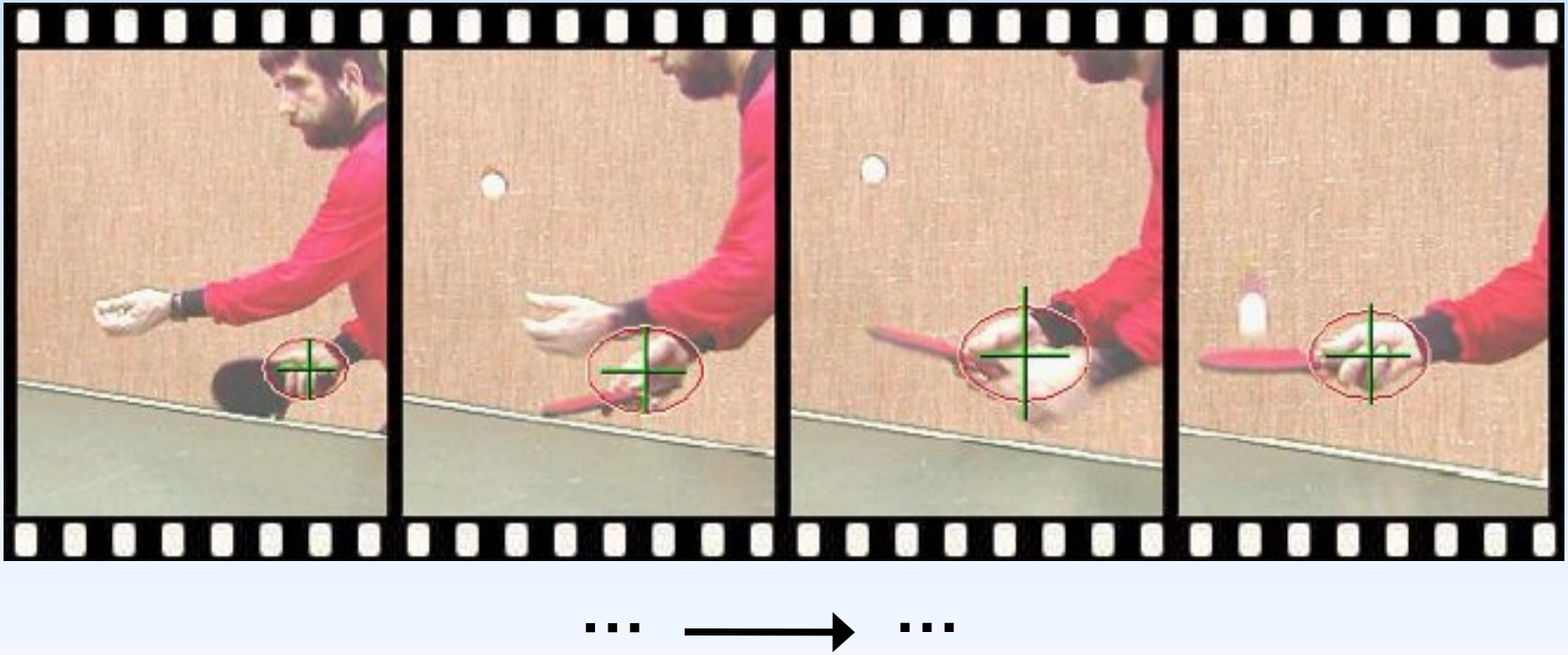


Final clusters

Not all trajectories in the attraction basin reach the same mode



Non-Rigid Object Tracking



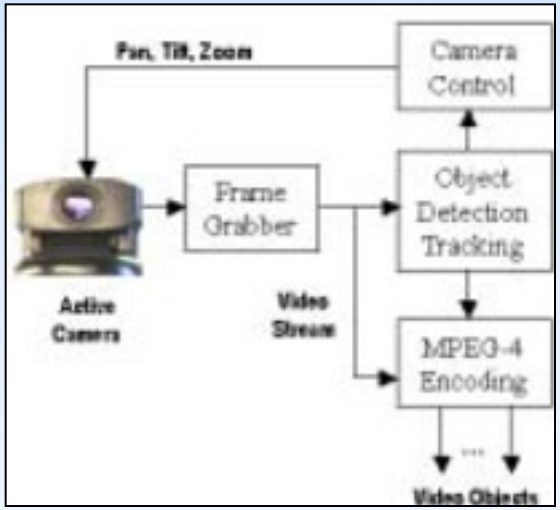
Non-Rigid Object Tracking

Real-Time

Surveillance

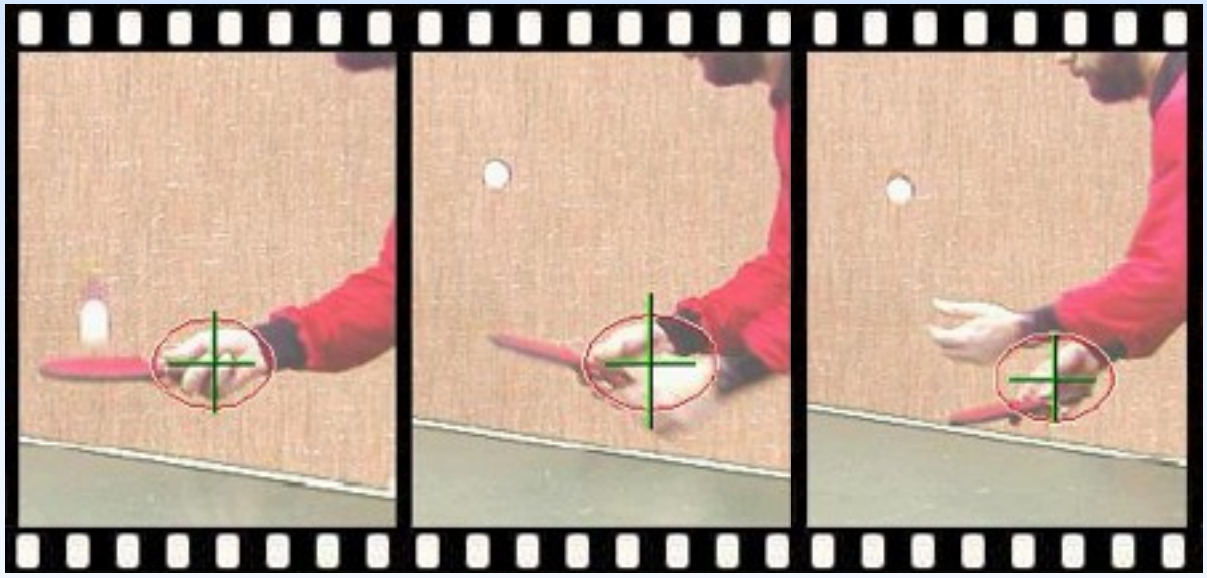
Driver Assistance

Object-Based Video Compression



Mean-Shift Object Tracking

General Framework: Target Representation

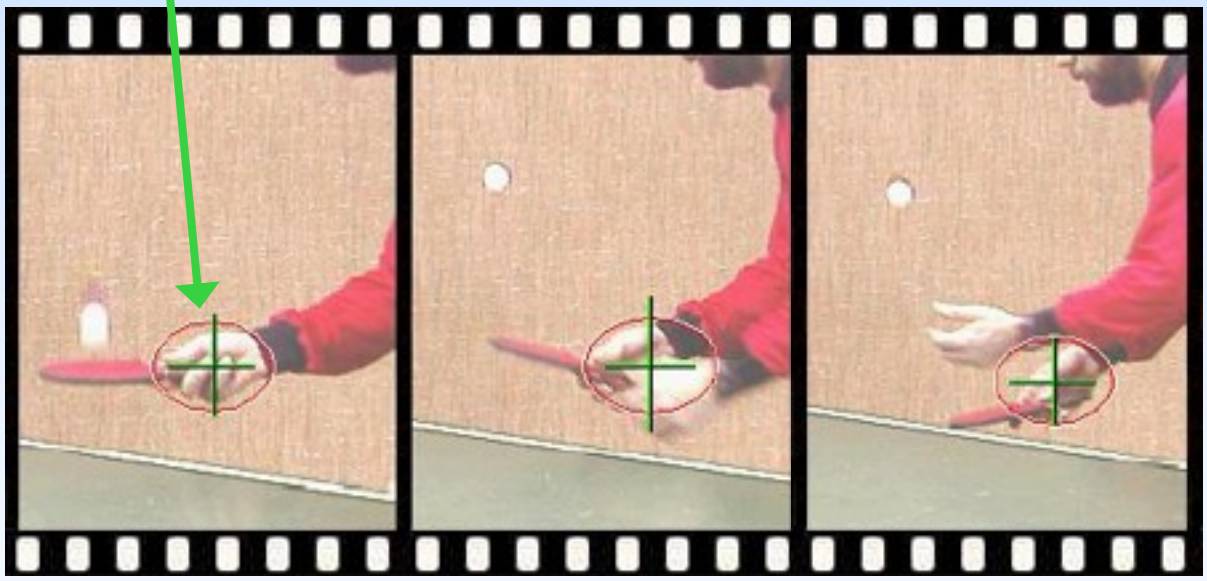


… Current frame → …

Mean-Shift Object Tracking

General Framework: Target Representation

Choose a reference model in the current frame



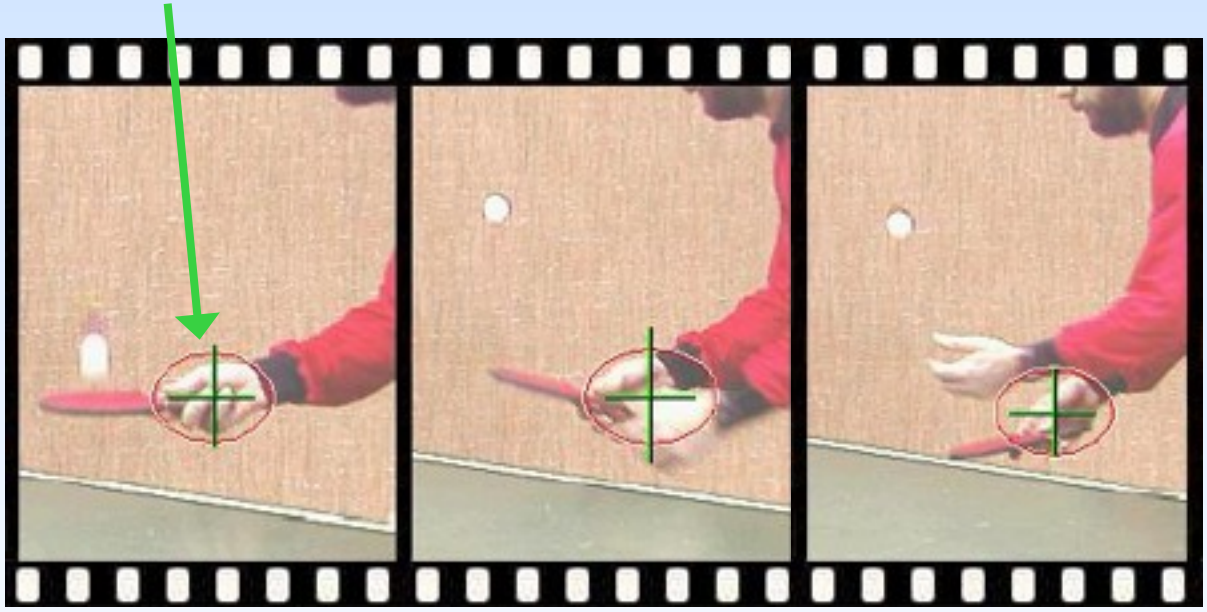
... Current frame → ...

Mean-Shift Object Tracking

General Framework: Target Representation

Choose a reference model in the current frame

Choose a feature space

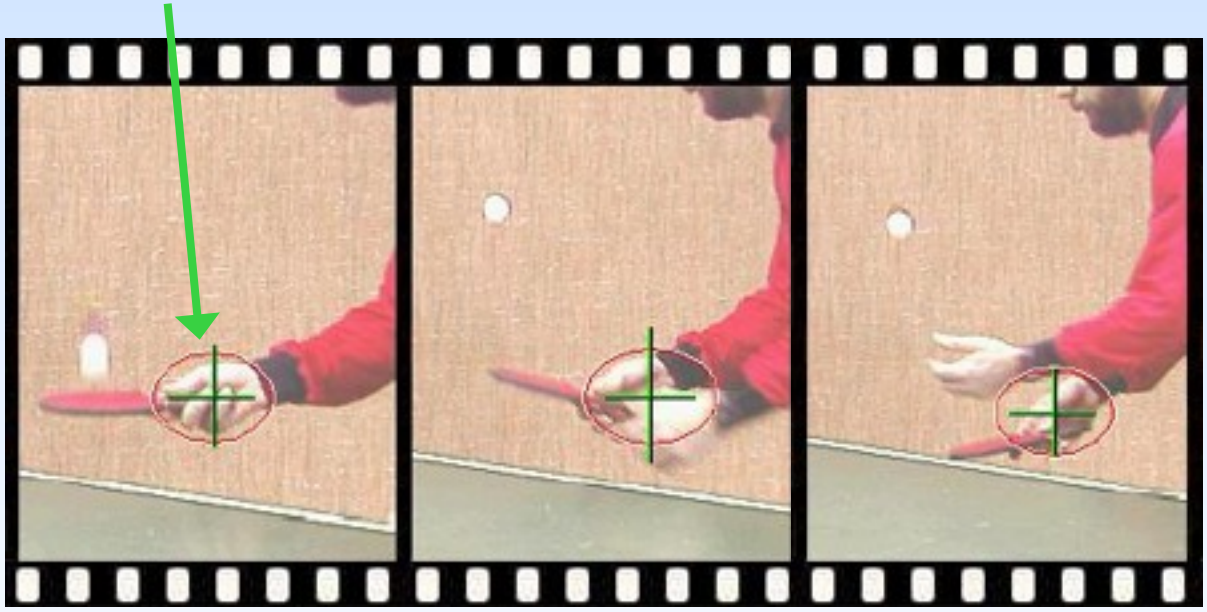


... Current frame ...

Mean-Shift Object Tracking

General Framework: Target Representation

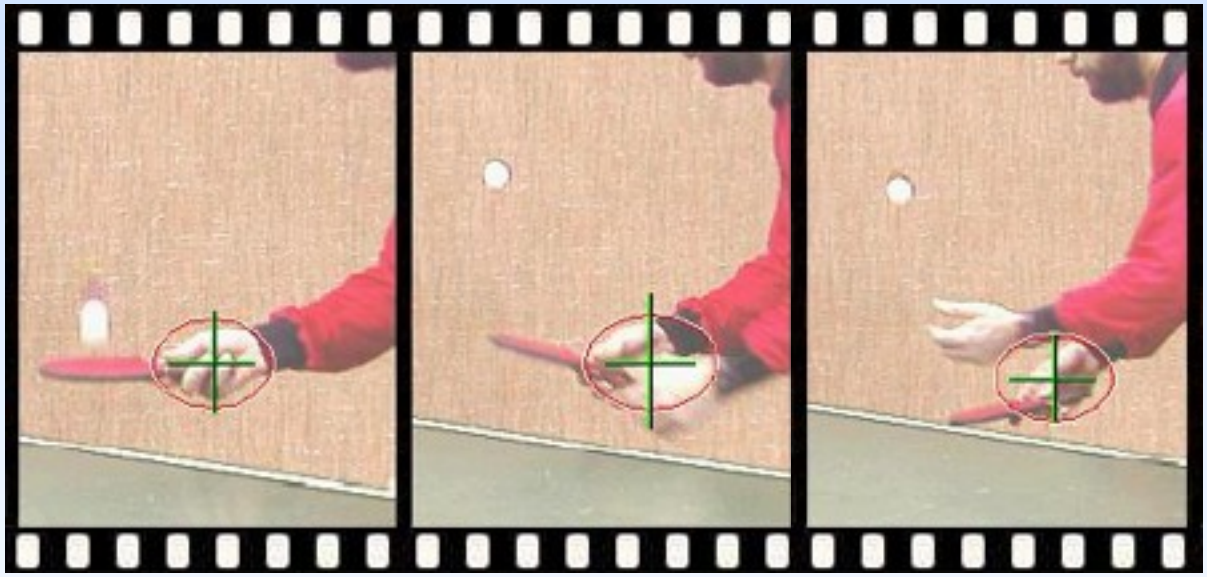
Choose a reference model in the current frame → Choose a feature space → Represent the model in the chosen feature space



… Current frame → …

Mean-Shift Object Tracking

General Framework: Target Representation



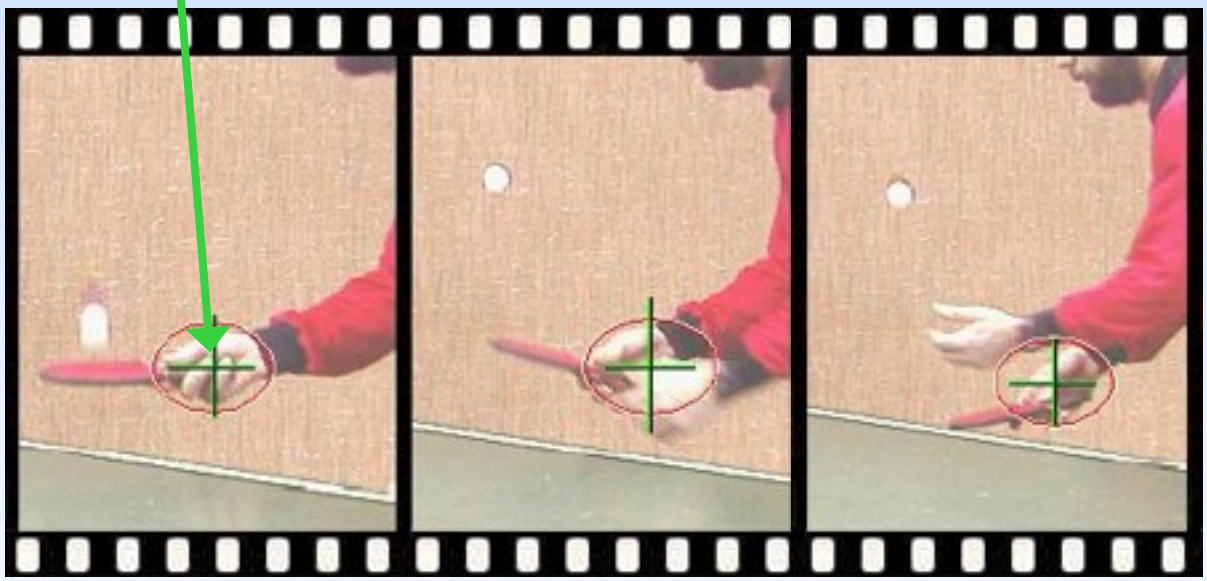
Model Candidate

… Current frame …

Mean-Shift Object Tracking

General Framework: Target Representation

Start from the position of the model in the current frame



Model Candidate

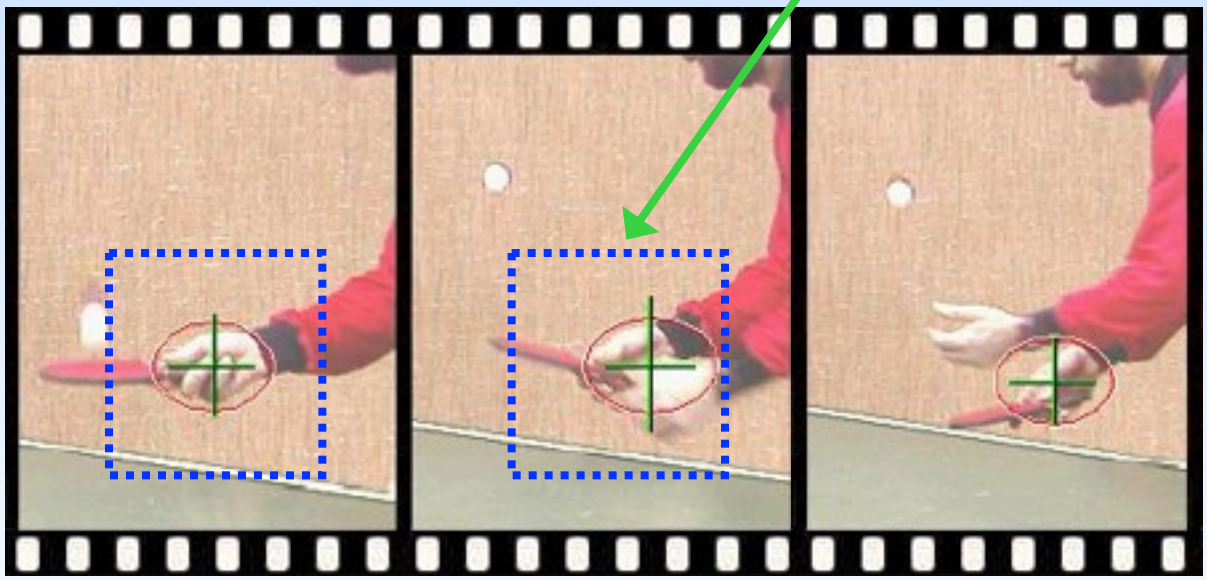
... Current frame ...

Mean-Shift Object Tracking

General Framework: Target Representation

Start from the position of the model in the current frame

Search in the model's neighborhood in next frame



Model

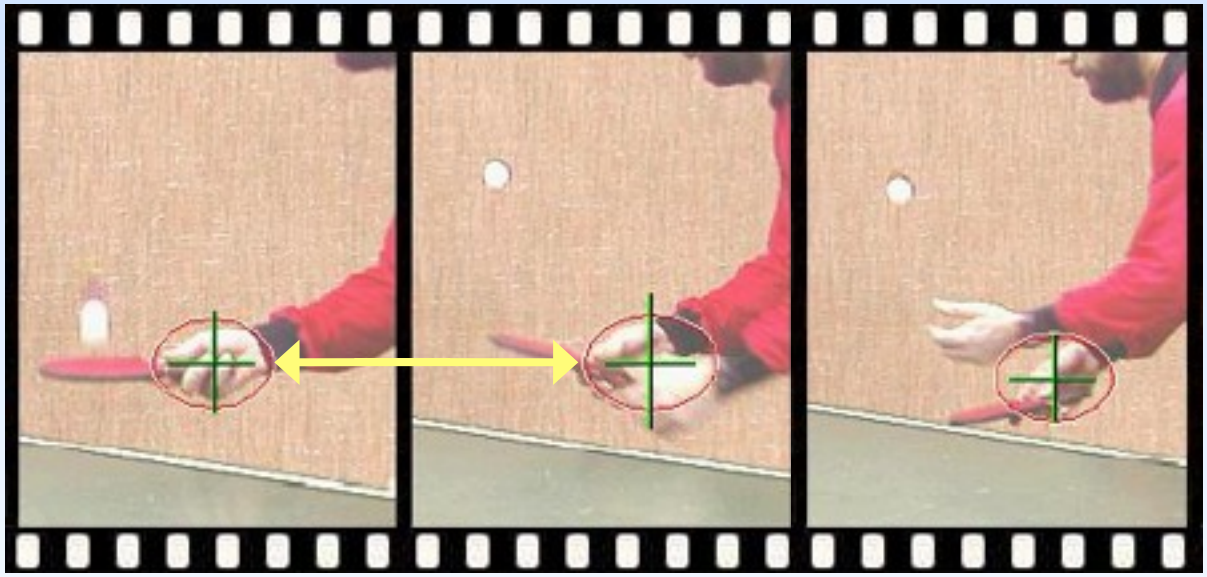
Candidate

... Current frame ...

Mean-Shift Object Tracking

General Framework: Target Representation

Start from the position of the model in the current frame → Search in the model's neighborhood in next frame → Find best candidate by maximizing a similarity func.



Model Candidate

... Current frame → ...

Mean-Shift Object Tracking

General Framework: Target Representation

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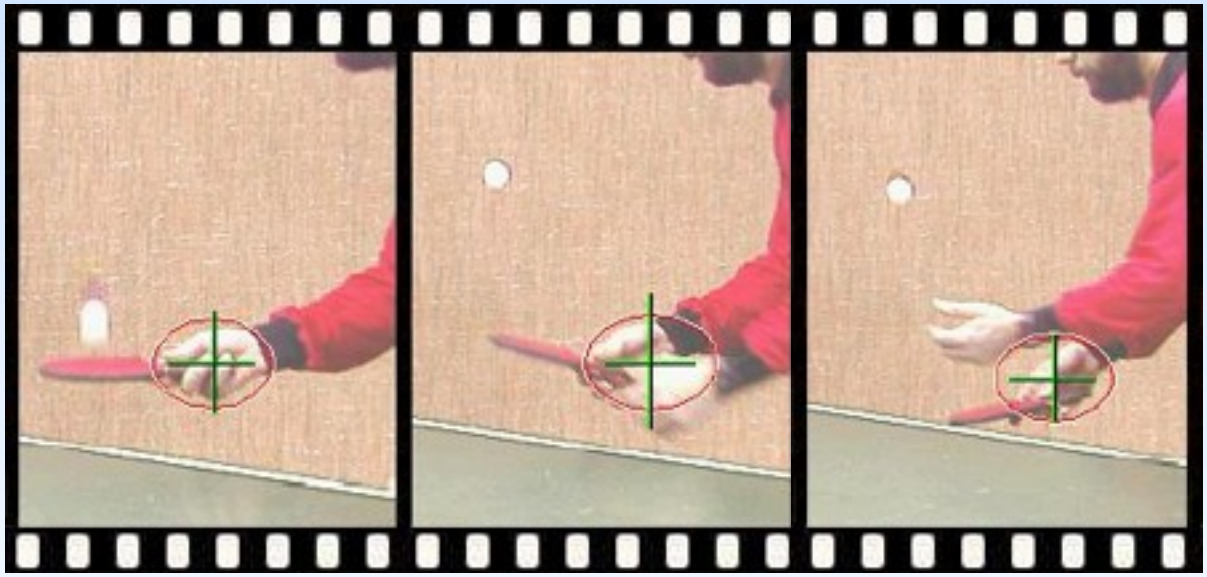


Model Candidate

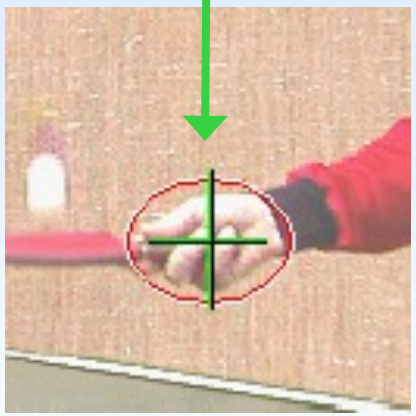
… Current frame → …

Repeat the same process in the next pair of frames

Mean-Shift Object Tracking

General Framework: Target Representation

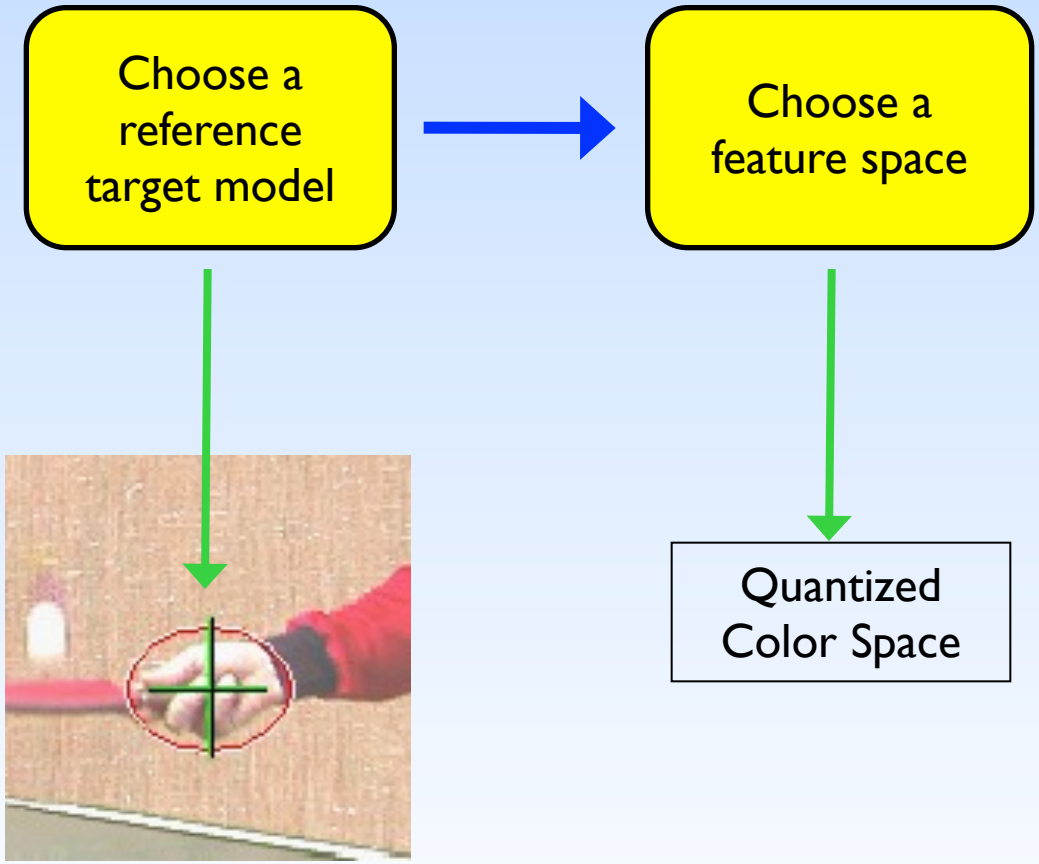
Choose a reference target model



Kernel Based Object Tracking, by Comaniniu, Ramesh, Meer

Mean-Shift Object Tracking

General Framework: Target Representation



Kernel Based Object Tracking, by Comaniniu, Ramesh, Meer

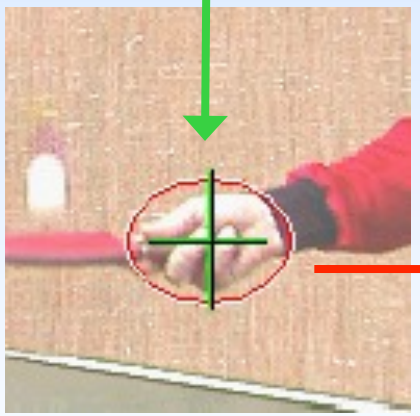
Mean-Shift Object Tracking

General Framework: Target Representation

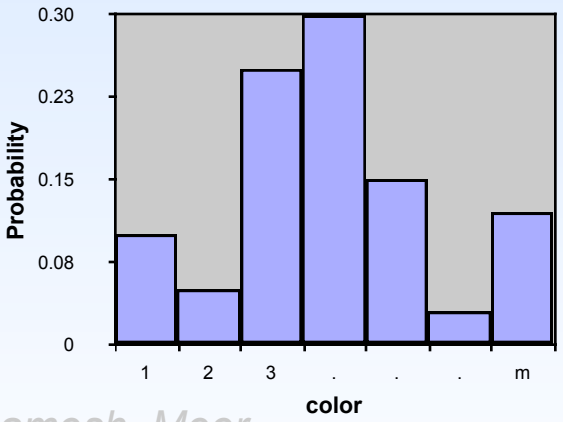
Choose a reference target model

Choose a feature space

Represent the model by its PDF in the feature space



Quantized Color Space

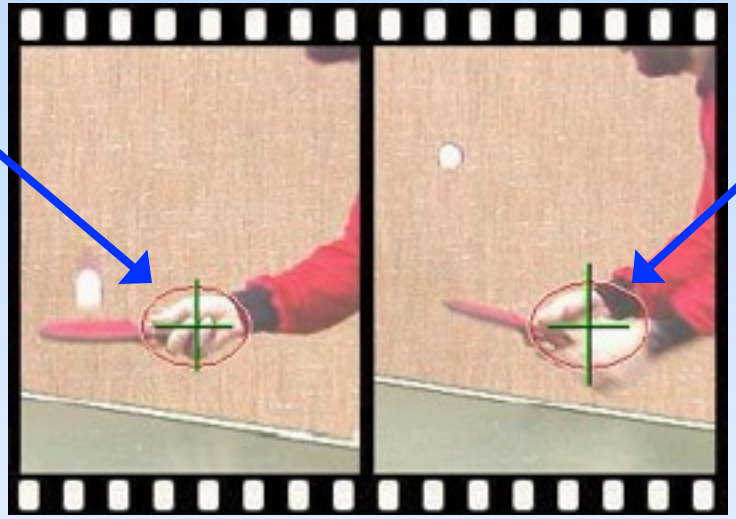


Kernel Based Object Tracking, by Comaniniu, Ramesh, Meer

Mean-Shift Object Tracking

PDF Representation

Target Model
(centered at 0)

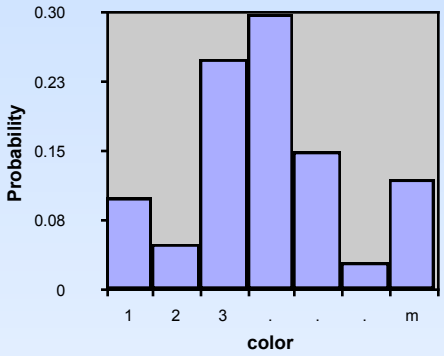


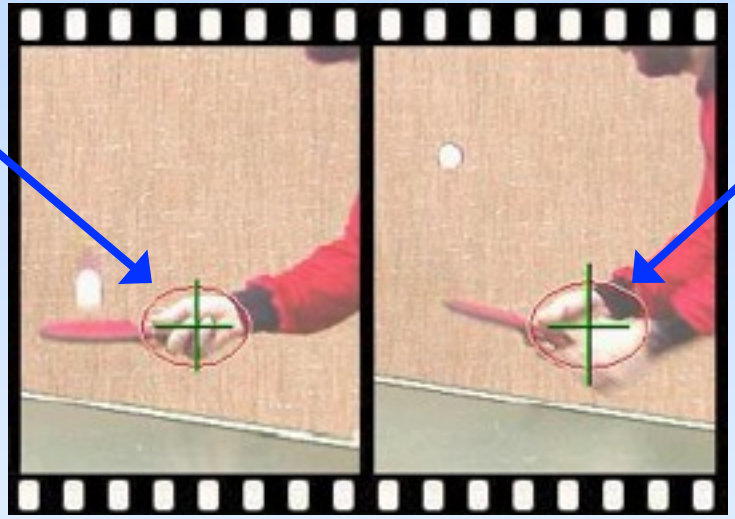
Target Candidate
(centered at y)

Mean-Shift Object Tracking

PDF Representation

Target Model
(centered at 0)





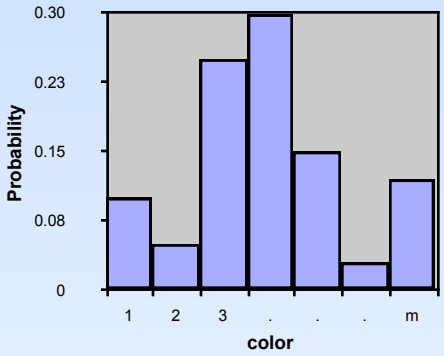
Target Candidate
(centered at y)

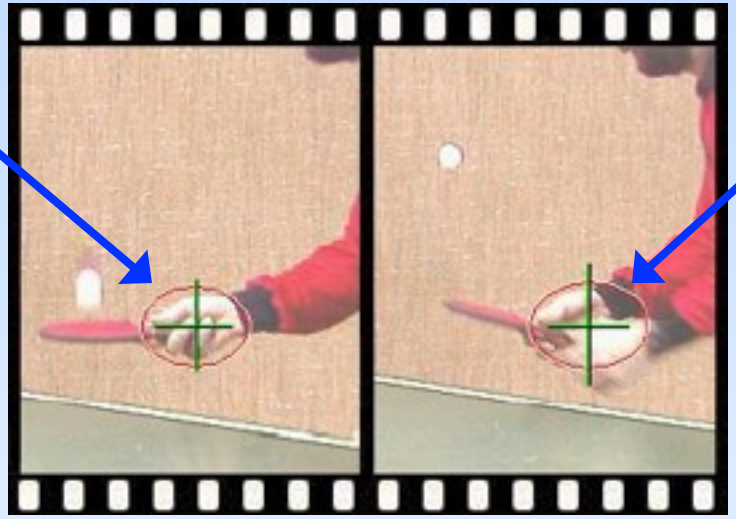
$$\vec{q} = \{q_u\}_{u=1..m} \qquad \sum_{u=1}^{m} q_u = 1$$

Mean-Shift Object Tracking

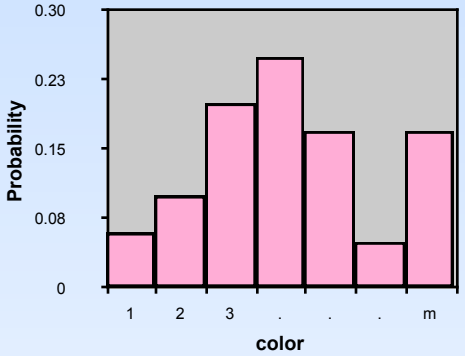
PDF Representation

Target Model
(centered at 0)





Target Candidate
(centered at y)



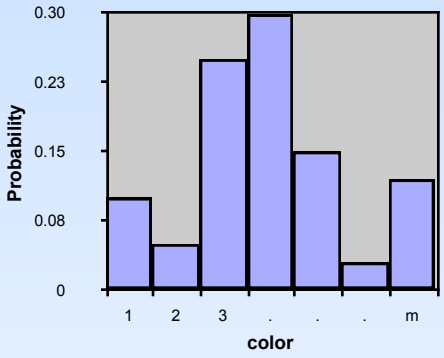
$$\vec{q} = \{q_u\}_{u=1..m} \qquad \sum_{u=1}^{m} q_u = 1$$

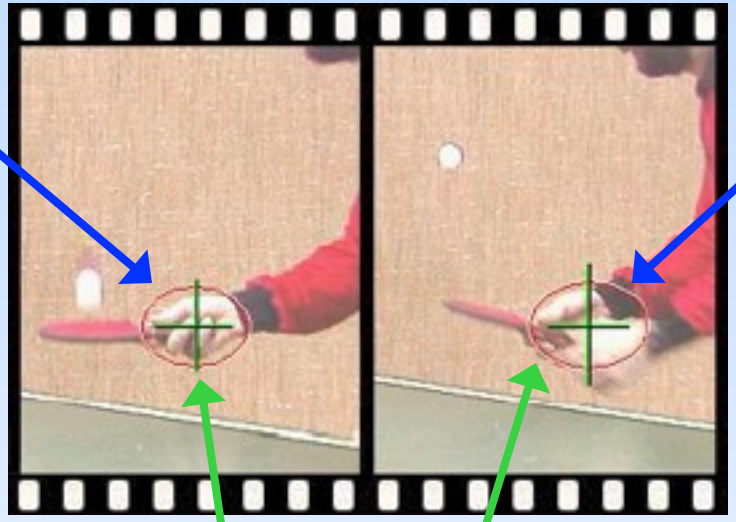
$$\vec{p}(y) = \{p_u(y)\}_{u=1..m} \qquad \sum_{u=1}^{m} p_u = 1$$

Mean-Shift Object Tracking

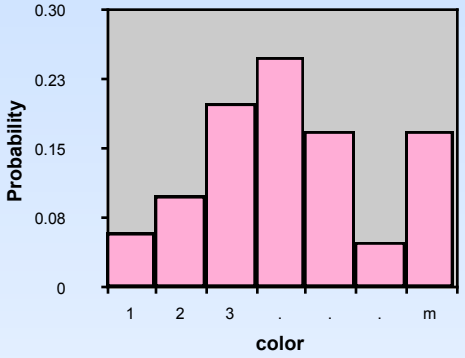
PDF Representation

Target Model
(centered at 0)





Target Candidate
(centered at y)



$$\vec{q} = \{q_u\}_{u=1..m} \qquad \sum_{u=1}^{m} q_u = 1$$

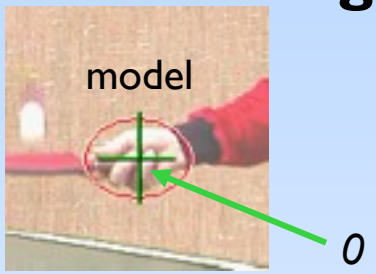
$$\vec{p}(y) = \{p_u(y)\}_{u=1..m} \qquad \sum_{u=1}^{m} p_u = 1$$

Similarity Function: $f(y) = f[\vec{q}, \vec{p}(y)]$

Mean-Shift Object Tracking

Finding the PDF of the target model

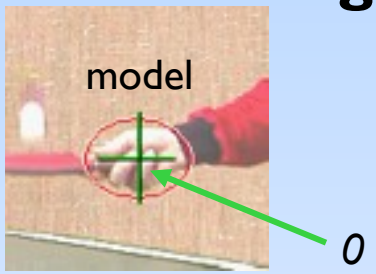
$\{x_i\}_{i=1..n}$ Target pixel locations



Mean-Shift Object Tracking

Finding the PDF of the target model

$\{x_i\}_{i=1..n}$ Target pixel locations



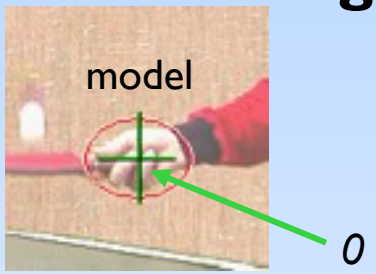
$k(x)$ A differentiable, isotropic, convex, monotonically decreasing kernel

- Peripheral pixels are affected by occlusion and background interference

Mean-Shift Object Tracking

Finding the PDF of the target model

$\{x_i\}_{i=1..n}$ Target pixel locations



$k(x)$ A differentiable, isotropic, convex, monotonically decreasing kernel

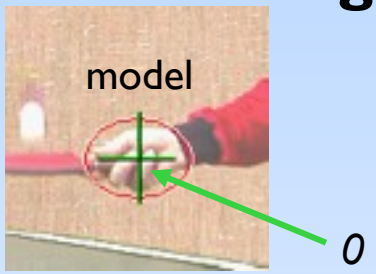
- Peripheral pixels are affected by occlusion and background interference

$b(x)$ The color bin index (1..*m*) of pixel *x*

Mean-Shift Object Tracking

Finding the PDF of the target model

$\{x_i\}_{i=1..n}$ Target pixel locations



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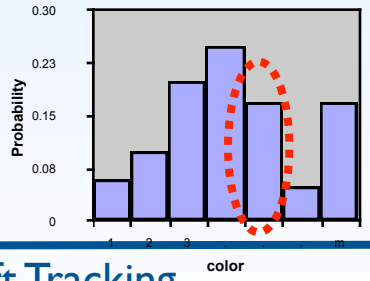
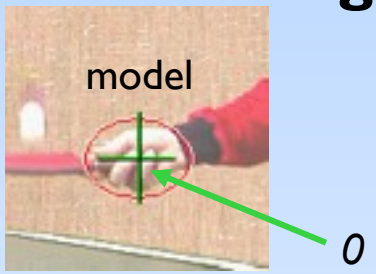
Probability of feature u in model



Mean-Shift Object Tracking

Finding the PDF of the target model

$\{x_i\}_{i=1..n}$ Target pixel locations



$k(x)$ A differentiable, isotropic, convex, monotonically decreasing kernel

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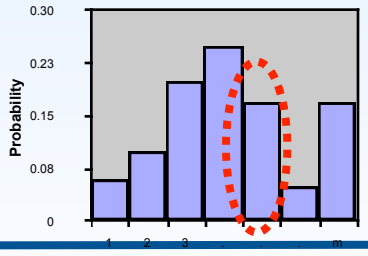
$b(x)$ The color bin index (1..*m*) of pixel *x*

Probability of feature u in model

$$q_u = C \sum_{b(x_i)=u} k\left(\|x_i\|^2\right)$$

Normalization factor

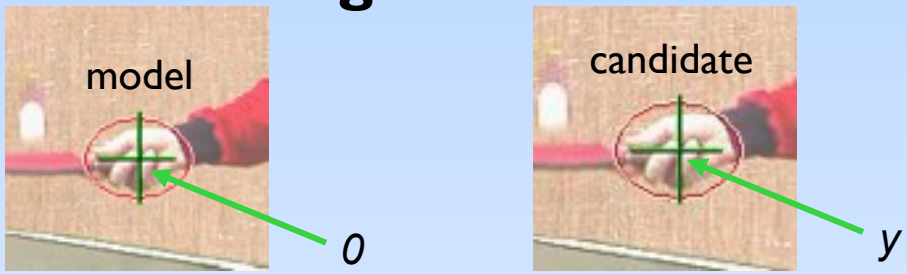
Pixel weight



Mean-Shift Object Tracking

Finding the PDF of the target model

$\{x_i\}_{i=1..n}$ Target pixel locations



$k(x)$ A differentiable, isotropic, convex, monotonically decreasing kernel

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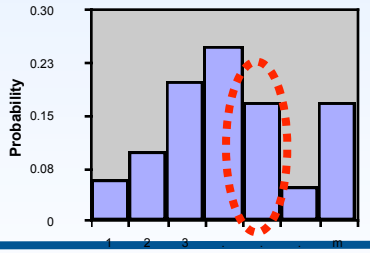
$b(x)$ The color bin index (1..*m*) of pixel *x*

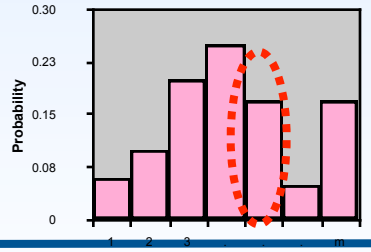
Probability of feature u in model **Probability of feature u in candidate**

$$q_u = C \sum_{b(x_i)=u} k\left(\|x_i\|^2\right)$$

Normalization factor

Pixel weight

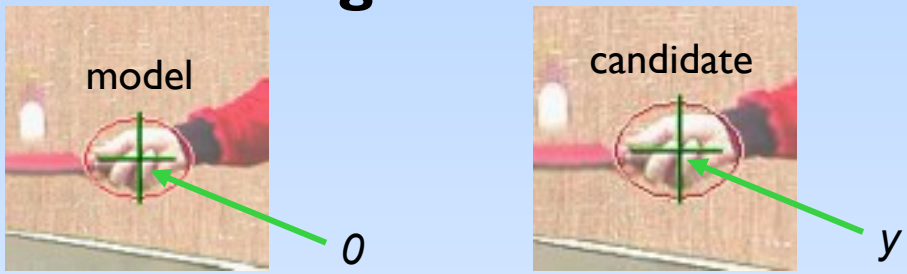




Mean-Shift Object Tracking

Finding the PDF of the target model

$\{x_i\}_{i=1..n}$ Target pixel locations



$k(x)$ A differentiable, isotropic, convex, monotonically decreasing kernel

- Peripheral pixels are affected by occlusion and background interference

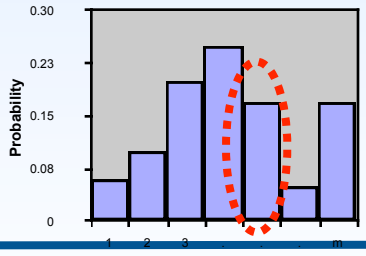
$b(x)$ The color bin index (1..*m*) of pixel *x*

Probability of feature u in model

$$q_u = C \sum_{b(x_i)=u} k\left(\|x_i\|^2\right)$$

Normalization factor

Pixel weight

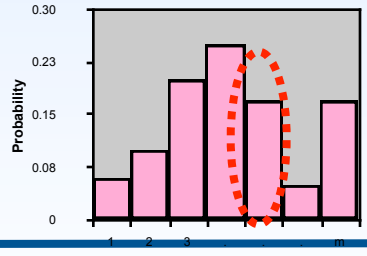


Probability of feature u in candidate

$$p_u(y) = C_h \sum_{b(x_i)=u} k\left(\left\|\frac{y - x_i}{h}\right\|^2\right)$$

Normalization factor

Pixel weight



Mean-Shift Object Tracking

Similarity Function

Target model: $\vec{q} = (q_1, \ldots, q_m)$

Target candidate: $\vec{p}(y) = (p_1(y), \ldots, p_m(y))$

Similarity function: $f(y) = f[\vec{p}(y), \vec{q}] = ?$

Mean-Shift Object Tracking

Similarity Function

Target model: $\vec{q} = (q_1, \ldots, q_m)$

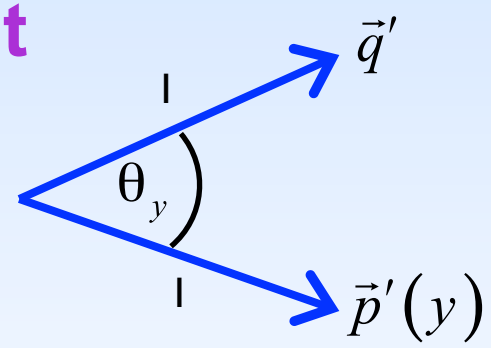
Target candidate: $\vec{p}(y) = (p_1(y), \ldots, p_m(y))$

Similarity function: $f(y) = f[\vec{p}(y), \vec{q}] = ?$

The Bhattacharyya Coefficient

$\vec{q}' = (\sqrt{q_1}, \ldots, \sqrt{q_m})$

$\vec{p}'(y) = (\sqrt{p_1(y)}, \ldots, \sqrt{p_m(y)})$



Mean-Shift Object Tracking

Similarity Function

Target model: $\vec{q} = (q_1, \ldots, q_m)$

Target candidate: $\vec{p}(y) = (p_1(y), \ldots, p_m(y))$

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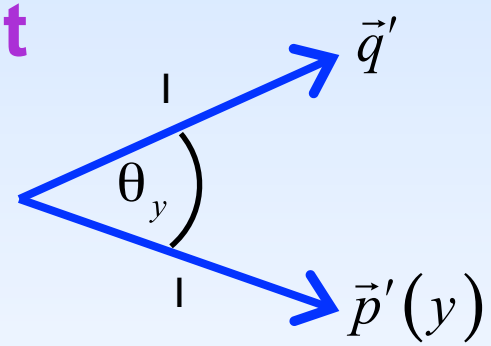
$$\vec{q}' = \left(\sqrt{q_1}, \ldots, \sqrt{q_m}\right)$$

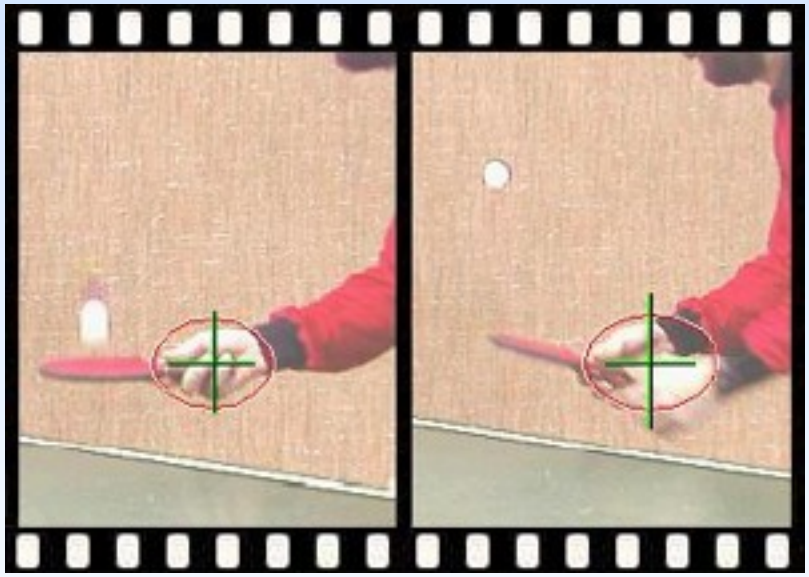
$$\vec{p}'(y) = \left(\sqrt{p_1(y)}, \ldots, \sqrt{p_m(y)}\right)$$



$$f(y) = \cos\theta_y = \frac{p'(y)^T q'}{\|p'(y)\| \times \|q'\|} = \sum_{u=1}^{m} \sqrt{p_u(y) q_u}$$

Mean-Shift Object Tracking

Target Localization Algorithm

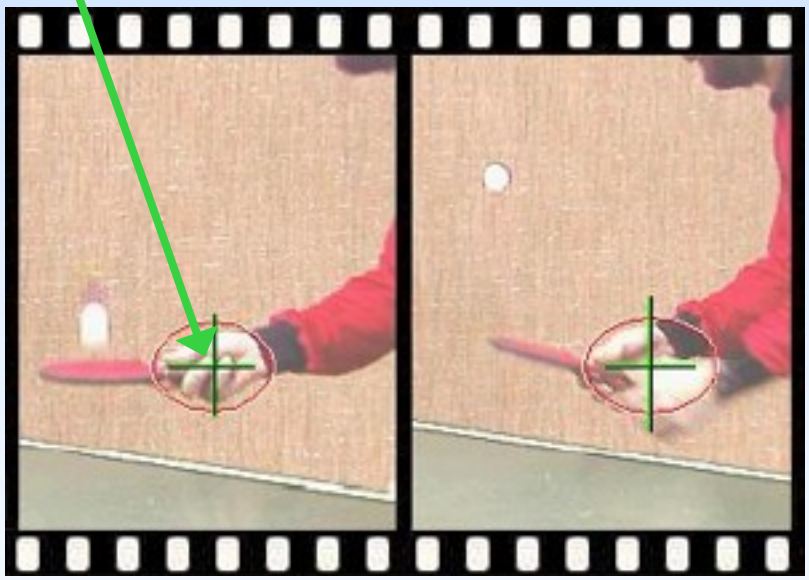


$\vec{q}$ $\vec{p}(y)$

Mean-Shift Object Tracking

Target Localization Algorithm

Start from the position of the model in the current frame



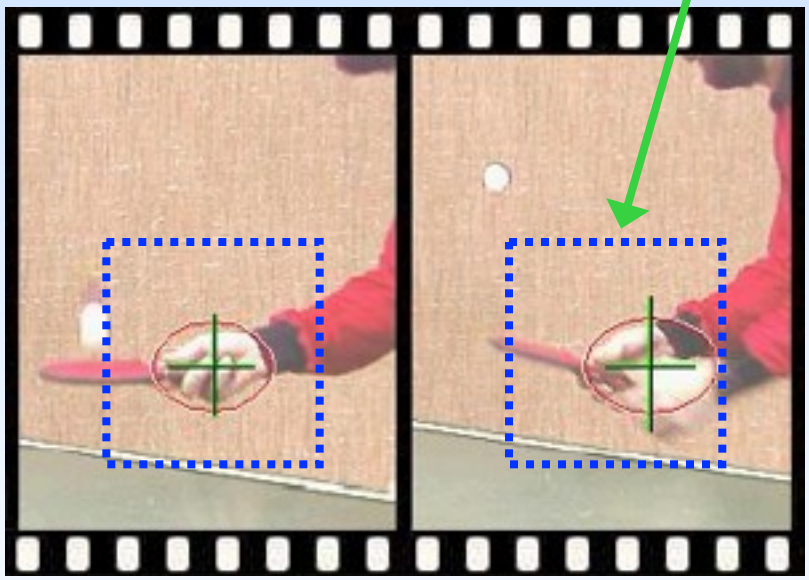
$\vec{q}$ $\vec{p}(y)$

Mean-Shift Object Tracking

Target Localization Algorithm

Start from the position of the model in the current frame

Search in the model's neighborhood in next frame



$\vec{q}$ $\vec{p}(y)$

Mean-Shift Object Tracking

Target Localization Algorithm

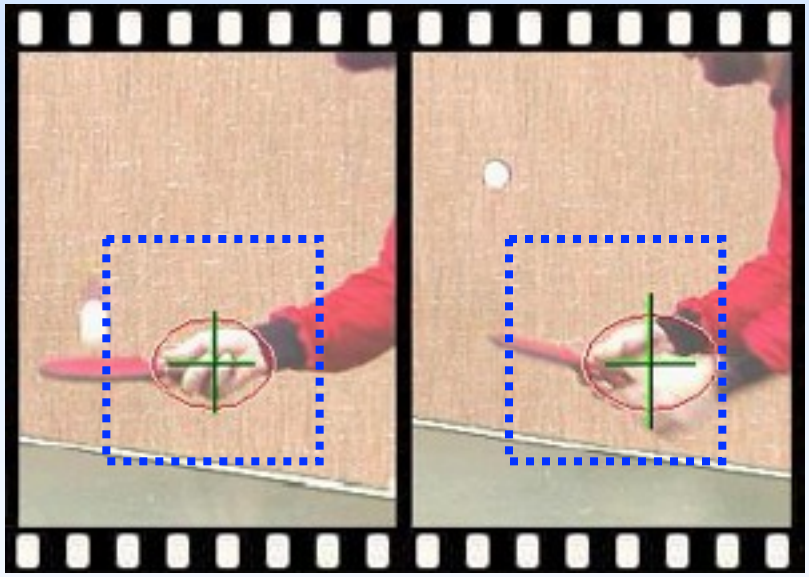
Start from the position of the model in the current frame

→

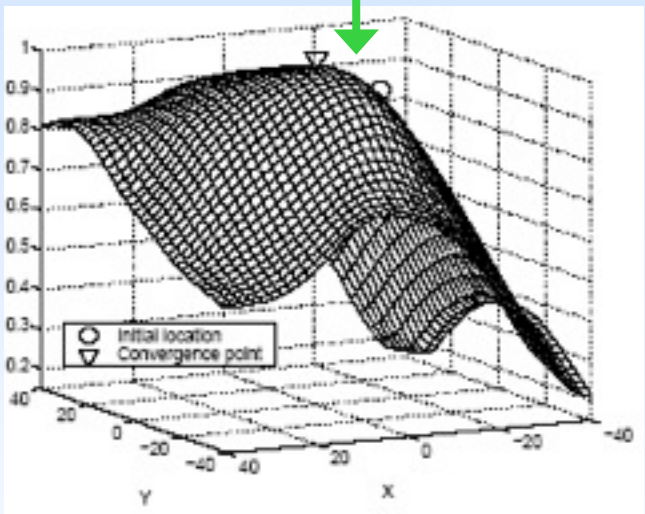
Search in the model's neighborhood in next frame

→

Find best candidate by maximizing a similarity func.



$\vec{q}$ $\vec{p}(y)$



$f\left[\vec{p}(y),\vec{q}\right]$

Mean-Shift Object Tracking

Approximating the Similarity Function

$$f(y) = \sum_{u=1}^{m} \sqrt{p_u(y) q_u}$$

Model location: y_0

Candidate location: y

Mean-Shift Object Tracking

Approximating the Similarity Function

$$f(y) = \sum_{u=1}^{m} \sqrt{p_u(y) q_u}$$

Model location: y_0

Candidate location: y

Linear approx. (around y_0)

$$f(y) \approx \frac{1}{2} \sum_{u=1}^{m} \sqrt{p_u(y_0) q_u} + \frac{1}{2} \sum_{u=1}^{m} p_u(y) \sqrt{\frac{q_u}{p_u(y_0)}}$$

Mean-Shift Object Tracking

Approximating the Similarity Function

$$f(y) = \sum_{u=1}^{m} \sqrt{p_u(y) q_u}$$

Model location: y_0

Candidate location: y

Linear approx. (around y_0)

$$f(y) \approx \underbrace{\frac{1}{2} \sum_{u=1}^{m} \sqrt{p_u(y_0) q_u}}_{\text{Independent of } y} + \frac{1}{2} \sum_{u=1}^{m} p_u(y) \sqrt{\frac{q_u}{p_u(y_0)}}$$

Mean-Shift Object Tracking

Approximating the Similarity Function

$$f(y) = \sum_{u=1}^{m} \sqrt{p_u(y) q_u}$$

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Mean-Shift Object Tracking

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$$p_u(y) = C_h \sum_{b(x_i)=u} k\left(\left\| \frac{y - x_i}{h} \right\|^2 \right)$$

Mean-Shift Object Tracking

Approximating the Similarity Function

$$f(y) = \sum_{u=1}^{m} \sqrt{p_u(y) q_u}$$

Model location: y_0

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$$\frac{C_h}{2} \sum_{i=1}^{n} w_i k\left(\left\| \frac{y - x_i}{h} \right\|^2 \right)$$

Mean-Shift Object Tracking

Approximating the Similarity Function

$$f(y) = \sum_{u=1}^{m} \sqrt{p_u(y) q_u}$$

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Linear approx. (around y_0)

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Mean-Shift Object Tracking

Approximating the Similarity Function

$$f(y) = \sum_{u=1}^{m} \sqrt{p_u(y) q_u}$$

Model location: y_0

Candidate location: y

Linear approx. (around y_0)

$$f(y) \approx \underbrace{\frac{1}{2} \sum_{u=1}^{m} \sqrt{p_u(y_0) q_u}}_{\text{Independent of } y} + \underbrace{\frac{1}{2} \sum_{u=1}^{m} p_u(y) \sqrt{\frac{q_u}{p_u(y_0)}}}$$

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Mean-Shift Object Tracking

Approximating the Similarity Function

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Linear approx. (around y_0)

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$$\frac{C_h}{2} \sum_{i=1}^{n} w_i k\left(\left\| \frac{y - x_i}{h} \right\|^2 \right)$$

Density estimate! (as a function of y)

Mean-Shift Object Tracking

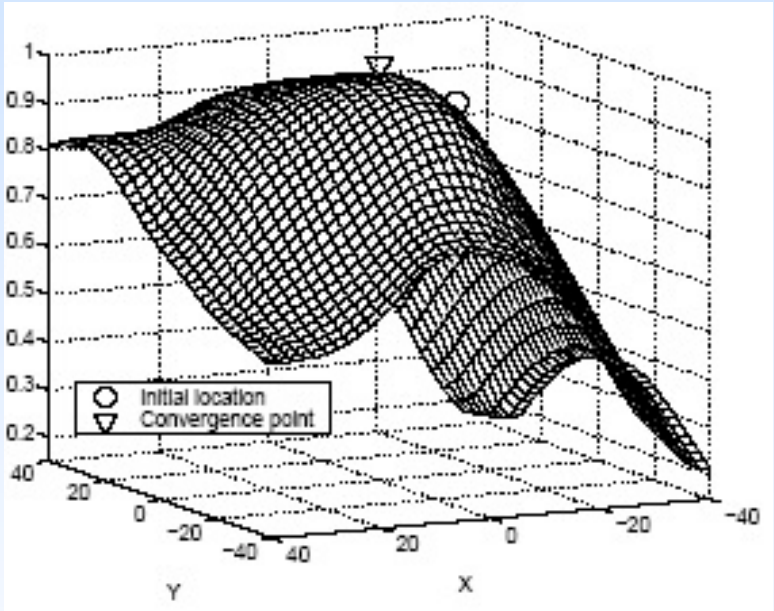
Maximizing the Similarity Function

The mode of $\frac{C_h}{2}\sum_{i=1}^{n} w_i k\left(\left\|\frac{y-x_i}{h}\right\|^2\right)$ = sought maximum

Mean-Shift Object Tracking

Maximizing the Similarity Function

The mode of $\frac{C_h}{2}\sum_{i=1}^{n} w_i k\left(\left\|\frac{y-x_i}{h}\right\|^2\right)$ = sought maximum

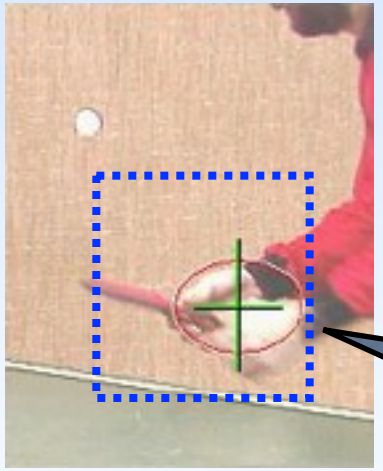


Mean-Shift Object Tracking

Maximizing the Similarity Function

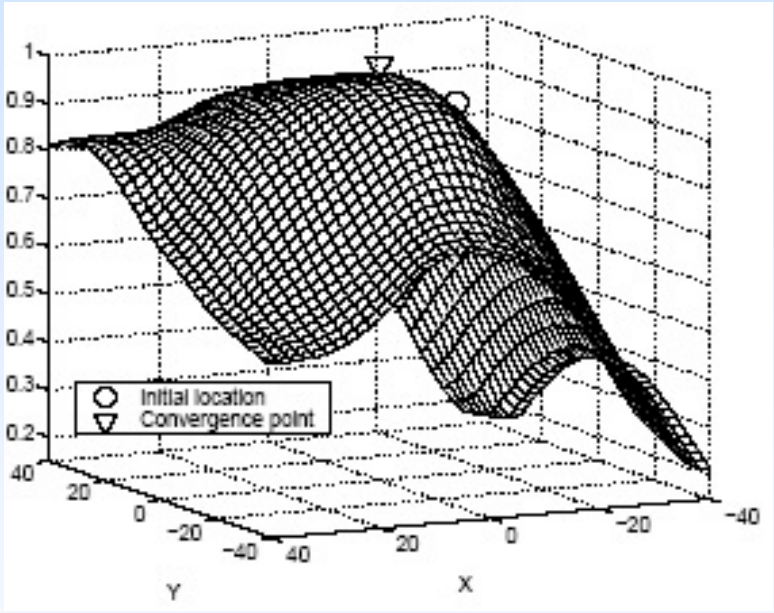
The mode of $\frac{C_h}{2}\sum_{i=1}^{n} w_i k\left(\left\|\frac{y-x_i}{h}\right\|^2\right)$ = sought maximum

Important Assumption:



The target representation provides sufficient discrimination

One mode in the searched neighborhood



Mean-Shift Object Tracking

Applying Mean-Shift

The mode of $\frac{C_h}{2}\sum_{i=1}^{n} w_i k\left(\left\|\frac{y-x_i}{h}\right\|^2\right)$ = sought maximum

Original Mean-Shift:

Find mode of $c\sum_{i=1}^{n} k\left(\left\|\frac{y-x_i}{h}\right\|^2\right)$ using $y_1 = \frac{\sum_{i=1}^{n} x_i g\left(\left\|\frac{y_0-x_i}{h}\right\|^2\right)}{\sum_{i=1}^{n} g\left(\left\|\frac{y_0-x_i}{h}\right\|^2\right)}$

Mean-Shift Object Tracking

Applying Mean-Shift

The mode of $\frac{C_h}{2}\sum_{i=1}^{n} w_i k\left(\left\|\frac{y-x_i}{h}\right\|^2\right)$ = sought maximum

Original Mean-Shift:

Find mode of $c\sum_{i=1}^{n} k\left(\left\|\frac{y-x_i}{h}\right\|^2\right)$ using $y_1 = \frac{\sum_{i=1}^{n} x_i g\left(\left\|\frac{y_0-x_i}{h}\right\|^2\right)}{\sum_{i=1}^{n} g\left(\left\|\frac{y_0-x_i}{h}\right\|^2\right)}$

Extended Mean-Shift:

Find mode of $c\sum_{i=1}^{n} w_i k\left(\left\|\frac{y-x_i}{h}\right\|^2\right)$ using $y_1 = \frac{\sum_{i=1}^{n} x_i w_i g\left(\left\|\frac{y_0-x_i}{h}\right\|^2\right)}{\sum_{i=1}^{n} w_i g\left(\left\|\frac{y_0-x_i}{h}\right\|^2\right)}$

Mean-Shift Object Tracking

Applying Mean-Shift

The mode of $\frac{C_h}{2}\sum_{i=1}^{n} w_i k\left(\left\|\frac{y-x_i}{h}\right\|^2\right)$ = sought maximum

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Find mode of $c\sum_{i=1}^{n} k\left(\left\|\frac{y-x_i}{h}\right\|^2\right)$ using $y_1 = \frac{\sum_{i=1}^{n} x_i g\left(\left\|\frac{y_0-x_i}{h}\right\|^2\right)}{\sum_{i=1}^{n} g\left(\left\|\frac{y_0-x_i}{h}\right\|^2\right)}$

Extended Mean-Shift:

Find mode of $c\sum_{i=1}^{n} w_i k\left(\left\|\frac{y-x_i}{h}\right\|^2\right)$ using $y_1 = \frac{\sum_{i=1}^{n} x w_i g\left(\left\|\frac{y_0-x_i}{h}\right\|^2\right)}{\sum_{i=1}^{n} w_i g\left(\left\|\frac{y_0-x_i}{h}\right\|^2\right)}$

Mean-Shift Object Tracking

About Kernels and Profiles

Extended Mean-Shift:

Find mode of $c\sum_{i=1}^{n} w_i k\left(\left\|\frac{y - x_i}{h}\right\|^2\right)$ using $y_1 = \frac{\sum_{i=1}^{n} x_i w_i g\left(\left\|\frac{y_0 - x_i}{h}\right\|^2\right)}{\sum_{i=1}^{n} w_i g\left(\left\|\frac{y_0 - x_i}{h}\right\|^2\right)}$

Mean-Shift Object Tracking

About Kernels and Profiles

A special class of radially symmetric kernels:

$$K(x) = ck\left(\|x\|^2\right)$$

The profile of kernel *K*

Extended Mean-Shift:

Find mode of $c\sum_{i=1}^{n} w_i k\left(\left\|\frac{y - x_i}{h}\right\|^2\right)$ using $y_1 = \frac{\sum_{i=1}^{n} x_i w_i g\left(\left\|\frac{y_0 - x_i}{h}\right\|^2\right)}{\sum_{i=1}^{n} w_i g\left(\left\|\frac{y_0 - x_i}{h}\right\|^2\right)}$

Mean-Shift Object Tracking

About Kernels and Profiles

A special class of radially symmetric kernels:

$$K(x) = ck\left(\|x\|^2\right)$$

The profile of kernel *K*

$$k'(x) = -g(x)$$

Extended Mean-Shift:

Find mode of $c\sum_{i=1}^{n} w_i k\left(\left\|\frac{y - x_i}{h}\right\|^2\right)$ using $y_1 = \frac{\sum_{i=1}^{n} x_i w_i g\left(\left\|\frac{y_0 - x_i}{h}\right\|^2\right)}{\sum_{i=1}^{n} w_i g\left(\left\|\frac{y_0 - x_i}{h}\right\|^2\right)}$

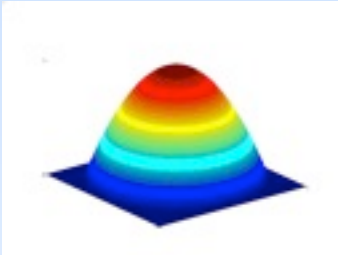
Mean-Shift Object Tracking

Choosing the Kernel

A special class of radially symmetric kernels:

$$K(x) = ck\left(\|x\|^2\right)$$

Epanechnikov kernel:



$$k(x) = \begin{Bmatrix} 1-x & \text{if } \|x\| \le 1 \\ 0 & \text{otherwise} \end{Bmatrix}$$

Mean-Shift Object Tracking

Choosing the Kernel

A special class of radially symmetric kernels:

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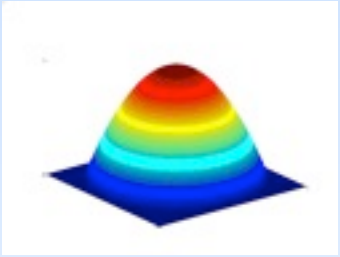
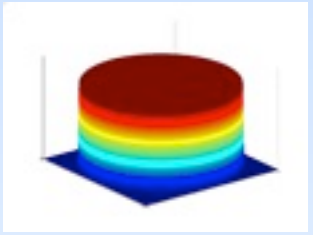


$$k(x) = \begin{cases} 1-x & \text{if } \|x\| \le 1 \\ 0 & \text{otherwise} \end{cases}$$

Uniform kernel:



$$g(x) = -k(x) = \begin{cases} 1 & \text{if } \|x\| \le 1 \\ 0 & \text{otherwise} \end{cases}$$

Mean-Shift Object Tracking

Choosing the Kernel

A special class of radially symmetric kernels:

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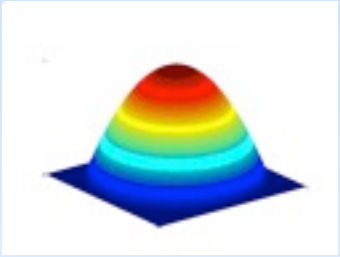
Epanechnikov kernel:



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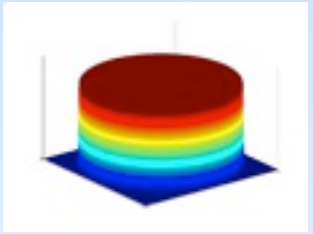


$$g(x) = -k(x) = \begin{cases} 1 & \text{if } \|x\| \le 1 \\ 0 & \text{otherwise} \end{cases}$$

$$y_1 = \frac{\sum_{i=1}^{n} x_i w_i g\left(\left\|\frac{y_0 - x_i}{h}\right\|^2\right)}{\sum_{i=1}^{n} w_i g\left(\left\|\frac{y_0 - x_i}{h}\right\|^2\right)} \rightarrow y_1 = \frac{\sum_{i=1}^{n} x_i w_i}{\sum_{i=1}^{n} w_i}$$

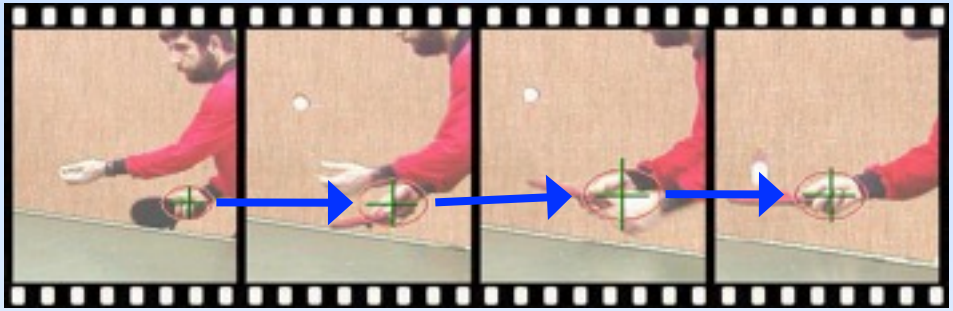
Mean-Shift Object Tracking

Adaptive Scale

Problem

:

The scale of the target changes in time → The scale (*h*) of the kernel must be adapted



Mean-Shift Object Tracking

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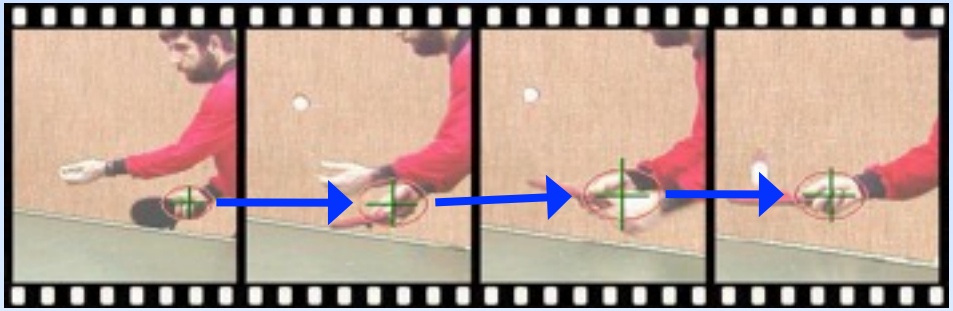

Solution:

Run localization 3 times with different *h*

Mean-Shift Object Tracking

Adaptive Scale

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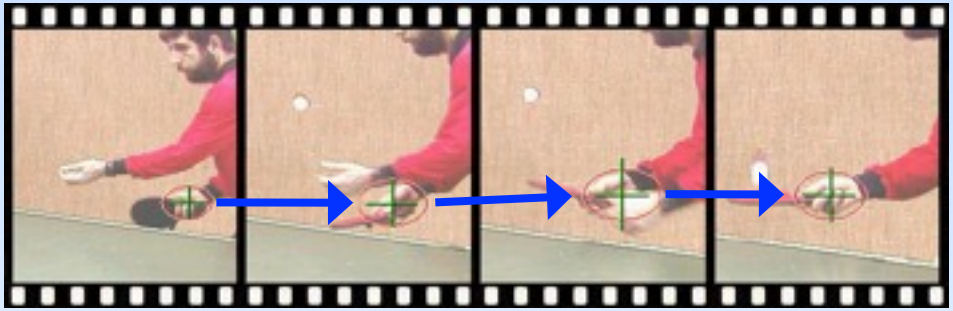
The scale of the target changes in time → The scale (*h*) of the kernel must be adapted



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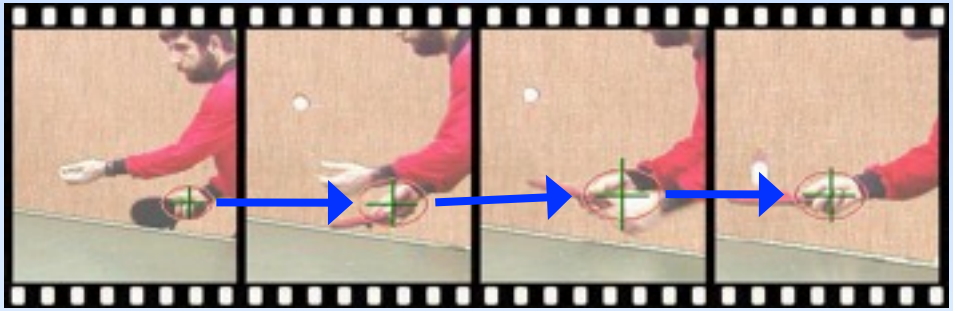

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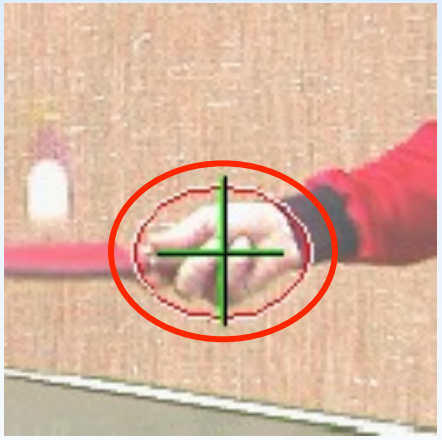

Mean-Shift Object Tracking

Adaptive Scale

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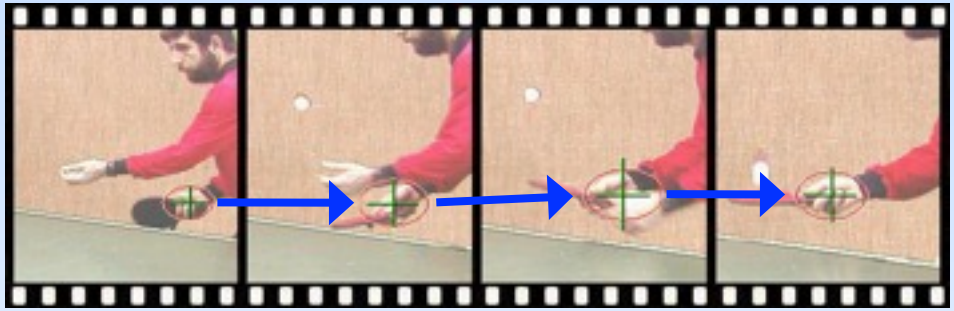

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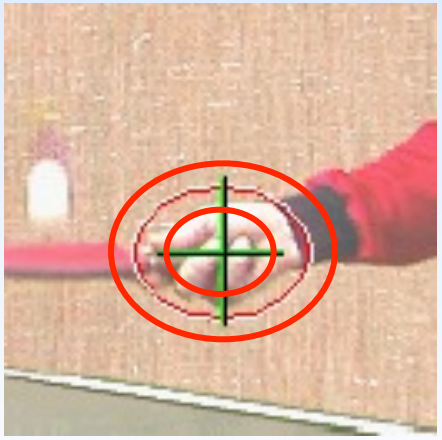

Mean-Shift Object Tracking

Adaptive Scale

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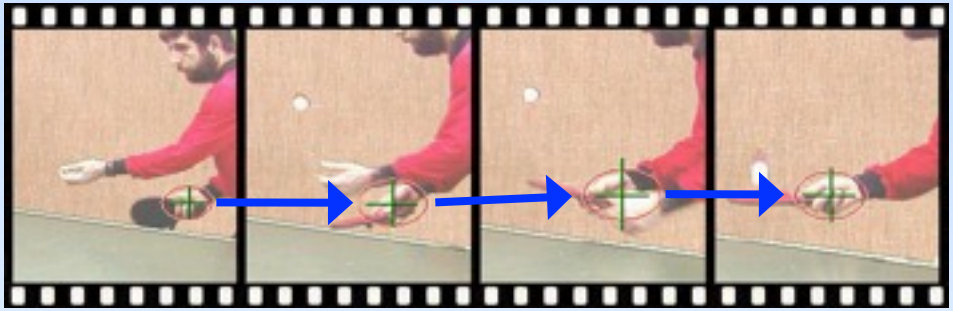
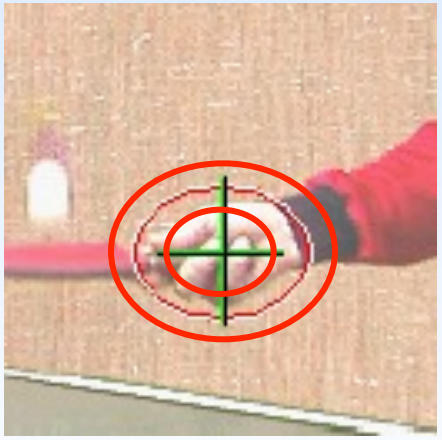


Solution

Run localization 3 times with different *h* → Choose *h* that achieves maximum similarity



Mean-Shift Object Tracking

Results

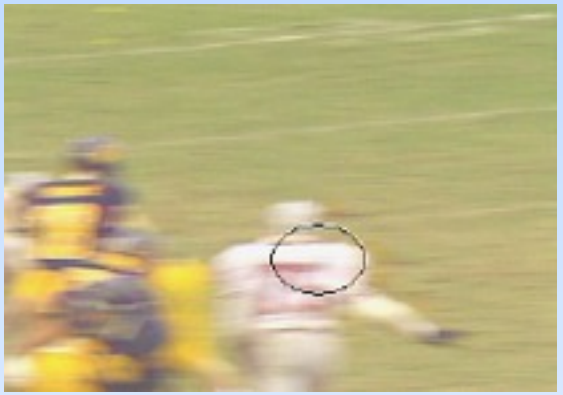
Feature space: 16×16×16 quantized RGB
Target: manually selected on 1st frame
Average mean-shift iterations: 4

Mean-Shift Object Tracking

Results

Partial occlusion

Distraction



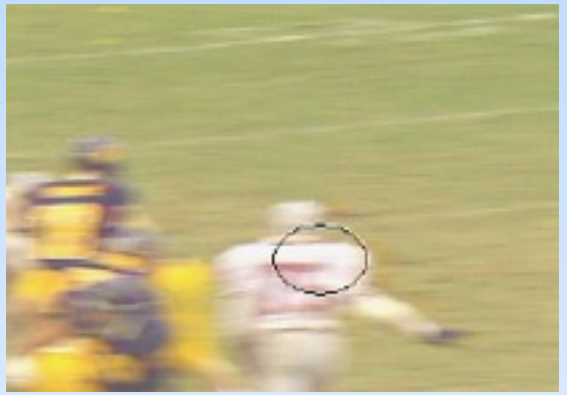
Motion blur

Mean-Shift Object Tracking

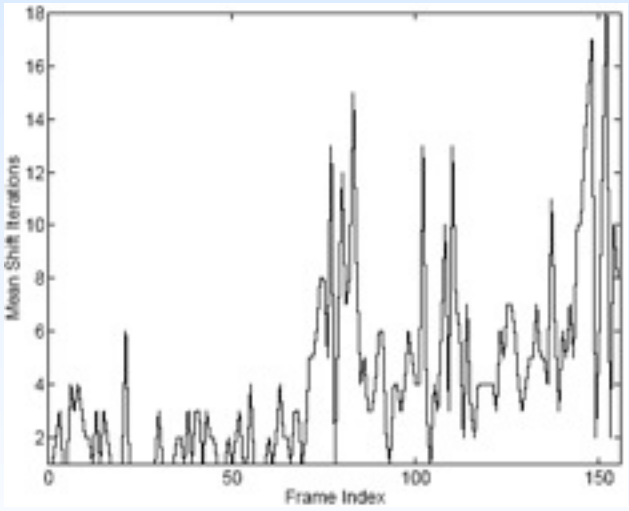
Results

Partial occlusion

Distraction



Motion blur



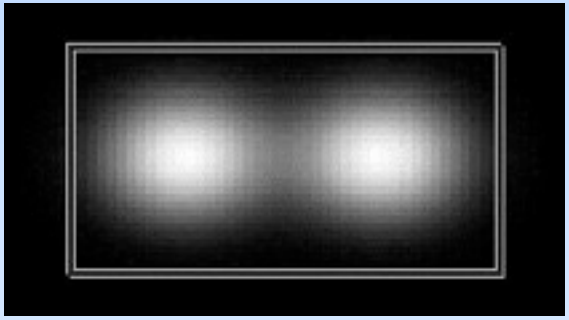
Mean-Shift Object Tracking

Results

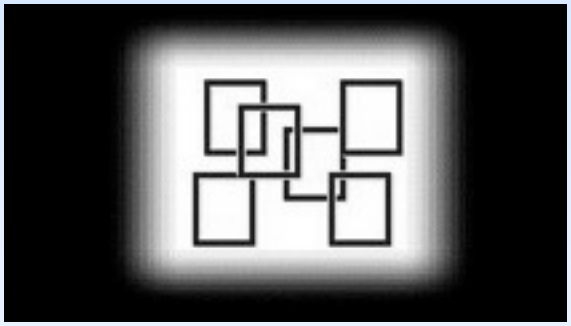
Mean-Shift Object Tracking

The Scale Selection Problem

Kernel too big



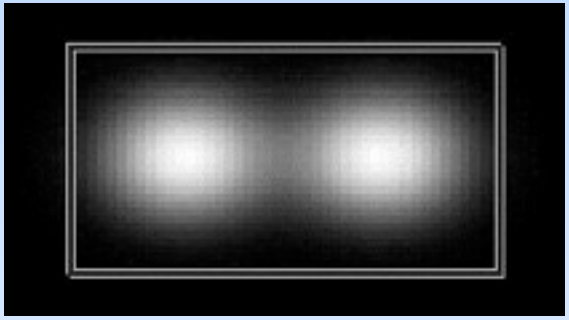
Kernel too small



Mean-Shift Object Tracking

The Scale Selection Problem

Kernel too big



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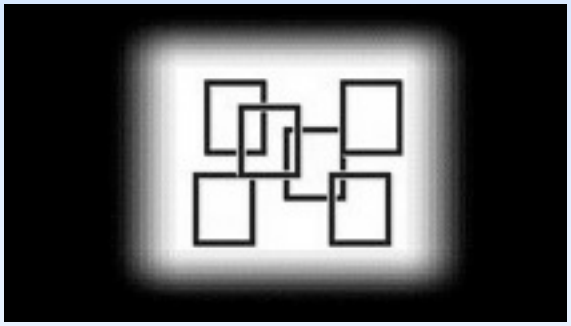

Poor localization

h mustn't get too big or too small!

Mean-Shift Object Tracking

The Scale Selection Problem

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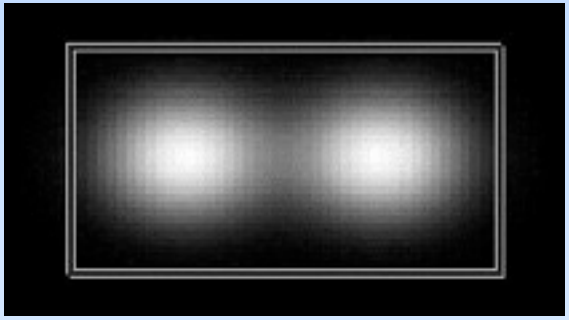

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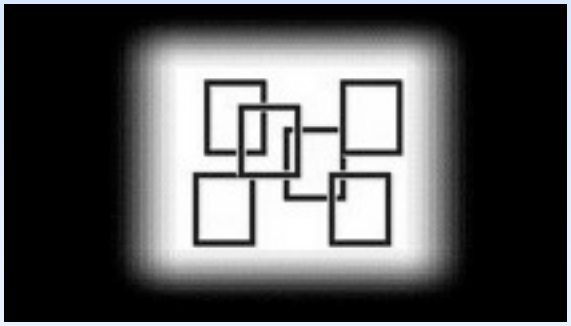


Poor localization → *h* mustn't get too big or too small!

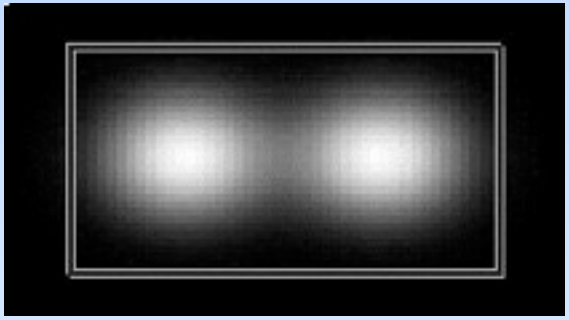
Problem:

In uniformly colored regions, similarity is invariant to *h*

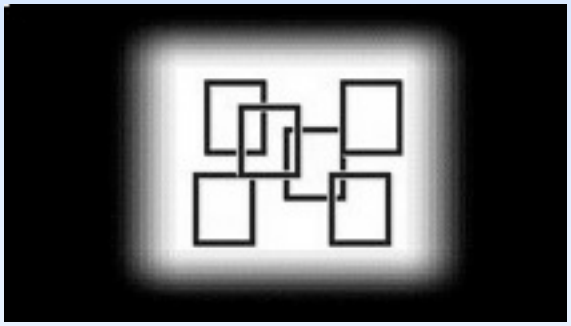
Mean-Shift Object Tracking

The Scale Selection Problem

Kernel too big



Kernel too small



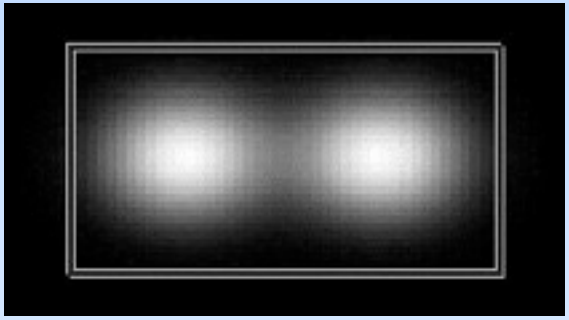
Poor localization → *h* mustn't get too big or too small!

Problem: In uniformly colored regions, similarity is invariant to *h* → Smaller *h* may achieve better similarity

Mean-Shift Object Tracking

The Scale Selection Problem

Kernel too big



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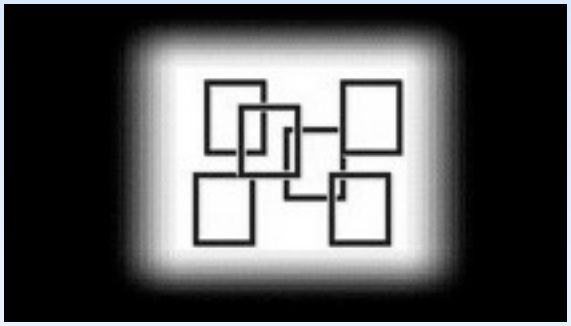


Poor localization → *h* mustn't get too big or too small!

Problem:

In uniformly colored regions, similarity is invariant to *h* → Smaller *h* may achieve better similarity → Nothing keeps *h* from shrinking too small!

Tracking Through Scale Space

Motivation

Spatial localization for several scales

Previous method

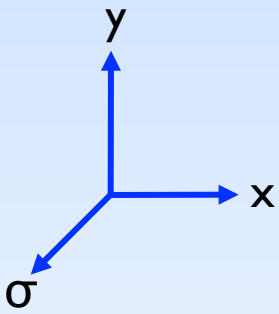
Simultaneous localization in space and scale

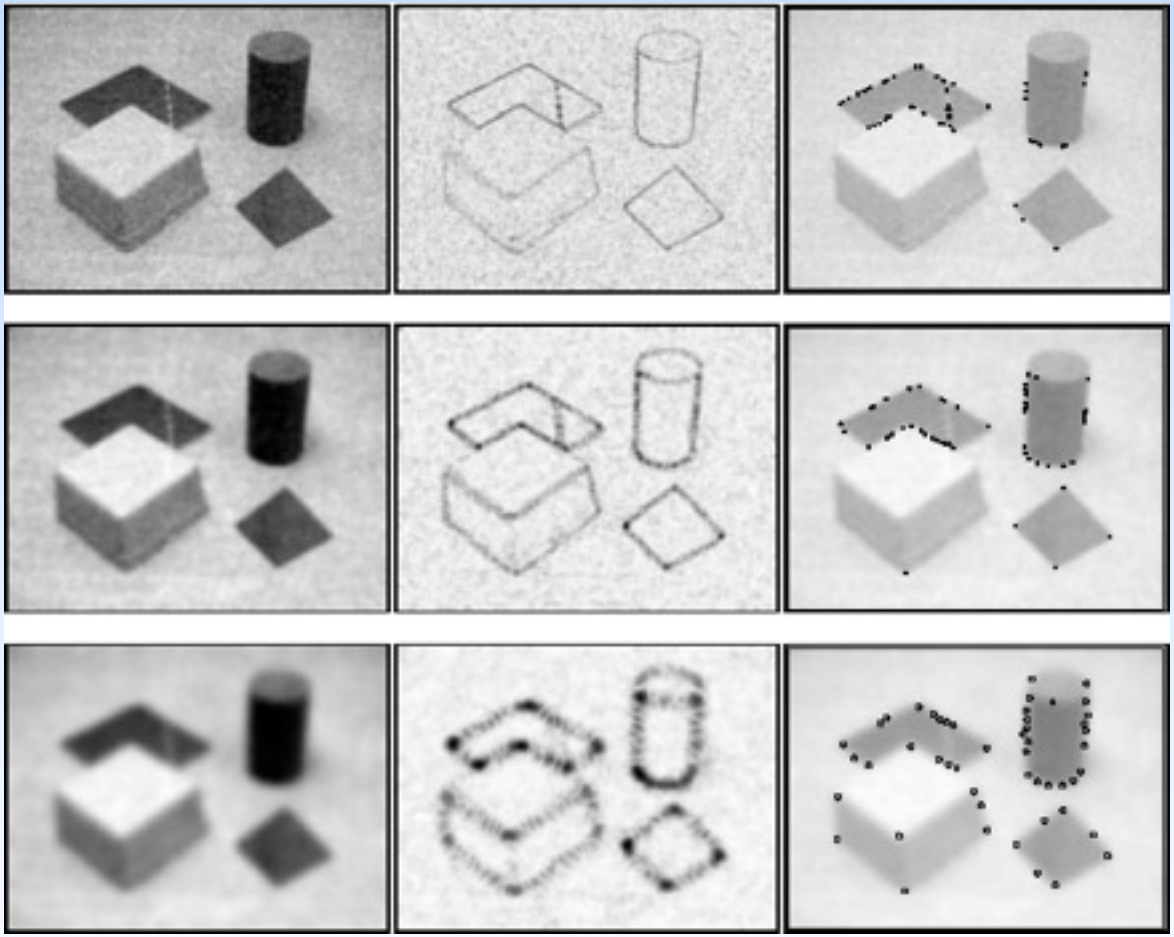
This method

Mean-shift Blob Tracking through Scale Space, by R. Collins

Lindeberg's Theory

Selecting the best scale for describing image features





Scale-space representation | Differential operator applied | 50 strongest responses

Lindeberg's Theory

Selecting the best scale for describing image features

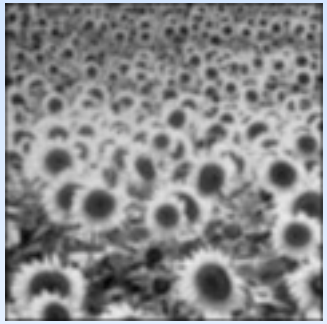
$f(x) =$ 

Lindeberg's Theory

Selecting the best scale for describing image features

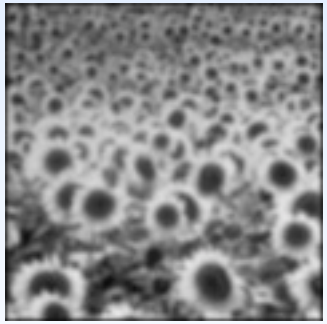
$G(x;\sigma_1)$

$G(x;\sigma_2)$



$\vdots$

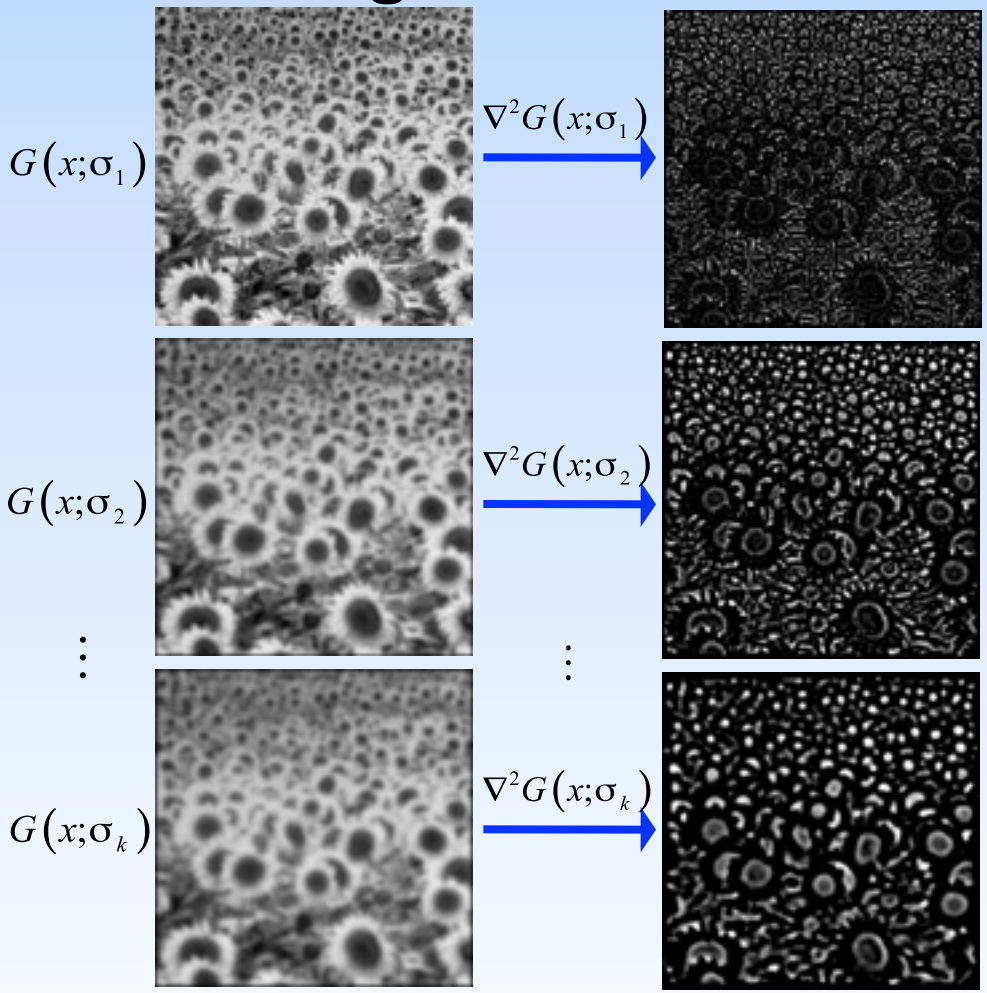
$G(x;\sigma_k)$



Scale-space representation

Lindeberg's Theory

Selecting the best scale for describing image features



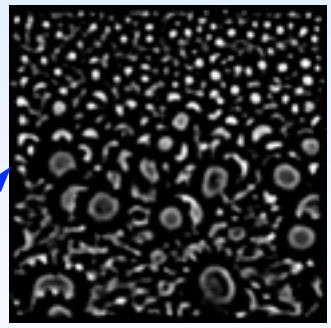
Lindeberg's Theory

Selecting the best scale for describing image features

$LOG(x;\sigma_1)$

$LOG(x;\sigma_2)$

$LOG(x;\sigma_k)$



Laplacian of Gaussian (LOG)

2D LOG filter with scale σ

$$LOG(x;\sigma) = \frac{2\sigma^2 - \|x\|^2}{2\pi\sigma^6} e^{\frac{-\|x\|^2}{2\sigma^2}}$$

Lindeberg's Theory

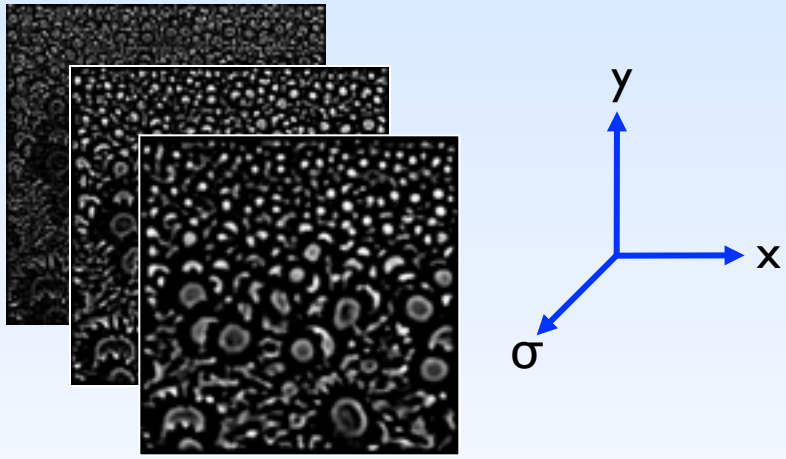
Selecting the best scale for describing image features

2D LOG filter with scale σ

$$LOG(x;\sigma) = \frac{2\sigma^2 - \|x\|^2}{2\pi\sigma^6} e^{\frac{-\|x\|^2}{2\sigma^2}}$$

3D scale-space representation

$$\forall x \in f,\ \forall \sigma_{1..k}:$$
$$L(x,\sigma) = LOG(x;\sigma) * f(x)$$



Lindeberg's Theory

Selecting the best scale for describing image features

2D LOG filter with scale σ

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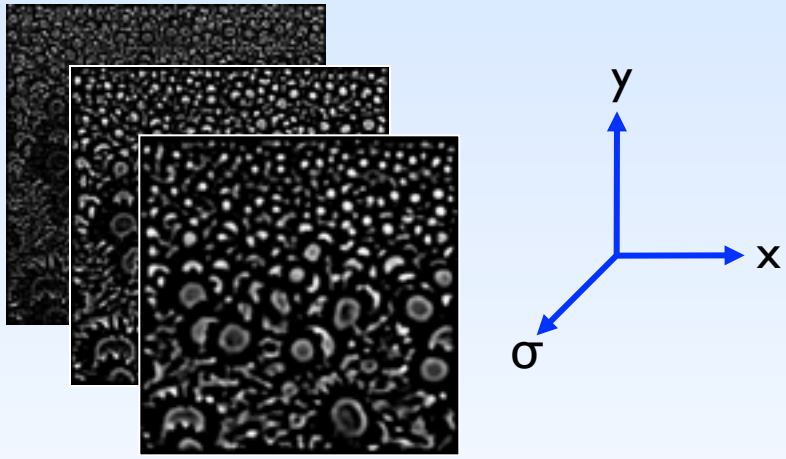


Best features are at (x,σ) that maximize L

Lindeberg's Theory

Multi-Scale Feature Selection Process

Original Image

$f(x)$

Lindeberg's Theory

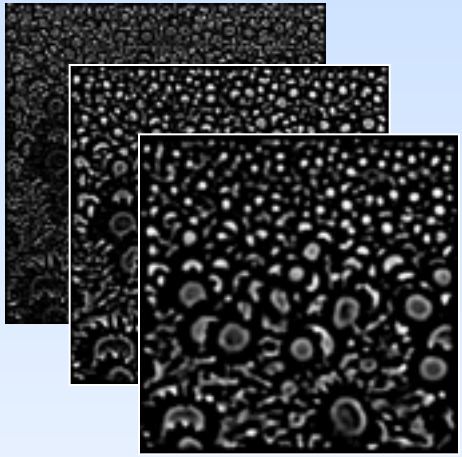
Multi-Scale Feature Selection Process

Original Image

$f(x)$

Convolve

3D scale-space function



$$L(x,\sigma) = LOG(x;\sigma) * f(x)$$

Lindeberg's Theory

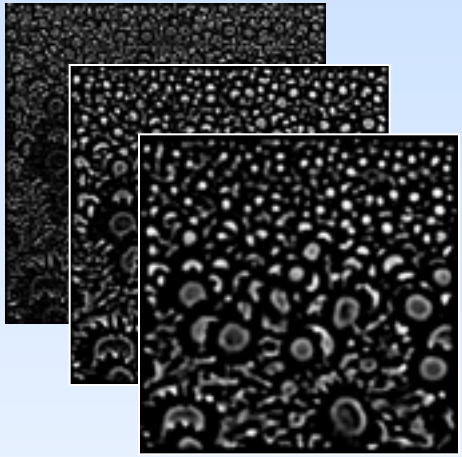
Multi-Scale Feature Selection Process

Original Image

$f(x)$

Convolve

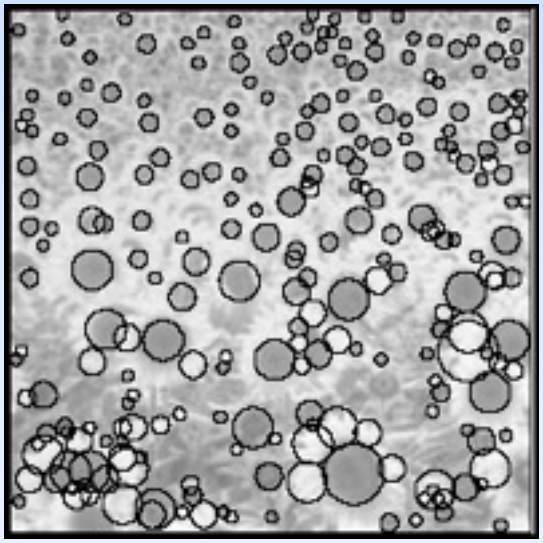
3D scale-space function



$$L(x,\sigma) = LOG(x;\sigma) * f(x)$$

Maximize

250 strongest responses
(Large circle = large scale)



Tracking Through Scale Space

Approximating LOG using DOG

$$LOG(x;\sigma) \approx DOG(x;\sigma) = G(x;\sigma) - G(x;1.6\sigma)$$

2D LOG filter with scale σ

2D DOG filter with scale σ

Tracking Through Scale Space

Approximating LOG using DOG

$$LOG(x;\sigma) \approx DOG(x;\sigma) = G(x;\sigma) - G(x;1.6\sigma)$$

2D LOG filter with scale σ

2D DOG filter with scale σ

2D Gaussian with μ=0 and scale σ

2D Gaussian with μ=0 and scale 1.6σ

Tracking Through Scale Space

Approximating LOG using DOG

$$LOG(x;\sigma) \approx DOG(x;\sigma) = G(x;\sigma) - G(x;1.6\sigma)$$

- 2D LOG filter with scale σ
- 2D DOG filter with scale σ
- 2D Gaussian with μ=0 and scale σ
- 2D Gaussian with μ=0 and scale 1.6σ

Why DOG?

- Gaussian pyramids are created faster
- Gaussian can be used as a mean-shift kernel

Tracking Through Scale Space

Approximating LOG using DOG

$$LOG(x;\sigma) \approx DOG(x;\sigma) = G(x;\sigma) - G(x;1.6\sigma)$$

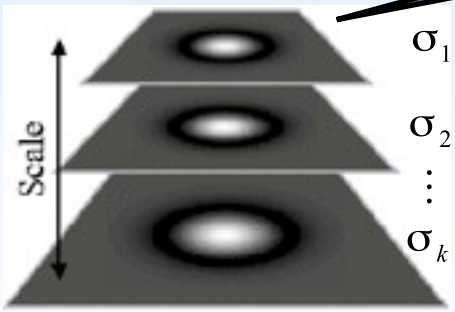
- 2D LOG filter with scale σ
- 2D DOG filter with scale σ
- 2D Gaussian with μ=0 and scale σ
- 2D Gaussian with μ=0 and scale 1.6σ

Why DOG?

- Gaussian pyramids are created faster
- Gaussian can be used as a mean-shift kernel

3D spatial kernel

$$K(x,\sigma) =$$



DOG filters at multiple scales

Tracking Through Scale Space

Approximating LOG using DOG

$$LOG(x;\sigma) \approx DOG(x;\sigma) = G(x;\sigma) - G(x;1.6\sigma)$$

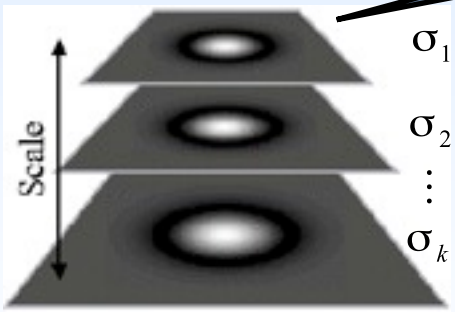
- 2D LOG filter with scale σ
- 2D DOG filter with scale σ
- 2D Gaussian with μ=0 and scale σ
- 2D Gaussian with μ=0 and scale 1.6σ

Why DOG?

- Gaussian pyramids are created faster
- Gaussian can be used as a mean-shift kernel

3D spatial kernel

$$K(x,\sigma) =$$



DOG filters at multiple scales

→ Scale-space filter bank

Tracking Through Scale Space

Using Lindeberg's Theory

Recall:

Model: $\vec{q} = (q_1, \ldots, q_m)$ at y_0

Candidate: $\vec{p}(y) = (p_1(y), \ldots, p_m(y))$

Color bin: $b(x)$

Pixel weight: $w(x) = \sqrt{\frac{q_{b(x)}}{p_{b(x)}(y_0)}}$

The likelihood that each candidate pixel belongs to the target

Tracking Through Scale Space

Using Lindeberg's Theory

Weight image

Recall:

Model: $\vec{q} = (q_1, \ldots, q_m)$ at y_0

Candidate: $\vec{p}(y) = (p_1(y), \ldots, p_m(y))$

Color bin: $b(x)$

Pixel weight: $w(x) = \sqrt{\frac{q_{b(x)}}{p_{b(x)}(y_0)}}$

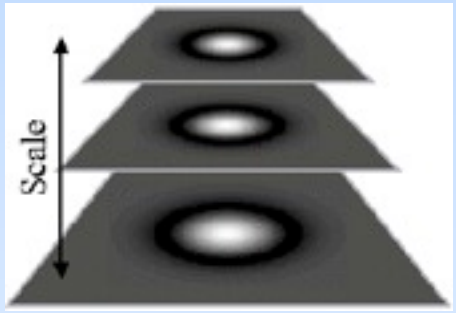
The likelihood that each candidate pixel belongs to the target

Tracking Through Scale Space

Using Lindeberg's Theory

Weight image

*



3D spatial kernel (DOG)

Centered at current location and scale

Recall:

Model: $\vec{q} = (q_1, \ldots, q_m)$ at y_0

Candidate: $\vec{p}(y) = (p_1(y), \ldots, p_m(y))$

Color bin: $b(x)$

Pixel weight: $w(x) = \sqrt{\frac{q_{b(x)}}{p_{b(x)}(y_0)}}$

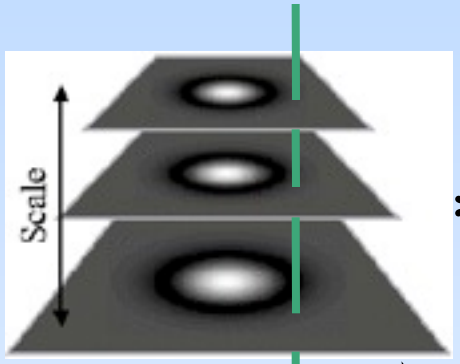
The likelihood that each candidate pixel belongs to the target

Tracking Through Scale Space

Using Lindeberg's Theory

Weight image

*



3D spatial kernel (DOG)

*

1D scale kernel (Epanechnikov)

Centered at current location and scale

Recall:

Model: $\vec{q} = (q_1, \ldots, q_m)$ at y_0

Candidate: $\vec{p}(y) = (p_1(y), \ldots, p_m(y))$

Color bin: $b(x)$

Pixel weight: $w(x) = \sqrt{\frac{q_{b(x)}}{p_{b(x)}(y_0)}}$

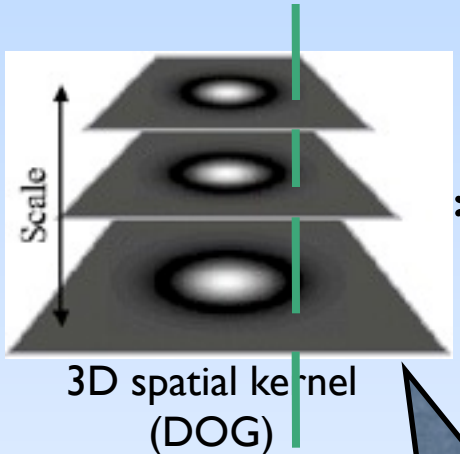
The likelihood that each candidate pixel belongs to the target

Tracking Through Scale Space

Using Lindeberg's Theory

Weight image

$*$



3D spatial kernel (DOG)

$*$

1D scale kernel (Epanechnikov)

$= E(x,\sigma)$

3D scale-space representation

Centered at current location and scale

Modes are blobs in the scale-space neighborhood

Recall:

Model: $\vec{q} = (q_1, \ldots, q_m)$ at y_0

Candidate: $\vec{p}(y) = (p_1(y), \ldots, p_m(y))$

Color bin: $b(x)$

Pixel weight: $w(x) = \sqrt{\frac{q_{b(x)}}{p_{b(x)}(y_0)}}$

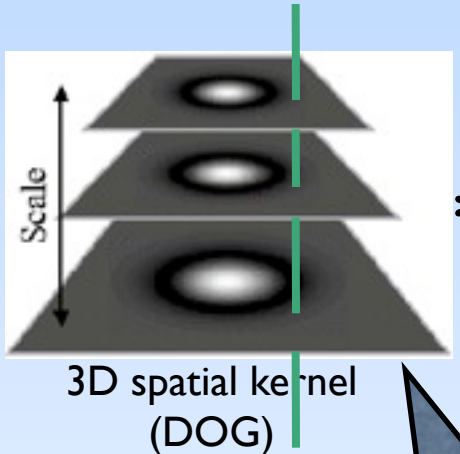
The likelihood that each candidate pixel belongs to the target

Tracking Through Scale Space

Using Lindeberg's Theory

Weight image

$*$



3D spatial kernel (DOG)

$*$

1D scale kernel (Epanechnikov)

$= E(x,\sigma)$

3D scale-space representation

Centered at current location and scale

Modes are blobs in the scale-space neighborhood

Need a mean-shift procedure that finds local modes in $E(x,\sigma)$

Recall:

Model: $\vec{q} = (q_1,\ldots,q_m)$ at y_0

Candidate: $\vec{p}(y) = (p_1(y),\ldots,p_m(y))$

Color bin: $b(x)$

Pixel weight: $w(x) = \sqrt{\dfrac{q_{b(x)}}{p_{b(x)}(y_0)}}$

The likelihood that each candidate pixel belongs to the target

Tracking Through Scale Space

Example

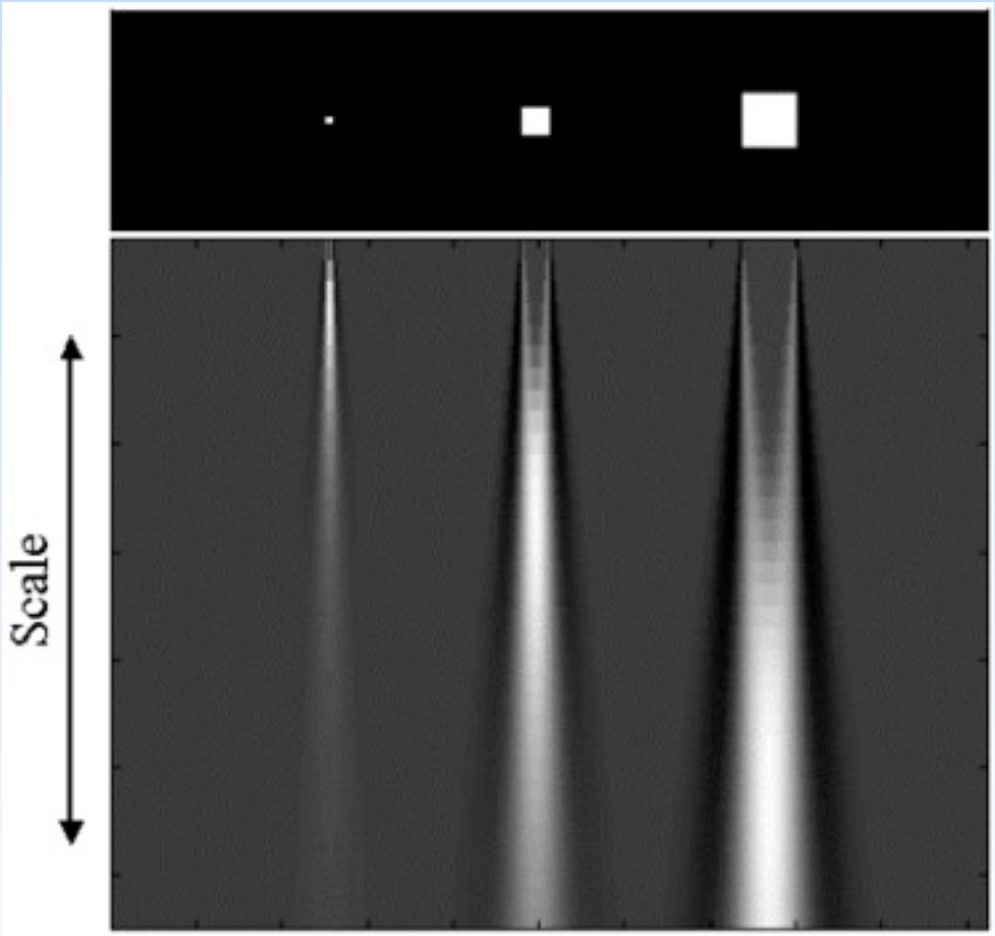


Image of 3 blobs

A slice through the 3D scale-space representation

Tracking Through Scale Space

Applying Mean-Shift

Use interleaved spatial/scale mean-shift

Spatial stage:

Fix σ and look for the best *x*

Scale stage:

Fix x and look for the best σ

Tracking Through Scale Space

Applying Mean-Shift

Use interleaved spatial/scale mean-shift

Spatial stage:

Fix σ and look for the best *x*

Scale stage:

Fix x and look for the best σ

Iterate stages until convergence of *x* and σ

Tracking Through Scale Space

Applying Mean-Shift

Use interleaved spatial/scale mean-shift

Spatial stage:

Fix σ and look for the best *x*

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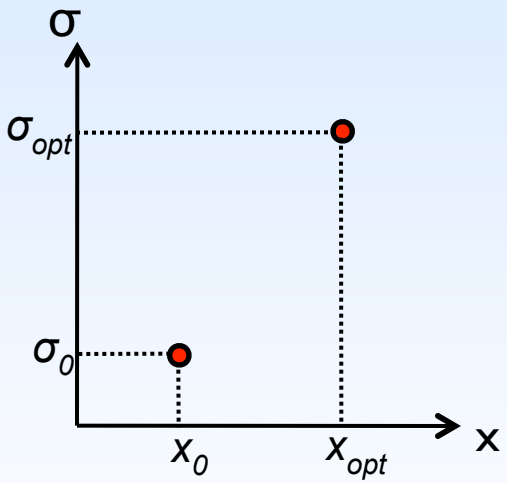

Iterate stages until convergence of *x* and σ

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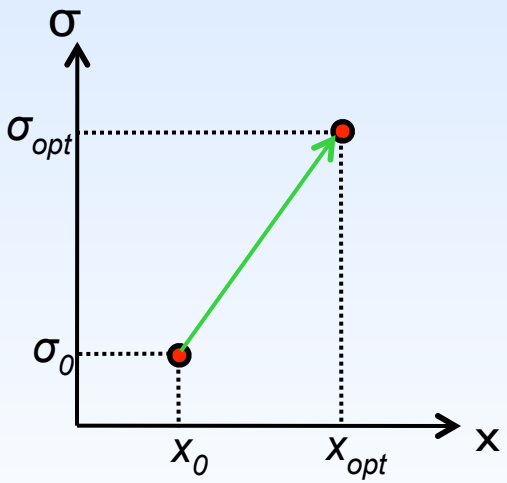

Iterate stages until convergence of *x* and σ

Tracking Through Scale Space

Applying Mean-Shift

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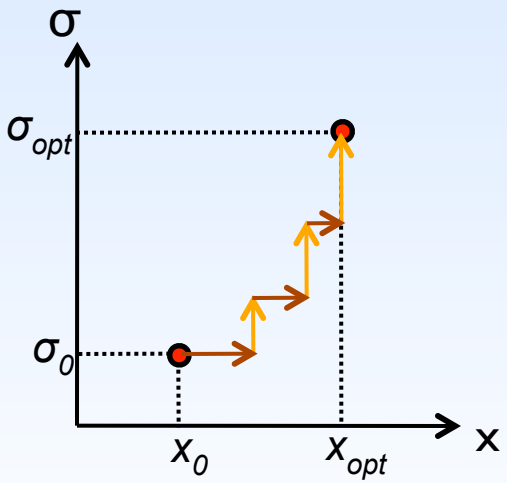
Iterate stages until convergence of *x* and σ



Tracking Through Scale Space

Results

Fixed-scale

Tracking Through Scale Space

Results

Fixed-scale

± 10% scale adaptation

Tracking Through Scale Space

Results

Fixed-scale

± 10% scale adaptation

Tracking through scale space