N-View Methods

Diana Mateus, Nassir Navab

Computer Aided Medical Procedures
Technische Universität München

3D Computer Vision II

Inspired by Slides from Adrien Bartoli

Outline

Structure From Motion (SFM) with Uncalibrated Cameras

Requirements A set of pictures taken from one camera in motion or multiple cameras.

Goal Compute from the pictures:

- a (Euclidean) 3D model of the scene structure.
- the camera parameters (intrinsic and extrinsic).

Assumptions

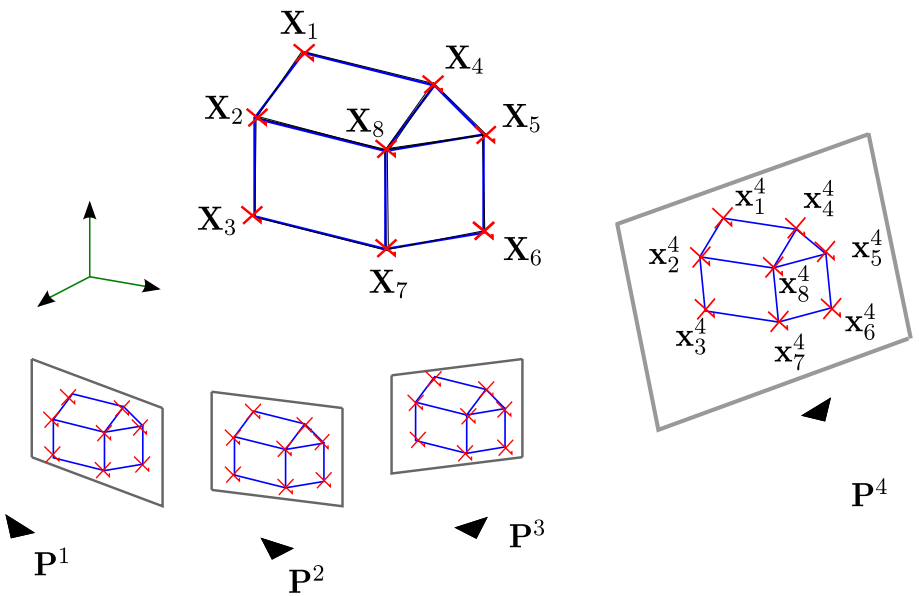
- The scene is rigid.
- Some of the camera intrinsic parameters are known or constant.


Microsoft Photosynth
Home
Explore
Forums
Blog
About
Create
Photosynth Tips
Click on the white boxes to see different photos.
Use the arrows to see more of the scene.
Use the buttons or mouse scroll wheel to zoom in & out.
Don't show again
More Info
West Angle
From a

Why Using Uncalibrated Cameras?

- Calibrated approaches require (at least) one image of a calibration object (i.e. of which the Euclidean 3D model is known).
- Uncalibrated approaches:
  - Do not require a special material, neither user interaction.
  - Impose only mild assumptions on the imaging conditions (thus applicable to images from unknown sources, e.g. internet).
  - Deal with on-line calibration (useful when camera intrinsic parameters vary, due to e.g. zooming).
  - Provide flexible and efficient methods for camera self-calibration.

Preliminaries



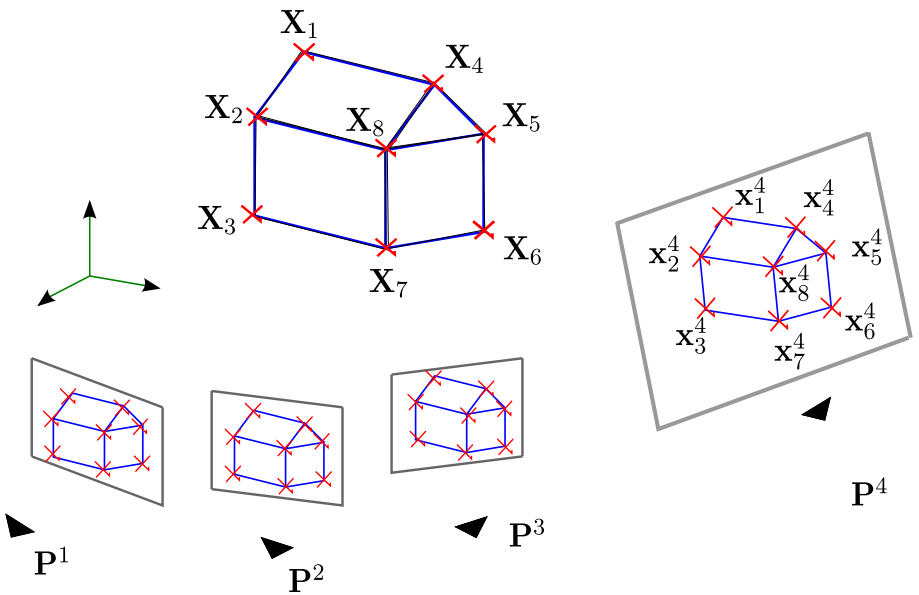
$$\mathbf{x}_j^i = \mathbf{P}^i \mathbf{X}_j$$

Scene structure: Set of 3D points $\mathbf{X}_j$, with $j = \{1, \ldots, n\}$. Other features can be used (e.g. lines, curves, planes).

Camera geometry: Affine or perspective (pin-hole) projection, represented by matrices $\mathbf{P}^i$, with $i = \{1, \ldots, m\}$.

Images: 2D points matched across the images with coordinates $\mathbf{x}_j^i$.

Preliminaries



$$\mathbf{x}_j^i = \mathbf{P}^i \mathbf{X}_j$$

Scene structure: (Unknown) Set of 3D points $\mathbf{X}_j$, with $j = \{1, \ldots, n\}$. Other features can be used (e.g. lines, curves, planes).

Camera geometry: (Unknown) Affine or perspective (pin-hole) projection, represented by matrices $\mathbf{P}^i$, with $i = \{1, \ldots, m\}$.

Images: (Given) 2D points matched across the images with coordinates $\mathbf{x}_j^i$.

Is it Possible?: A Counting Argument

- Look for cameras $\mathbf{P}^i$ and 3D points $\mathbf{X}_j$ that explain image points:

$$\mathbf{x}_j^i = \mathbf{P}^i \mathbf{X}_j \quad i = \{1, \ldots, m\} \quad j = \{1, \ldots, n\}$$

- For n points projected in m images :

Is it Possible?: A Counting Argument

- Look for cameras $\mathbf{P}^i$ and 3D points $\mathbf{X}_j$ that explain image points:

$$\mathbf{x}_j^i = \mathbf{P}^i \mathbf{X}_j \quad i = \{1, \ldots, m\} \quad j = \{1, \ldots, n\}$$

- For n points projected in m images :
  - Each point has 2 coordinates.

Is it Possible?: A Counting Argument

- Look for cameras $\mathbf{P}^i$ and 3D points $\mathbf{X}_j$ that explain image points:

$$\mathbf{x}_j^i = \mathbf{P}^i \mathbf{X}_j \quad i = \{1, \ldots, m\} \quad j = \{1, \ldots, n\}$$

- For n points projected in m images :
  - Each point has 2 coordinates.
  - We can build $2nm$ constraints.

Is it Possible?: A Counting Argument

- Look for cameras $\mathbf{P}^i$ and 3D points $\mathbf{X}_j$ that explain image points:

$$\mathbf{x}_j^i = \mathbf{P}^i \mathbf{X}_j \quad i = \{1, \ldots, m\} \quad j = \{1, \ldots, n\}$$

- For n points projected in m images :
  - Each point has 2 coordinates.
  - We can build $2nm$ constraints.
  - We want to recover m projection matrices P^i (Each 3×4 with 11 d.o.f. under affine assumption).

Is it Possible?: A Counting Argument

- Look for cameras $\mathbf{P}^i$ and 3D points $\mathbf{X}_j$ that explain image points:

$$\mathbf{x}_j^i = \mathbf{P}^i \mathbf{X}_j \quad i = \{1, \ldots, m\} \quad j = \{1, \ldots, n\}$$

- For n points projected in m images :
  - Each point has 2 coordinates.
  - We can build $2nm$ constraints.
  - We want to recover m projection matrices P^i (Each 3×4 with 11 d.o.f. under affine assumption).
  - $11m + 3n$ unknowns ?($3n$ due to 3D point coordinates).

Is it Possible?: A Counting Argument

- Look for cameras $\mathbf{P}^i$ and 3D points $\mathbf{X}_j$ that explain image points:

$$\mathbf{x}_j^i = \mathbf{P}^i \mathbf{X}_j \quad i = \{1, \ldots, m\} \quad j = \{1, \ldots, n\}$$

- For n points projected in m images :
  - Each point has 2 coordinates.
  - We can build $2nm$ constraints.
  - We want to recover m projection matrices P^i (Each 3×4 with 11 d.o.f. under affine assumption).
  - $11m + 3n$ unknowns ?($3n$ due to 3D point coordinates).
  - Projective reconstruction up to a projective transformation (4×4 matrix with 15 DoFs).

Is it Possible?: A Counting Argument

- Look for cameras $\mathbf{P}^i$ and 3D points $\mathbf{X}_j$ that explain image points:

$$\mathbf{x}_j^i = \mathbf{P}^i \mathbf{X}_j \quad i = \{1, \ldots, m\} \quad j = \{1, \ldots, n\}$$

- For n points projected in m images :
  - Each point has 2 coordinates.
  - We can build $2nm$ constraints.
  - We want to recover m projection matrices P^i (Each 3×4 with 11 d.o.f. under affine assumption).
  - $11m + 3n$ unknowns ?($3n$ due to 3D point coordinates).
  - Projective reconstruction up to a projective transformation (4×4 matrix with 15 DoFs).
  - $11m + 3n - 15$ unknowns

Is it Possible?: A Counting Argument

- Look for cameras $\mathbf{P}^i$ and 3D points $\mathbf{X}_j$ that explain image points:

$$\mathbf{x}_j^i = \mathbf{P}^i \mathbf{X}_j \quad i = \{1, \ldots, m\} \quad j = \{1, \ldots, n\}$$

- For n points projected in m images :
  - Each point has 2 coordinates.
  - We can build $2nm$ constraints.
  - We want to recover m projection matrices P^i (Each 3×4 with 11 d.o.f. under affine assumption).
  - $11m + 3n$ unknowns ?($3n$ due to 3D point coordinates).
  - Projective reconstruction up to a projective transformation (4×4 matrix with 15 DoFs).
  - $11m + 3n - 15$ unknowns
- The problem can be solved (in general) $2nm \geq 11m + 3n - 15$ e.g. $n = 10$ $m = 3$,

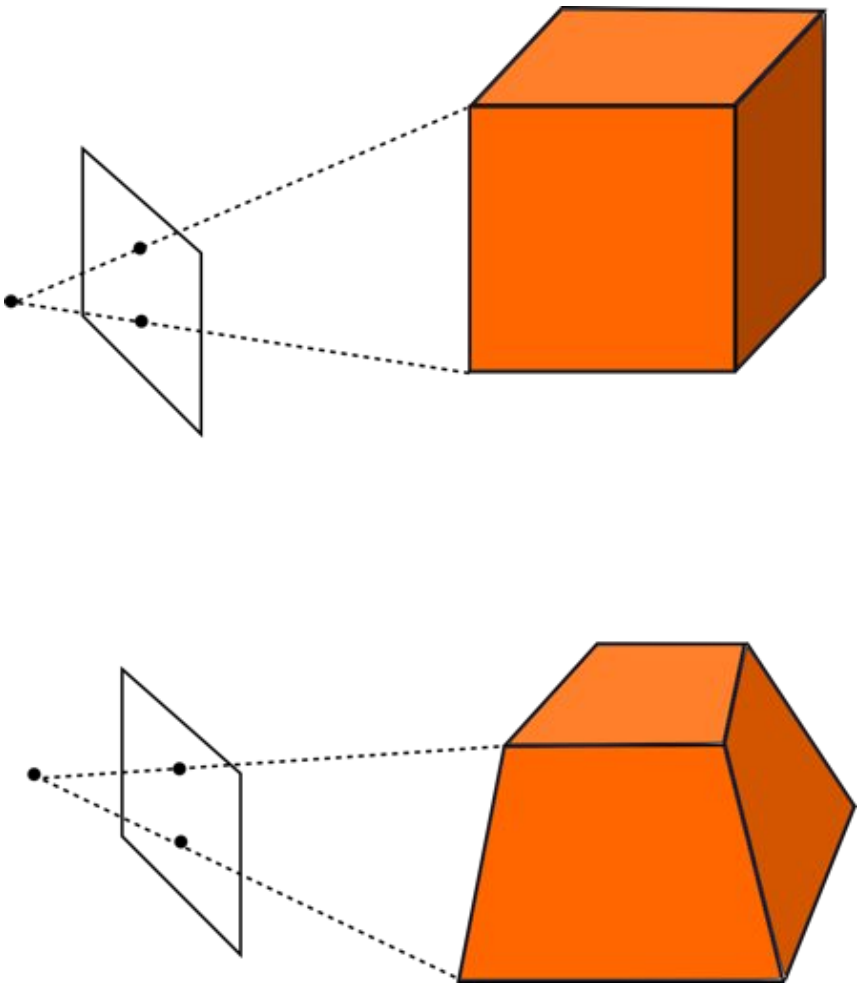
Is it Possible?: A Counting Argument

- Look for cameras $\mathbf{P}^i$ and 3D points $\mathbf{X}_j$ that explain image points:

$$\mathbf{x}_j^i = \mathbf{P}^i \mathbf{X}_j \quad i = \{1, \ldots, m\} \quad j = \{1, \ldots, n\}$$

- For n points projected in m images :
  - Each point has 2 coordinates.
  - We can build $2nm$ constraints.
  - We want to recover m projection matrices P^i (Each 3×4 with 11 d.o.f. under affine assumption).
  - $11m + 3n$ unknowns ?($3n$ due to 3D point coordinates).
  - Projective reconstruction up to a projective transformation (4×4 matrix with 15 DoFs).
  - $11m + 3n - 15$ unknowns
- The problem can be solved (in general) $2nm \geq 11m + 3n - 15$ e.g. $n = 10$ $m = 3$, then $60 \geq 33 + 30 - 15 = 48$.

Projective Reconstruction



SFM Method Overview

1. Find correspondences across the images.

SFM Method Overview

1. Find correspondences across the images.
2. Initialization with a "good enough" sub-optimal estimate, e.g. using a Factorization-based algorithm:
   - Batch.
   - Sequential.
   - Hierarchical.

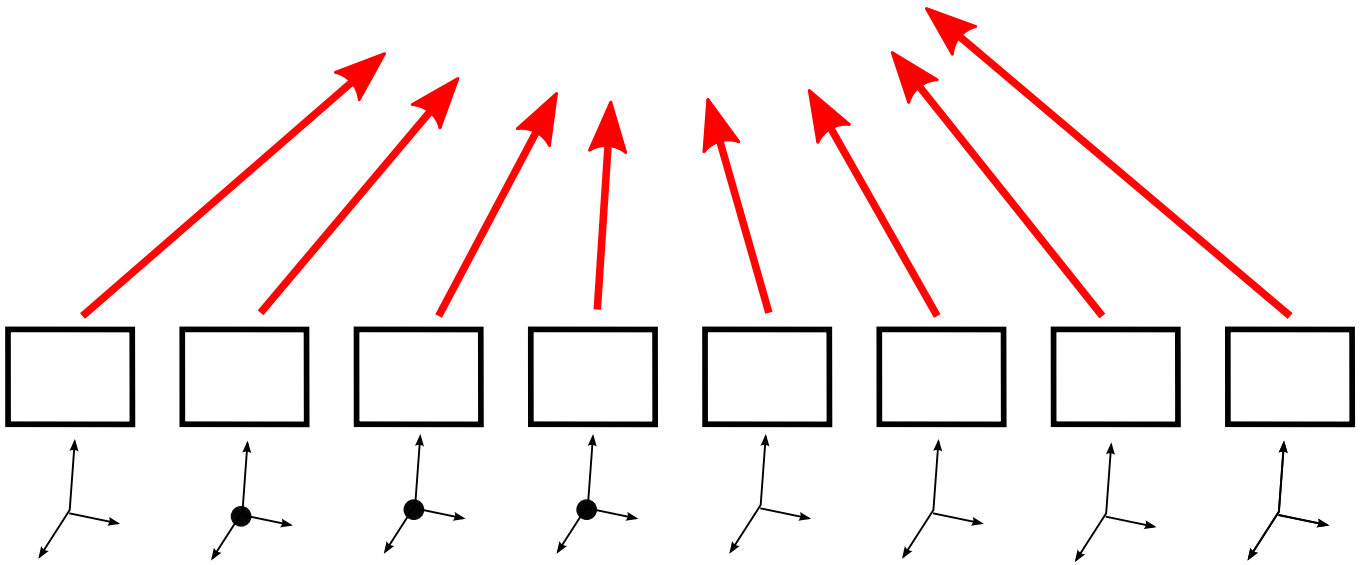
SFM Method Overview

1. Find correspondences across the images.
2. Initialization with a "good enough" sub-optimal estimate, e.g. using a Factorization-based algorithm:
   - Batch.
   - Sequential.
   - Hierarchical.
3. Bundle Adjustment (Optional):
   - Non-linear minimization of the reprojection error (discrepancy between measured and predicted feature coordinates).

SFM Method Overview

1. Find correspondences across the images.
2. Initialization with a "good enough" sub-optimal estimate, e.g. using a Factorization-based algorithm:
   - Batch.
   - Sequential.
   - Hierarchical.
3. Bundle Adjustment (Optional):
   - Non-linear minimization of the reprojection error (discrepancy between measured and predicted feature coordinates).
4. Self-Calibration (Optional).
   - Estimation of the intrinsic parameters of the camera using only the information available in the images taken by that camera.

Batch Algorithms



Batch Algorithms

- Uniform and simultaneous use of all data.
- Two type of approaches:

Factorization

- Simple and fast.
- All points need to be seen in all images.
- measurements (points) need to be valid.
- Orthographic [Tomasi, 1992].
- Affine [Kahl and Heyden, 1998].
- Projective [Sturm, 1996], iterative [Triggs, 1996; Heyden, 1997].

Closure constraints Handles unseen and erroneous points, but suffers from algebraic approximation, e.g. [Kahl and Heyden, 1998].

Outline

Affine Factorization

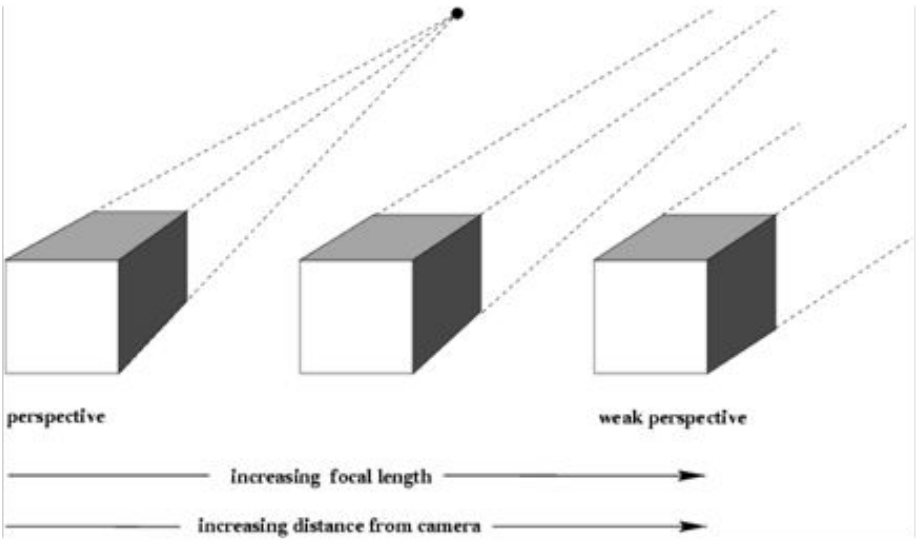
Assumptions:

- All points are seen in all views.
- There are no blunders.
- Affine camera model (uncalibrated) 8 d.o.f.:

$$\underbrace{\begin{pmatrix} x^i_{j;1} \\ x^i_{j;2} \end{pmatrix}}_{\mathbf{x}^i_j} = \underbrace{\begin{pmatrix} P^i_{11} & P^i_{12} & P^i_{13} \\ P^i_{21} & P^i_{22} & P^i_{23} \end{pmatrix}}_{\mathbf{P}^i} \underbrace{\begin{pmatrix} X_{j;1} \\ X_{j;2} \\ X_{j;3} \end{pmatrix}}_{\mathbf{X}_j} + \underbrace{\begin{pmatrix} t^i_1 \\ t^i_2 \end{pmatrix}}_{\mathbf{t}^i}$$

- $\mathbf{x}^i_j$ and $\mathbf{X}_j$ in inhomogeneous coordinates.

Affine Factorization: Camera model



Affine Factorization: Goal

Find a reconstruction of the scene that minimizes the geometric error in the image coordinate measurements. That is, estimate:

- Cameras $\{\mathbf{P}^i, \mathbf{t}^i\}$
- 3D points $\{\mathbf{X}_j\}$

such that the distance between the estimated image points $\hat{\mathbf{x}}_j^i = \mathbf{P}^i\mathbf{X}_j + \mathbf{t}^i$ and measured image points $\mathbf{x}_j^i$ is minimized:

$$\min_{\mathbf{P}^i,\mathbf{t}^i,\mathbf{X}_j} \sum_{i=1}^{m}\sum_{j=1}^{n} ||\mathbf{x}_j^i - \hat{\mathbf{x}}_j^i||^2$$

$$\min_{\mathbf{P}^i,\mathbf{t}^i,\mathbf{X}_j} \sum_{i=1}^{m}\sum_{j=1}^{n} ||\mathbf{x}_j^i - (\mathbf{P}^i\mathbf{X}_j + \mathbf{t}^i)||^2$$

Affine Factorization: Algorithm Overview

1. Compute the translations $\mathbf{t}^i$.
2. Center the data.
3. Construct the $2m \times n$ measurement matrix $\mathbf{M}$ (a single matrix equation).
4. Factorize $\mathbf{M}$, i.e. find the closest rank 3 matrix which is closest to $\mathbf{M}$ in Frobenious norm (SVD of $\mathbf{M}$ truncated to rank 3).

1. Compute the Translations

- Calculate the average of the points seen e.g. in image 1:

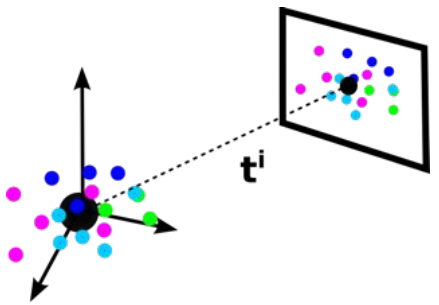
$$\begin{aligned} \mathbf{x}_1^1 &= \mathbf{P}^1\mathbf{X}_1 + \mathbf{t}^1 \\ &\vdots \\ \mathbf{x}_n^1 &= \mathbf{P}^1\mathbf{X}_n + \mathbf{t}^1 \\ \\ \frac{1}{n}\sum_{j=1}^{n}\mathbf{x}_j^1 &= \frac{1}{n}\mathbf{P}^1\sum_{j=1}^{n}\mathbf{X}_j + \mathbf{t}^1 \end{aligned}$$

1. Compute the Translations

- Calculate the average of the points seen e.g. in image 1:

$$\begin{aligned} \mathbf{x}_1^1 &= \mathbf{P}^1\mathbf{X}_1 + \mathbf{t}^1 \\ &\vdots \\ \mathbf{x}_n^1 &= \mathbf{P}^1\mathbf{X}_n + \mathbf{t}^1 \end{aligned}$$

$$\frac{1}{n}\sum_{j=1}^{n}\mathbf{x}_j^1 = \frac{1}{n}\mathbf{P}^1\sum_{j=1}^{n}\mathbf{X}_j + \mathbf{t}^1$$



- Affine cameras map the centroid of a set of 3D points to the centroid of their projection. Set the (arbitrary) origin of the 3D coordinate frame to the centroid of the unknown 3D points ($\frac{1}{n}\mathbf{P}^a\sum_{j=1}^{n}\mathbf{X}_j \overset{!}{=} 0$):

$$\mathbf{t}^i = \frac{1}{n}\sum_{j=1}^{n}\mathbf{x}_j^i$$

2. Center Image Points

- Transform the image measurements by subtracting the centroid of the points in each image.

$$\mathbf{x}_j^i = \mathbf{P}^i \mathbf{X}_j + \mathbf{t}^i \quad \Rightarrow \quad \begin{aligned} \mathbf{m}_j^i &\triangleq \mathbf{x}_j^i - \mathbf{t}^i \\ &= (\mathbf{P}^i \mathbf{X}_j + \mathbf{t}^i) - \mathbf{t}^i \\ &= \mathbf{P}^i \mathbf{X}_j \end{aligned}$$

- The resultant centered projection model is bilinear.
- Removing the translation, simplifies the problem:

$$\min_{\mathbf{P}^i, \mathbf{X}_j} \sum_{i=1}^{m} \sum_{j=1}^{n} ||\mathbf{m}_j^i - (\mathbf{P}_j^i \mathbf{X}_j)||^2$$

- All points must be seen in all cameras. (why?)

3. A Single Matrix Equation

- Gather the centered projections for image 1 in a single matrix equation:

$$\underbrace{\begin{pmatrix} \mathbf{m}_1^1 & \mathbf{m}_2^1 & \dots & \mathbf{m}_n^1 \end{pmatrix}}_{2 \times n} = \underbrace{\mathbf{P}^1}_{2 \times 3} \underbrace{\begin{pmatrix} \mathbf{X}_1 & \mathbf{X}_2 & \dots & \mathbf{X}_n \end{pmatrix}}_{3 \times n}$$

- Further, gather the equations for all images

$$\underbrace{\begin{pmatrix} \mathbf{m}_1^1 & \mathbf{m}_2^1 & \dots & \mathbf{m}_n^1 \\ \mathbf{m}_1^2 & \mathbf{m}_2^2 & \dots & \mathbf{m}_n^2 \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{m}_1^m & \mathbf{m}_2^m & \dots & \mathbf{m}_n^m \end{pmatrix}}_{\mathbf{M}} = \underbrace{\begin{pmatrix} \mathbf{P}^1 \\ \mathbf{P}^2 \\ \vdots \\ \mathbf{P}^m \end{pmatrix}}_{\mathbf{P}} \underbrace{\begin{pmatrix} \mathbf{X}_1 & \mathbf{X}_2 & \dots & \mathbf{X}_n \end{pmatrix}}_{\mathbf{S}}$$

3. A Single Matrix Equation

$$\underbrace{\begin{pmatrix} \mathbf{m}_1^1 & \mathbf{m}_2^1 & \dots & \mathbf{m}_n^1 \\ \mathbf{m}_1^2 & \mathbf{m}_2^2 & \dots & \mathbf{m}_n^2 \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{m}_1^m & \mathbf{m}_2^m & \dots & \mathbf{m}_n^m \end{pmatrix}}_{\mathbf{M}} = \underbrace{\begin{pmatrix} \mathbf{P}^1 \\ \mathbf{P}^2 \\ \vdots \\ \mathbf{P}^m \end{pmatrix}}_{\mathbf{P}} \underbrace{\begin{pmatrix} \mathbf{X}_1 & \mathbf{X}_2 & \dots & \mathbf{X}_n \end{pmatrix}}_{\mathbf{S}}$$

- The $(2m \times n)$ matrix $\mathbf{M}$ is the measurement matrix.
- The $(2m \times 3)$ matrix $\mathbf{P}$ is the (affine) projection matrix.
- The $(3 \times n)$ matrix $\mathbf{S}$ is the structure matrix.

3. A Single Matrix Equation

$$\underbrace{\begin{pmatrix} \mathbf{m}_1^1 & \mathbf{m}_2^1 & \dots & \mathbf{m}_n^1 \\ \mathbf{m}_1^2 & \mathbf{m}_2^2 & \dots & \mathbf{m}_n^2 \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{m}_1^m & \mathbf{m}_2^m & \dots & \mathbf{m}_n^m \end{pmatrix}}_{\mathbf{M}} = \underbrace{\begin{pmatrix} \mathbf{P}^1 \\ \mathbf{P}^2 \\ \vdots \\ \mathbf{P}^m \end{pmatrix}}_{\mathbf{P}} \underbrace{\begin{pmatrix} \mathbf{X}_1 & \mathbf{X}_2 & \dots & \mathbf{X}_n \end{pmatrix}}_{\mathbf{S}}$$

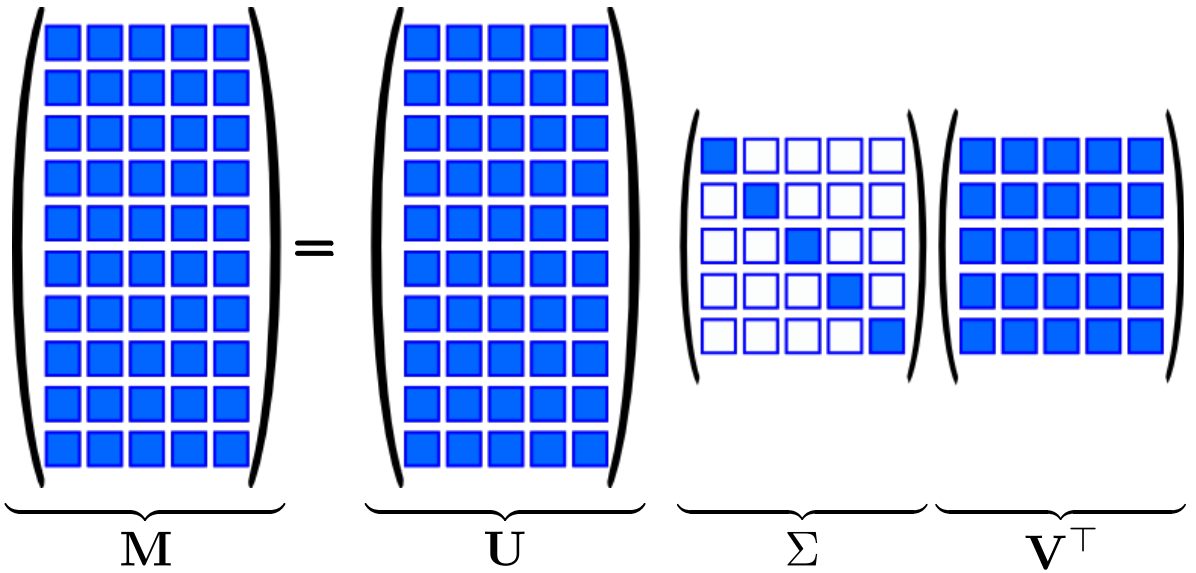
- The $(2m \times n)$ matrix $\mathbf{M}$ is the measurement matrix.
- The $(2m \times 3)$ matrix $\mathbf{P}$ is the (affine) projection matrix.
- The $(3 \times n)$ matrix $\mathbf{S}$ is the structure matrix.
- In the absence of noise, the measurement matrix has rank 3 at most.

3. A Single Matrix Equation

- In the presence of noise $\mathbf{M} = \mathbf{PS}$ is not exactly verified.
- We look for the matrix $\hat{\mathbf{M}}$ that is the closest possible to $\mathbf{M}$ in Frobenious norm sense.

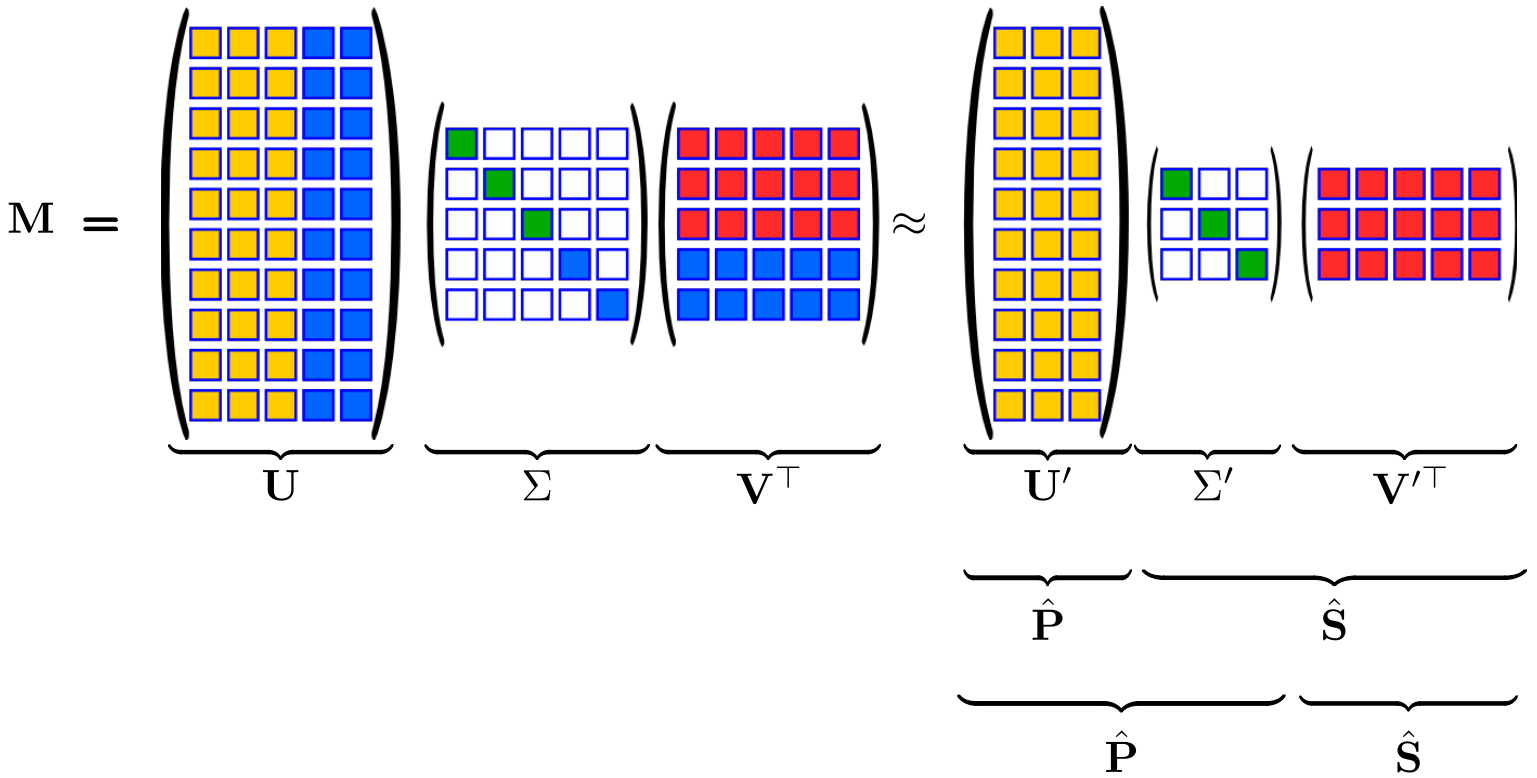
$$
\begin{aligned}
||\mathbf{M} - \hat{\mathbf{M}}||_F^2 &= \sum_{ij} (M_{ij} - \hat{M}_{ij})^2 \\
&= \sum_{ij} ||(\mathbf{m}_j^i - \hat{\mathbf{m}}_j^i)||^2 \\
&= \sum_{ij} ||(\mathbf{m}_j^i - \hat{\mathbf{P}}^i \hat{\mathbf{X}}_j)||^2
\end{aligned}
$$

4. Factorize M



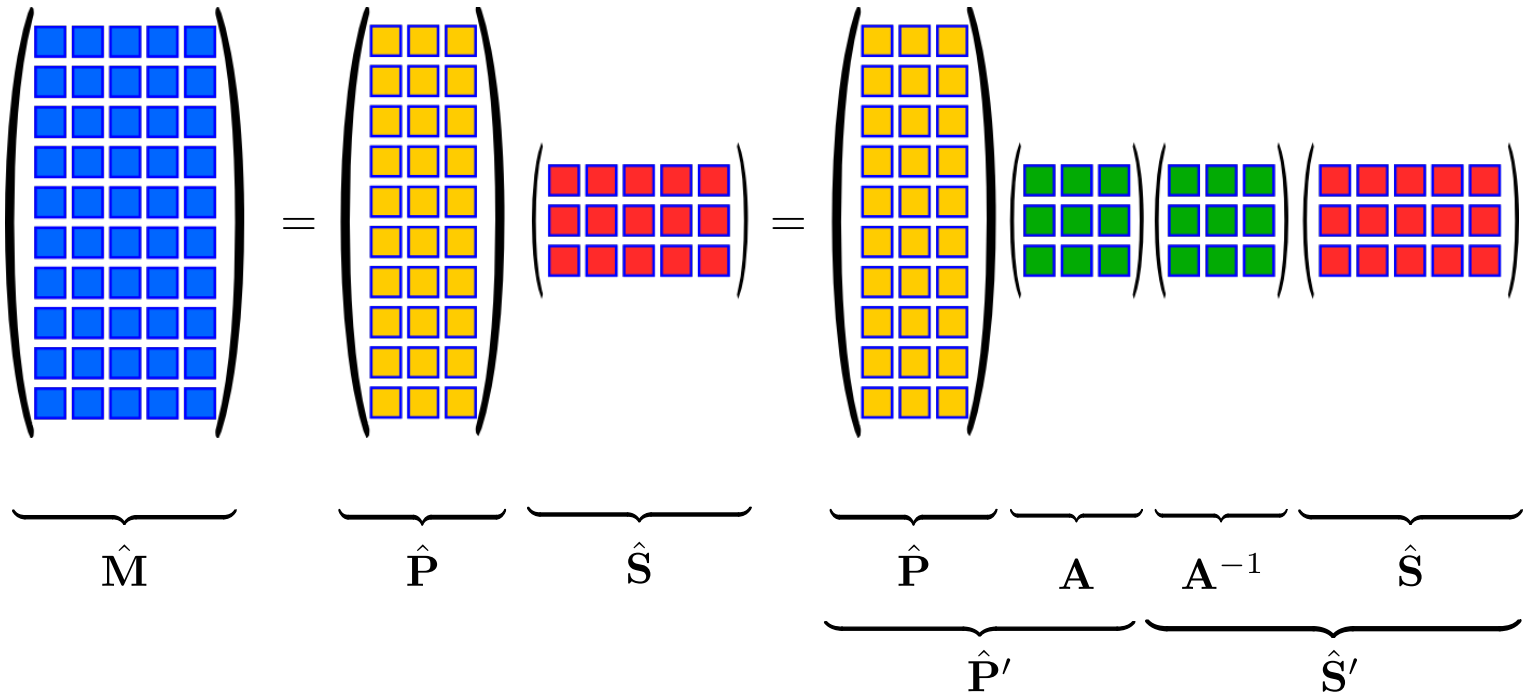
- The measurement matrix is factorized with SVD as $\mathbf{M} = \mathbf{U}\Sigma\mathbf{V}^\top$.

4. Factorize M



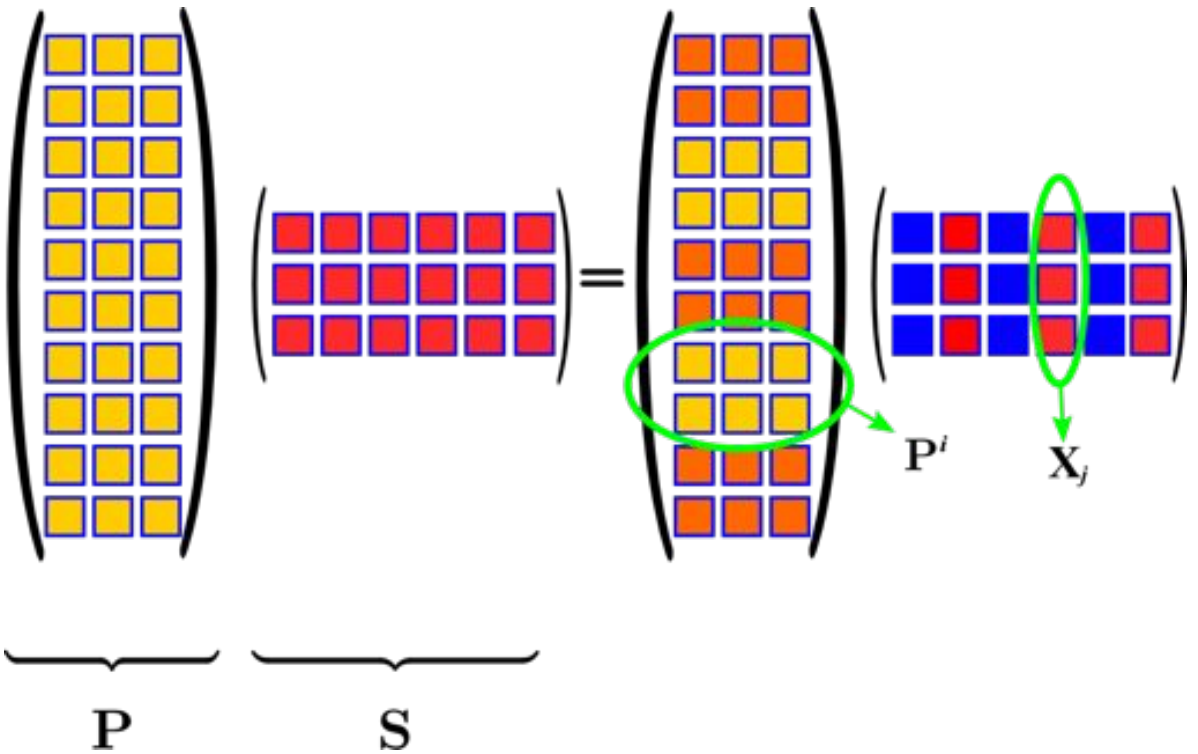
- The estimate of the joint **projection matrix** $\hat{\mathbf{P}}$ is the first 3 columns of $\mathbf{U}$.
- The estimate of the **structure matrix** $\hat{\mathbf{S}}$ is the first 3 rows of $\mathbf{V}^\top$.

4. Factorize M



- Closest rank-3 approximation $\hat{\mathbf{M}}$ yields Maximum Likelihood Estimate (assumption of isotropic mean-zero Gaussian noise).
- Affine ambiguity.

Identify the estimated parameters



Summary of the Algorithm

Input: n points in m images $\mathbf{x}_j^i$.

Outputs: 3D affine reconstruction defined by 3D points $\mathbf{X}_j^i$ and cameras $(\mathbf{P}^i, \mathbf{t}^i)$.

1. Compute the **translation** vectors $\mathbf{t}^i = \frac{1}{n}\sum_{j=1}^{n} \mathbf{x}_j^i$.
2. **Center** the image points $\mathbf{m}_j^i = \mathbf{x}_j^i - \mathbf{t}^i$.
3. Form and factorize the centered **measurement matrix** using SVD: $\mathbf{M} = \mathbf{U}\Sigma\mathbf{V}^\top$.
4. Extract the joint **projection matrix** $\widehat{\mathbf{P}}$ as the first 3 columns of $\mathbf{U}$.
5. Extract the joint **structure matrix** $\widehat{\mathbf{S}}$ as the first 3 rows of $\mathbf{V}^\top$.
6. Extract the individual projection matrices $\mathbf{P}^i$ from $\widehat{\mathbf{P}}$ and the individual 3D points $\mathbf{X}_j$ from $\widehat{\mathbf{S}}$.

Outline

Orthographic Factorization

[Tomasi Kanade, 1992] method first estimates the affine structure and then **upgrades it to Euclidean**. It operates on a batch of at least 3 frames with known correspondences (i.e., with tracked features).

- Factorize $\mathbf{M}$ through singular value decomposition:

$$\mathbf{M} = \mathbf{U}\Sigma\mathbf{V}^\top$$

- The affine reconstruction can be computed as:

$$\tilde{\mathbf{P}} = \mathbf{U}, \qquad \tilde{\mathbf{S}} = \mathbf{V}^\top$$

- Upgrade to a metric reconstruction, by finding an invertible matrix $\mathbf{A}$, such that:

$$\hat{\mathbf{P}} = \tilde{\mathbf{P}}\mathbf{A}^{-1}, \qquad \hat{\mathbf{S}} = \mathbf{A}\tilde{\mathbf{S}}$$

Orthographic Factorization

To find $\mathbf{A}$ additionally consider metric constraints under an orthographic camera model:

$$\mathcal{P}^i_{3\times 4} = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1/f & f \end{bmatrix}_{3\times 4} \begin{bmatrix} \mathbf{R} & \mathbf{t} \\ \mathbf{0}^\top & 1 \end{bmatrix}_{4\times 4}$$

- This model appears as the limit of the general perspective projection as the focal length f becomes large with respect to the distance of the camera to the object.

Orthographic Factorization

To find $\mathbf{A}$ additionally consider metric constraints under an orthographic camera model:

$$\mathcal{P}^i_{3\times 4} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{3\times 4} \begin{bmatrix} \mathbf{R} & \mathbf{t} \\ \mathbf{0}^\top & 1 \end{bmatrix}_{4\times 4}$$

- This model appears as the limit of the general perspective projection as the focal length f becomes large with respect to the distance of the camera to the object.

Orthographic Factorization

To find $\mathbf{A}$ additionally consider metric constraints under an orthographic camera model:

$$\mathcal{P}^i_{3\times 4} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{3\times 4} \begin{bmatrix} \mathbf{R} & \mathbf{t} \\ \mathbf{0}^\top & 1 \end{bmatrix}_{4\times 4}$$

$$= \begin{bmatrix} \mathbf{r}_1^{i\top} & t_1^i \\ \mathbf{r}_2^{i\top} & t_2^i \\ \mathbf{0}^\top & 1 \end{bmatrix}_{3\times 4}$$

- This model appears as the limit of the general perspective projection as the focal length f becomes large with respect to the distance of the camera to the object.
- 5 d.o.f.s (3 due to the rotation and 2 due to the translation)

Orthographic Factorization

$$\widehat{\mathbf{P}}^i = \begin{bmatrix} \widehat{\mathbf{p}}_1^{i\top} \\ \widehat{\mathbf{p}}_2^{i\top} \end{bmatrix} = \begin{bmatrix} \mathbf{r}_1^{i\top} & 0 \\ \mathbf{r}_2^{i\top} & 0 \end{bmatrix}$$

Remove the translation as in the affine case. Then, the orthographic projection matrix is characterized by two orthonormal vectors $\mathbf{r}_1^i$ and $\mathbf{r}_2^i$:

$$\begin{aligned} \widehat{\mathbf{p}}_1^{i\top}\hat{\mathbf{p}}_1^i &= 1 \\ \widehat{\mathbf{p}}_2^{i\top}\hat{\mathbf{p}}_2^i &= 1 \quad \rightarrow \\ \widehat{\mathbf{p}}_1^{i\top}\hat{\mathbf{p}}_2^i &= 0 \end{aligned}$$

Orthographic Factorization

$$\widehat{\mathbf{P}}^i = \begin{bmatrix} \widehat{\mathbf{p}}_1^{i\top} \\ \widehat{\mathbf{p}}_2^{i\top} \end{bmatrix} = \begin{bmatrix} \mathbf{r}_1^{i\top} & 0 \\ \mathbf{r}_2^{i\top} & 0 \end{bmatrix}$$

Remove the translation as in the affine case. Then, the orthographic projection matrix is characterized by two orthonormal vectors $\mathbf{r}_1^i$ and $\mathbf{r}_2^i$:

$$\begin{aligned} \widehat{\mathbf{p}}_1^{i\top}\widehat{\mathbf{p}}_1^i &= 1 \\ \widehat{\mathbf{p}}_2^{i\top}\widehat{\mathbf{p}}_2^i &= 1 \\ \widehat{\mathbf{p}}_1^{i\top}\widehat{\mathbf{p}}_2^i &= 0 \end{aligned} \quad \rightarrow \quad \begin{aligned} \tilde{\mathbf{p}}_1^{i\top}\,\mathbf{A}^{-1}\mathbf{A}^{-\top}\,\tilde{\mathbf{p}}_1^i &= 1 \\ \tilde{\mathbf{p}}_2^{i\top}\,\mathbf{A}^{-1}\mathbf{A}^{-\top}\,\tilde{\mathbf{p}}_2^i &= 1 \\ \tilde{\mathbf{p}}_1^{i\top}\,\mathbf{A}^{-1}\mathbf{A}^{-\top}\,\tilde{\mathbf{p}}_2^i &= 0 \end{aligned}$$

Remember that $\hat{\mathbf{P}} = \tilde{\mathbf{P}}\mathbf{A}^{-1}$

Orthographic Factorization

$$\widehat{\mathbf{P}}^i = \begin{bmatrix} \widehat{\mathbf{p}}_1^{i\top} \\ \widehat{\mathbf{p}}_2^{i\top} \end{bmatrix} = \begin{bmatrix} \mathbf{r}_1^{i\top} & 0 \\ \mathbf{r}_2^{i\top} & 0 \end{bmatrix}$$

Remove the translation as in the affine case. Then, the orthographic projection matrix is characterized by two orthonormal vectors $\mathbf{r}_1^i$ and $\mathbf{r}_2^i$:

$$\begin{aligned} \widehat{\mathbf{p}}_1^{i\top}\widehat{\mathbf{p}}_1^i &= 1 \\ \widehat{\mathbf{p}}_2^{i\top}\widehat{\mathbf{p}}_2^i &= 1 \\ \widehat{\mathbf{p}}_1^{i\top}\widehat{\mathbf{p}}_2^i &= 0 \end{aligned} \quad \rightarrow \quad \begin{aligned} \tilde{\mathbf{p}}_1^{i\top}\,\mathbf{C}\,\tilde{\mathbf{p}}_1^i &= 1 \\ \tilde{\mathbf{p}}_2^{i\top}\,\mathbf{C}\,\tilde{\mathbf{p}}_2^i &= 1 \\ \tilde{\mathbf{p}}_1^{i\top}\,\mathbf{C}\,\tilde{\mathbf{p}}_2^i &= 0 \end{aligned}$$

Orthographic Factorization

$$\begin{array}{lll} \tilde{\mathbf{p}}_1^{i\top} & \mathbf{C} & \tilde{\mathbf{p}}_1^i = 1 \\ \tilde{\mathbf{p}}_2^{i\top} & \mathbf{C} & \tilde{\mathbf{p}}_2^i = 1 \\ \tilde{\mathbf{p}}_1^{i\top} & \mathbf{C} & \tilde{\mathbf{p}}_2^i = 0 \end{array}$$

- The orthographic constraints lead to 3 linear equations on symmetric matrix $\mathbf{C}$ (6DoF)
- $\mathbf{A}$ can be obtained from $\mathbf{C}$ through Cholesky factorization and inversion.
  - The Cholesky factorization decomposes of a symmetric, positive-definite matrix into the product of a lower triangular matrix and its conjugate transpose $\mathbf{C} = \mathbf{L}\mathbf{L}^\top$
  - $\mathbf{A} = \mathbf{L}^{-1}$

Outline

Perspective Factorization

According to perspective camera model (after centering the points):

$$\lambda_j^i \mathbf{m}_j^i = \mathbf{P}^i \mathbf{X}_j = \begin{bmatrix} \mathbf{p}_1^{i\top} \\ \mathbf{p}_2^{i\top} \\ \mathbf{p}_3^{i\top} \end{bmatrix}_{3\times 4} \mathbf{X}_j$$

- Each $\mathbf{P}^i$ is 3×4
- $\mathbf{X}_j$ is a (4×1) homogenous vector.
- $\mathbf{m}_j^i = \begin{bmatrix} x_j^i \\ y_j^i \\ 1 \end{bmatrix}$, with x_j^i, y_j^i the centered image coordinates.
- λ_j^i is a scale factor for each point: the "projective depth".

Perspective Factorization

$$\lambda_j^i \mathbf{m}_j^i = \mathbf{P}^i \mathbf{X}_j = \begin{bmatrix} \mathbf{P}_1^{i\top} \\ \mathbf{P}_2^{i\top} \\ \mathbf{P}_3^{i\top} \end{bmatrix} \mathbf{X}_j$$

- The single matrix equation becomes:

$$\underbrace{\begin{pmatrix} \lambda_1^1 \mathbf{m}_1^1 & \lambda_2^1 \mathbf{m}_2^1 & \dots & \lambda_n^1 \mathbf{m}_n^1 \\ \lambda_1^2 \mathbf{m}_1^2 & \lambda_2^2 \mathbf{m}_2^2 & \dots & \lambda_n^2 \mathbf{m}_n^2 \\ \vdots & \vdots & \ddots & \vdots \\ \lambda_1^m \mathbf{m}_1^m & \lambda_2^m \mathbf{m}_2^m & \dots & \lambda_n^m \mathbf{m}_n^m \end{pmatrix}}_{\mathbf{M}} = \underbrace{\begin{pmatrix} \mathbf{P}^1 \\ \mathbf{P}^2 \\ \vdots \\ \mathbf{P}^m \end{pmatrix}}_{\mathbf{P}} \underbrace{\begin{pmatrix} \mathbf{X}_1 & \mathbf{X}_2 & \dots & \mathbf{X}_n \end{pmatrix}}_{\mathbf{S}}$$

Perspective Factorization

$$\underbrace{\begin{pmatrix} \lambda_1^1\mathbf{m}_1^1 & \lambda_2^1\mathbf{m}_2^1 & \dots & \lambda_n^1\mathbf{m}_n^1 \\ \lambda_1^2\mathbf{m}_1^2 & \lambda_2^2\mathbf{m}_2^2 & \dots & \lambda_n^2\mathbf{m}_n^2 \\ \vdots & \vdots & \ddots & \vdots \\ \lambda_1^m\mathbf{m}_1^m & \lambda_2^m\mathbf{m}_2^m & \dots & \lambda_n^m\mathbf{m}_n^m \end{pmatrix}}_{\mathbf{M}} = \underbrace{\begin{pmatrix} \mathbf{P}^1 \\ \mathbf{P}^2 \\ \vdots \\ \mathbf{P}^m \end{pmatrix}}_{\mathbf{P}} \underbrace{\begin{pmatrix} \mathbf{X}_1 & \mathbf{X}_2 & \dots & \mathbf{X}_n \end{pmatrix}}_{\mathbf{S}}$$

Can we still use factorization?

Perspective Factorization

$$\underbrace{\begin{pmatrix} \lambda_1^1\mathbf{m}_1^1 & \lambda_2^1\mathbf{m}_2^1 & \dots & \lambda_n^1\mathbf{m}_n^1 \\ \lambda_1^2\mathbf{m}_1^2 & \lambda_2^2\mathbf{m}_2^2 & \dots & \lambda_n^2\mathbf{m}_n^2 \\ \vdots & \vdots & \ddots & \vdots \\ \lambda_1^m\mathbf{m}_1^m & \lambda_2^m\mathbf{m}_2^m & \dots & \lambda_n^m\mathbf{m}_n^m \end{pmatrix}}_{\mathbf{M}} = \underbrace{\begin{pmatrix} \mathbf{P}^1 \\ \mathbf{P}^2 \\ \vdots \\ \mathbf{P}^m \end{pmatrix}}_{\mathbf{P}} \underbrace{\begin{pmatrix} \mathbf{X}_1 & \mathbf{X}_2 & \dots & \mathbf{X}_n \end{pmatrix}}_{\mathbf{S}}$$

- Collect measurements in one image $\mathbf{M}^i = [\mathbf{m}_1^i \quad \mathbf{m}_2^i \quad \dots \quad \mathbf{m}_n^i]$
- Collect projective depths per camera $\Lambda^i = \mathrm{diag}(\lambda_1^i, \lambda_2^i, \dots, \lambda_n^i)$.

$$\begin{matrix} (\mathbf{M}^1) \\ (\mathbf{M}^2) \\ \vdots \\ (\mathbf{M}^m) \end{matrix} \quad \begin{matrix} (\Lambda^1) \\ (\Lambda^2) \\ \vdots \\ (\Lambda^m) \end{matrix} = \underbrace{\begin{matrix} (\mathbf{P}^1) \\ (\mathbf{P}^2) \\ \vdots \\ (\mathbf{P}^m) \end{matrix}}_{\mathbf{P}} \underbrace{\begin{pmatrix} \mathbf{X}_1 & \mathbf{X}_2 & \dots & \mathbf{X}_n \end{pmatrix}}_{\mathbf{S}}$$

Perspective Factorization

$$\begin{array}{ccc} \mathbf{M}^1\Lambda^1 & = & \mathbf{P}^1\mathbf{S} \\ \mathbf{M}^2\Lambda^2 & = & \mathbf{P}^2\mathbf{S} \\ \vdots & \vdots & \vdots \\ \mathbf{M}^m\Lambda^m & = & \mathbf{P}^m\mathbf{S} \end{array}$$

- $\mathbf{M}^i$ are known, but Λ^i,$\mathbf{P}^i$ and $\mathbf{S}$ are unknown.
- $\mathbf{PS}$ has $\mathrm{rank}\ 4$: product of a $3m \times 4$ and a $4 \times n$ matrices .
- Assume that Λ^i are known, then $\mathbf{PS}$ can be computed.
  - Use the singular value decomposition: $\mathbf{PS} = \mathbf{U}\Sigma\mathbf{V}^\top$
  - In the noise-free case $\Sigma = \mathrm{diag}(\sigma_1, \sigma_2, \sigma_3, \sigma_4, 0, \ldots, 0)$
  - A reconstruction is obtained by setting:
    - $\mathbf{P} =$ the first 4 columns of $\mathbf{U}\Sigma$.
    - $\mathbf{S} =$ the first 4 rows of $\mathbf{V}^\top$.

Perspective Factorization

$$\begin{pmatrix} \mathbf{M}^1\Lambda^1 \\ \mathbf{M}^2\Lambda^2 \\ \vdots \\ \mathbf{M}^m\Lambda^m \end{pmatrix} = \mathbf{PS}$$

- $\mathbf{M}^i$ are known, but Λ^i,$\mathbf{P}^i$ and $\mathbf{S}$ are unknown.
- $\mathbf{PS}$ has $\mathrm{rank}\ 4$: product of a $3m \times 4$ and a $4 \times n$ matrices .
- Assume that Λ^i are known, then $\mathbf{PS}$ can be computed.
  - Use the singular value decomposition: $\mathbf{PS} = \mathbf{U}\Sigma\mathbf{V}^\top$
  - In the noise-free case $\Sigma = \mathrm{diag}(\sigma_1, \sigma_2, \sigma_3, \sigma_4, 0, \ldots, 0)$
  - A reconstruction is obtained by setting:
    - $\mathbf{P} =$ the first 4 columns of $\mathbf{U}\Sigma$.
    - $\mathbf{S} =$ the first 4 rows of $\mathbf{V}^\top$.

Iterative Perspective Factorization Algorithm

- When Λ^i are unknown the following algorithm can be used:
  1. Normalize image data using isotropic scaling.

Iterative Perspective Factorization Algorithm

- When Λ^i are unknown the following algorithm can be used:
  1. Normalize image data using isotropic scaling.
  2. Initialize $\lambda_j^i = 1$ (e.g. set $\lambda_j^i = 1$, affine approximation).

Iterative Perspective Factorization Algorithm

- When Λ^i are unknown the following algorithm can be used:
  1. Normalize image data using isotropic scaling.
  2. Initialize $\lambda_j^i = 1$ (e.g. set $\lambda_j^i = 1$, affine approximation).
  3. Normalize the depths $(\alpha^i \beta_j \lambda_j^i)\mathbf{m}_j^i = (\alpha^i \mathbf{P}^i)(\beta_j \mathbf{X}_j)$.
     The closer $\lambda_j^i = 1$, the error being minimized represents better the geometric distance. Heuristic:
     - multiply each row of $\mathbf{P}^i$ by an α^i so that it has unit norm.
     - Do the same with β_j and the columns of $\mathbf{X}_j$

Iterative Perspective Factorization Algorithm

- When Λ^i are unknown the following algorithm can be used:
  1. Normalize image data using isotropic scaling.
  2. Initialize $\lambda_j^i = 1$ (e.g. set $\lambda_j^i = 1$, affine approximation).
  3. Normalize the depths $(\alpha^i \beta_j \lambda_j^i)\mathbf{m}_j^i = (\alpha^i \mathbf{P}^i)(\beta_j \mathbf{X}_j)$.
  4. Factorize $\mathbf{M}$ and obtain an estimate of $\mathbf{P}$ and $\mathbf{S}$.

Iterative Perspective Factorization Algorithm

- When Λ^i are unknown the following algorithm can be used:
  1. Normalize image data using isotropic scaling.
  2. Initialize $\lambda_j^i = 1$ (e.g. set $\lambda_j^i = 1$, affine approximation).
  3. Normalize the depths $(\alpha^i \beta_j \lambda_j^i)\mathbf{m}_j^i = (\alpha^i \mathbf{P}^i)(\beta_j \mathbf{X}_j)$.
  4. Factorize $\mathbf{M}$ and obtain an estimate of $\mathbf{P}$ and $\mathbf{S}$.
  5. If σ_5 is sufficiently small then `STOP`.

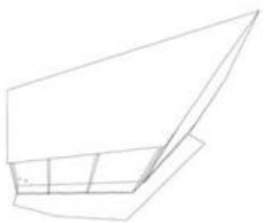
Iterative Perspective Factorization Algorithm

- When Λ^i are unknown the following algorithm can be used:
  1. Normalize image data using isotropic scaling.
  2. Initialize $\lambda_j^i = 1$ (e.g. set $\lambda_j^i = 1$, affine approximation).
  3. Normalize the depths $(\alpha^i \beta_j \lambda_j^i)\mathbf{m}_j^i = (\alpha^i \mathbf{P}^i)(\beta_j \mathbf{X}_j)$.
  4. Factorize $\mathbf{M}$ and obtain an estimate of $\mathbf{P}$ and $\mathbf{S}$.
  5. If σ_5 is sufficiently small then `STOP`.
  6. Use $\mathbf{m}$, $\mathbf{P}$ and $\mathbf{S}$ to estimate Λ^i from the camera equations (linearly) $\mathbf{m}^i \Lambda^i = \mathbf{P}^i \mathbf{S}$

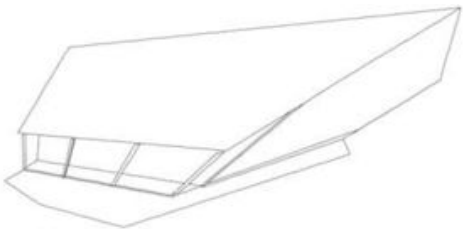
Iterative Perspective Factorization Algorithm

- When Λ^i are unknown the following algorithm can be used:
  1. Normalize image data using isotropic scaling.
  2. Initialize $\lambda_j^i = 1$ (e.g. set $\lambda_j^i = 1$, affine approximation).
  3. Normalize the depths $(\alpha^i \beta_j \lambda_j^i)\mathbf{m}_j^i = (\alpha^i \mathbf{P}^i)(\beta_j \mathbf{X}_j)$.
  4. Factorize $\mathbf{M}$ and obtain an estimate of $\mathbf{P}$ and $\mathbf{S}$.
  5. If σ_5 is sufficiently small then `STOP`.
  6. Use $\mathbf{m}$, $\mathbf{P}$ and $\mathbf{S}$ to estimate Λ^i from the camera equations (linearly) $\mathbf{m}^i \Lambda^i = \mathbf{P}^i \mathbf{S}$
  7. `GOTO` 3.
- In general the algorithm minimizes a proximity measure $\mathrm{Prox}(\Lambda, \mathbf{P}, \mathbf{S}) = \sigma_5$.
- Convergence is not guaranteed.
- Note that structure and motion are recovered up to an arbitrary projective transformation.

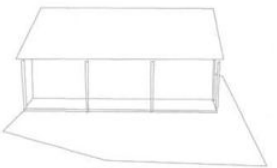
Euclidean Update



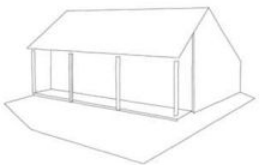
Projective



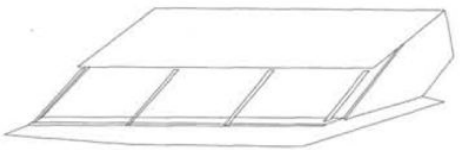
Projective



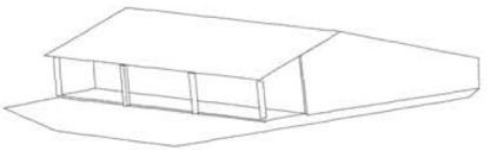
Euclidean



Euclidean



Affine



Affine

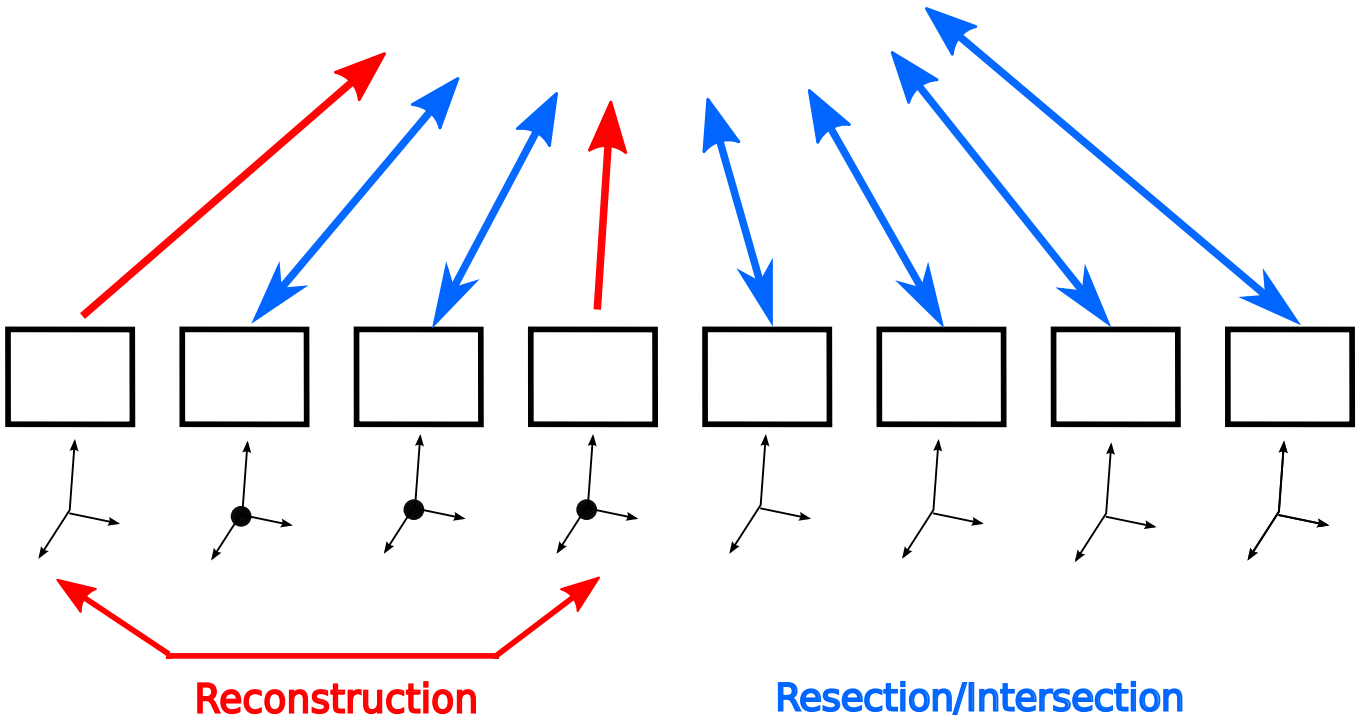
Euclidean

Euclidean

- The affine and perspective reconstructions may be upgraded to a metric reconstruction by:
  - Supplying metric information on the scene.
  - Using auto-calibration methods.
  - A combination of the two.
  - Hartley Zisserman - Chap 10.

Outline

Sequential Algorithms



Sequential Algorithms

- Register each view in turn.
- Pick a "small" number of views and compute a partial 3D model using e.g. factorization.
- Repeat until the 3D model is complete.
  - Pick an unregistered view and apply resection, i.e. compute the camera matrix. The view must share sufficiently many points with the partial 3D model.
  - Apply triangulation (intersection) to reconstruct new 3D features appearing in the latest registered view.

Sequential Algorithms

Pros

- Robustness to blunders is easily incorporated
- On-line processing

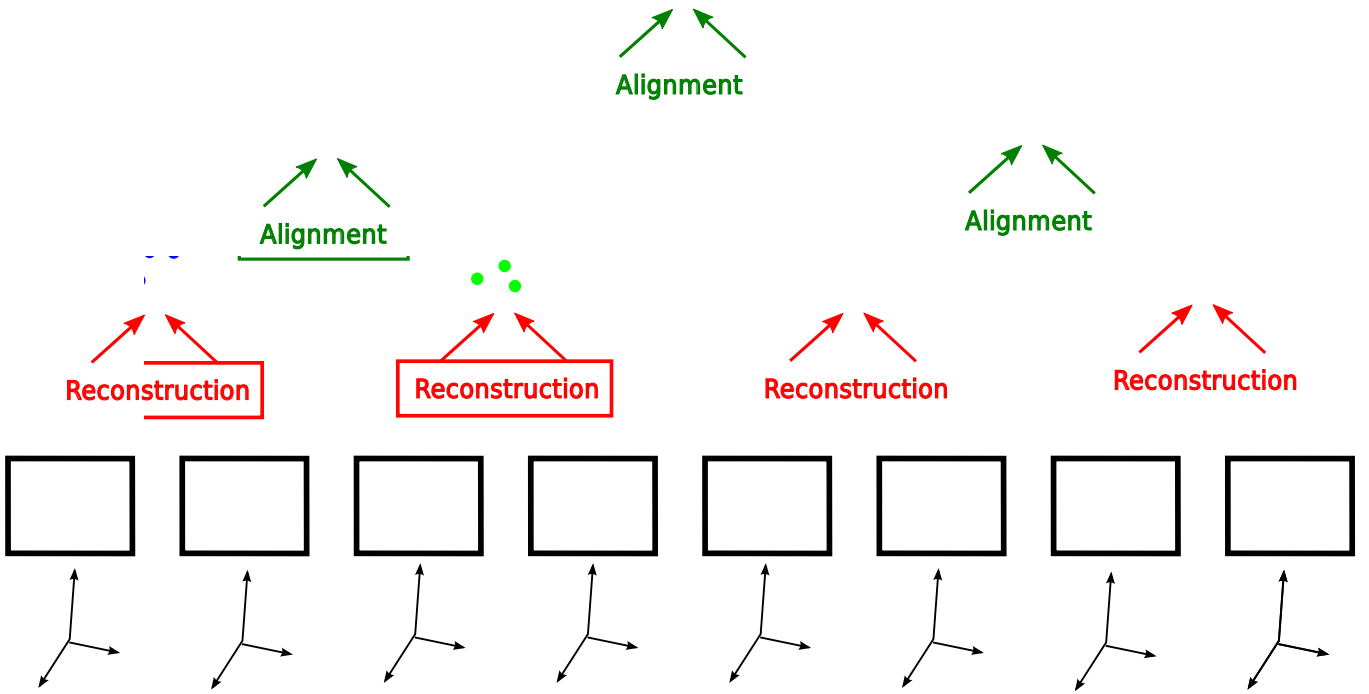
Cons

- The error in the initial partial 3D model conditions the final complete 3D model, i.e. it may make the 3D model drift away from the optimal one.

Note: If computational cost is not an important criteria, Bundle Adjustment is launched in the inner loop to prevent drifting.

Outline

Hierarchical Algorithms



Hierarchical Algorithms

- **Split** the image sequence into subsequences [Fitzgibbon et al., 1998 ; Nistr 2000].
- **Reconstruct** a partial 3D model from each subsequence using e.g. factorization.
- Hierarchically **merge** the partial 3D models (by computing a 3D homography, e.g. [Csurka et al., 1999]).
- This approach distributes the error over the entire sequence and can be easily made **robust**.

Extensions

- Extension to **paraperspective** cameras [Poelman and Kanade, 1993].
- Other **features** (lines and conics) [Kahl and Heyden, 1998 ; Quan, 1997].
- Extension to **projective** cameras [Sturm and Triggs, 1996]. The projective depths of each image point are recovered before factorization and the algorithm does not longer minimize the reprojection error.
- Missing data [Jacobs, 1997 ; Martinec and Pajdla, 2002].
- Multibody [Costeira and Kanade, 1995].
- Deformable [Bregler et al., 2000].

Recall: SFM Method Overview

1. Initialization: Compute a "good enough" sub-optimal estimate.
   - Batch.
   - Sequential.
   - Hierarchical algorithms.
2. Bundle Adjustment (Optional):
   - Non-linear minimization of the reprojection error (discrepancy between measured and predicted feature coordinates).
3. Self-Calibration.

Problem Statement

- 3D Points $\mathbf{X}_j$, with
  - with $j = \{1, \ldots, n\}$
  - n points.
- viewed by a set of cameras $\mathbf{P}^i$, with
  - $i = \{1, \ldots, m\}$
  - m views.

Problem: Given the projections $\mathbf{x}_j^i$ of the n points on the m cameras, find:

- The set of camera matrices $\widehat{\mathbf{P}}^i$
- and the set of points $\widehat{\mathbf{X}}_j$

such that:

$$\widehat{\mathbf{P}}^i \widehat{\mathbf{X}}_j = \mathbf{x}_j^i$$

Noisy Measurements

- If the image measurements are noisy, then the projection equation is not satisfied exactly.

$$\widehat{\mathbf{P}}^i \widehat{\mathbf{X}}_j \neq \mathbf{x}_j^i$$

- Use Maximum Likelihood estimation assuming noise is Gaussian.

Maximum Likelihood

- Estimate projection matrices $\hat{\mathbf{P}}^i$ and 3D points $\hat{\mathbf{X}}_j$ which project **exactly** to corrected image points $\hat{\mathbf{x}}_j^i$
- while minimizing the distance between the estimated $\hat{\mathbf{x}}_j^i$ and the measured $\mathbf{x}_j^i$ points for every view.

$$\min_{\hat{\mathbf{P}}^i, \hat{\mathbf{X}}_j} \sum_{ij} d(\, \hat{\mathbf{P}}^i \hat{\mathbf{X}}_j, \, \mathbf{x}_j^i \,)^2$$

- where $d(\mathbf{x}, \mathbf{y})$ is the geometric image distance between homogeneous points $\mathbf{x}$ and $\mathbf{y}$.

Bundle Adjustment

$$\min_{\hat{\mathbf{P}}^i, \hat{\mathbf{X}}_j} \sum_{ij} d(\, \hat{\mathbf{P}}^i \hat{\mathbf{X}}_j,\, \mathbf{x}_j^i \,)^2$$

$\min_{\hat{\mathbf{P}}^i, \hat{\mathbf{X}}_j} \ldots$
Adjust the bundle of rays between each camera center and the set of 3D points.

$\min_{\hat{\mathbf{P}}^i, \hat{\mathbf{X}}_j} \ldots$
Adjust the bundle of rays between each 3D point and the set of camera centers.

Non-linear Least Squares

Assume a problem can be modeled with a parameterized function f (parameters $\mathbf{p}$) such that:

$$\hat{\mathbf{x}} = f(\mathbf{p})$$

LSQ optimization seeks the parameters $\mathbf{p}$ that minimize the difference between the measure $\mathbf{x}$ and estimation $\hat{\mathbf{x}}$:

$$\arg\min_{\mathbf{p}} ||\mathbf{x} - f(\mathbf{p})||$$

- Iterative optimization algorithms:
  - Newton.
  - Levenberg-Marquardt
  - Sparse Levenberg-Marquardt

Number of parameters to optimize

- Each camera matrix $\mathbf{P}^i$ has 11 DoFs.
- There are m views.
- Each point $\mathbf{X}_j$ has 3 DoFs.
- There are n points.

There are $3n + 11m$ parameters to optimize !!!!

(This is a lower bound, it can be more due to over-parameterization)

- e.g. when using Levenberg Marquardt, matrices of $(3n + 11m) \times (3n + 11m)$ have to be factored or inverted, which is computationally expensive.

Practical Implementation

- Initialization
  - Use factorization for initialization.
  - Use available information:
    e.g. coplanarity leads to a closed form solution.
- Tackle the size of the problem by:
  - Reducing the number of views m or the number of points n.
    - Do not include all the points or all the views. These can be then computed by resection or triangulation.
    - Partition the data, e.g. hierarchical approach.
  - Interleave: iterate between minimizing the projection error by:
    - varying the entries of the cameras $\hat{\mathbf{P}}^i$.
    - varying the entries of the points $\hat{\mathbf{X}}^j$
  - Use sparse methods.

Bundle Adjustment

Used as a final step of any reconstruction.

Pros

- Provides ML estimate.
- Tolerant to missing data.
- Assigns covariances to each measure.
- Can be extended to include priors and constraints on camera parameters and point positions.

Cons:

- It requires a good initialization.
- It can become an extremely large minimization problem.

Recall

A 3×4 projection matrix $\mathbf{P}$ can be decomposed into

$$\mathbf{P} \propto \mathbf{K}[\mathbf{R} \quad \mathbf{t}]$$

$(\mathbf{R}; \mathbf{t})$ is the pose or extrinsic parameters
$\mathbf{K}$ is the calibration or intrinsic parameters

$$\mathbf{K} = \begin{pmatrix} \alpha f & sf & u_0 \\ 0 & f & v_0 \\ 0 & 0 & 1 \end{pmatrix}$$

f is the focal length
(u_0, v_0) is the principal point
α is the aspect ratio
s is the skew

Problem Statement

- The projective 3D model is defined up to a projective transformation

$$\mathbf{P}^i\mathbf{X}_j \quad \propto \quad \mathbf{P}^i\mathbf{H}\mathbf{H}^{-1}\mathbf{X}_j \quad = \quad \mathbf{P}^{i0}\mathbf{X}_j^0$$

  where $\mathbf{H}$ is a 3D (4×4) homography, with $\det(\mathbf{H}) = \mathbf{0}$ and 15 parameters.

- The goal is to obtain a Euclidean 3D model defined up to a similarity (7 parameters).
- The number of unknowns is thus $15 - 7 = 8$
- We are looking for a projective transformation $\mathbf{Z}$ such that:
  - $\mathbf{P}^i\mathbf{Z}$ are projection matrices
  - $\mathbf{Z}^{-1}\mathbf{X}_j$ are 3D points

  in a Euclidean coordinate frame.

Internal Camera Calibration

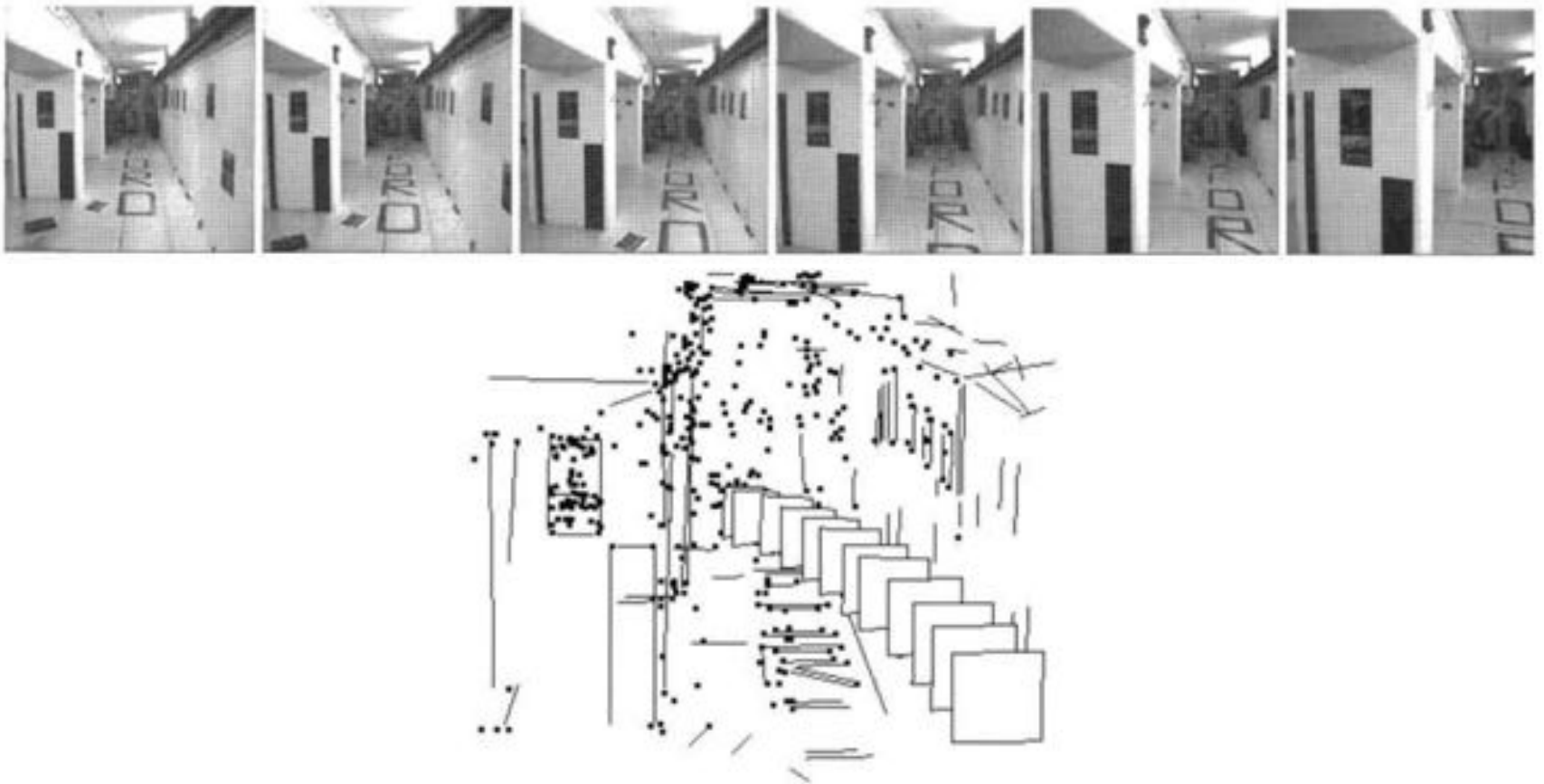
The internal camera calibration can be computed from prior knowledge (Chapter 19):

- on the scene, e.g. right angles, equal lengths.
- on the camera motion, e.g. pure translation, stereo rig.
- on the intrinsic parameters themselves:
  - Assume they are all unknown and constant.
  - Varying focal length calibration [Triggs, 1997 ; Pollefeys et al., 1998].

The estimated internal camera parameters allow us to compute a metric reconstruction from initially uncalibrated images

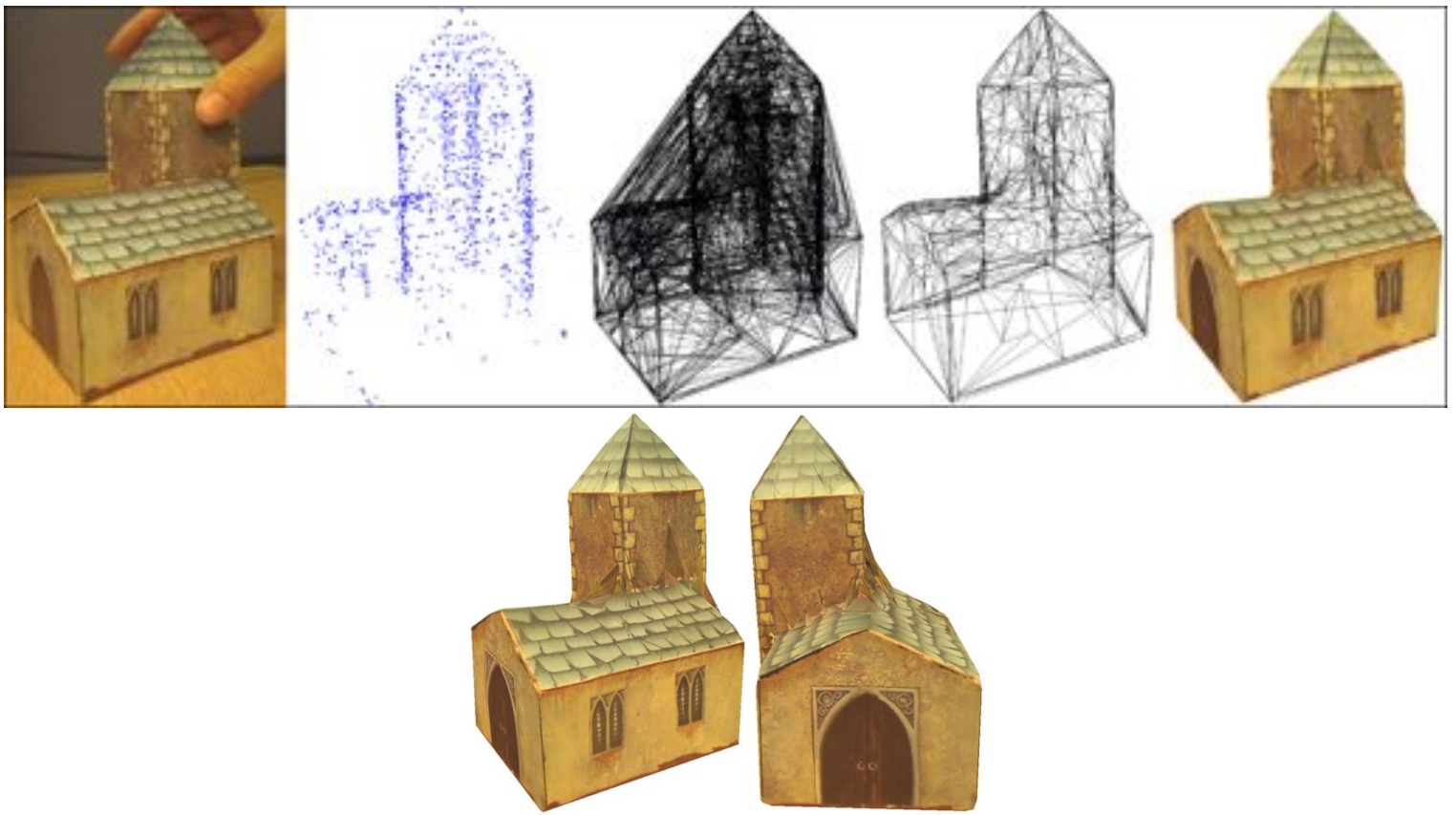
Outline

Example Applications



Automatic reconstruction of piecewise planar models from multiple views [Baillard,Zisserman-CVPR1999]

Example Applications

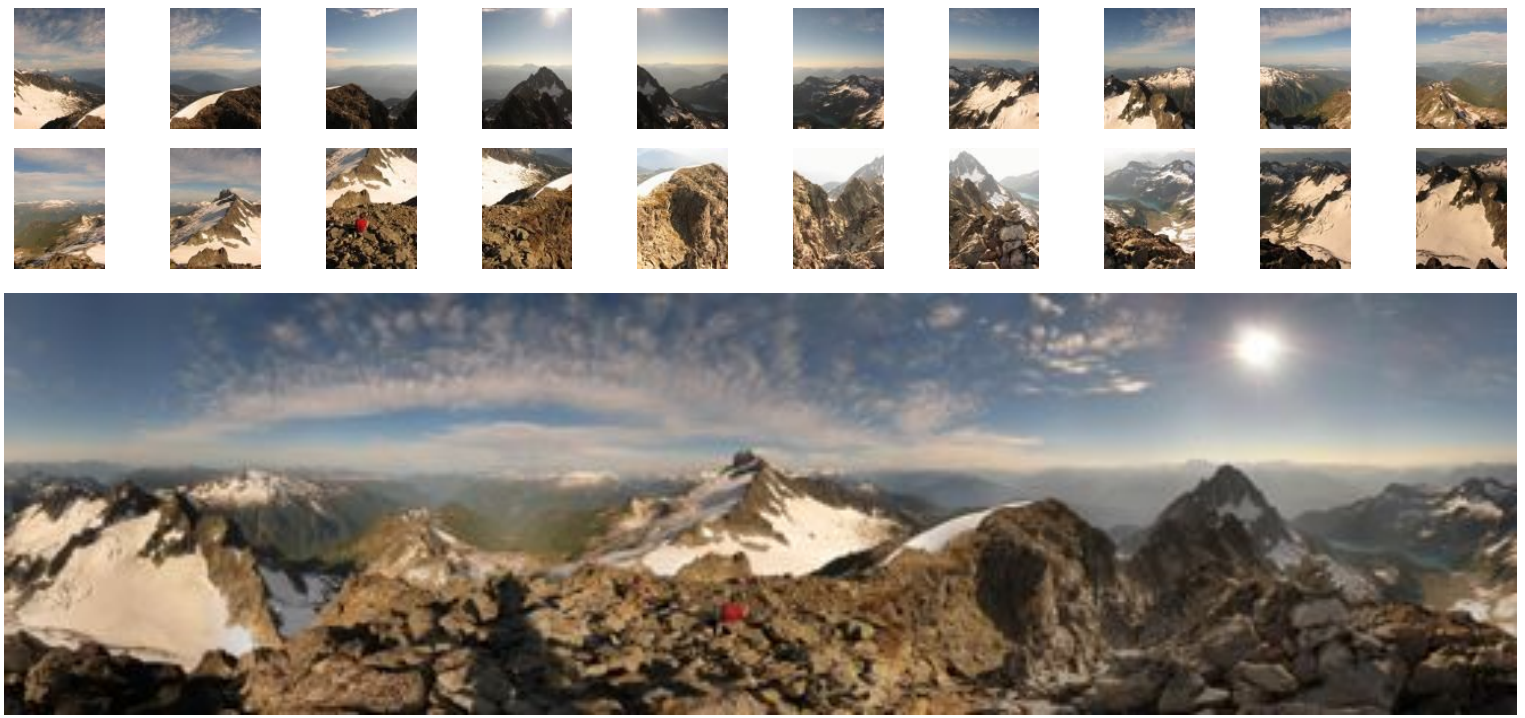


Creating 3D models with a simple webcam [Pan, Reitmayr, Drummond - BMVC 2009]

Comercial Software

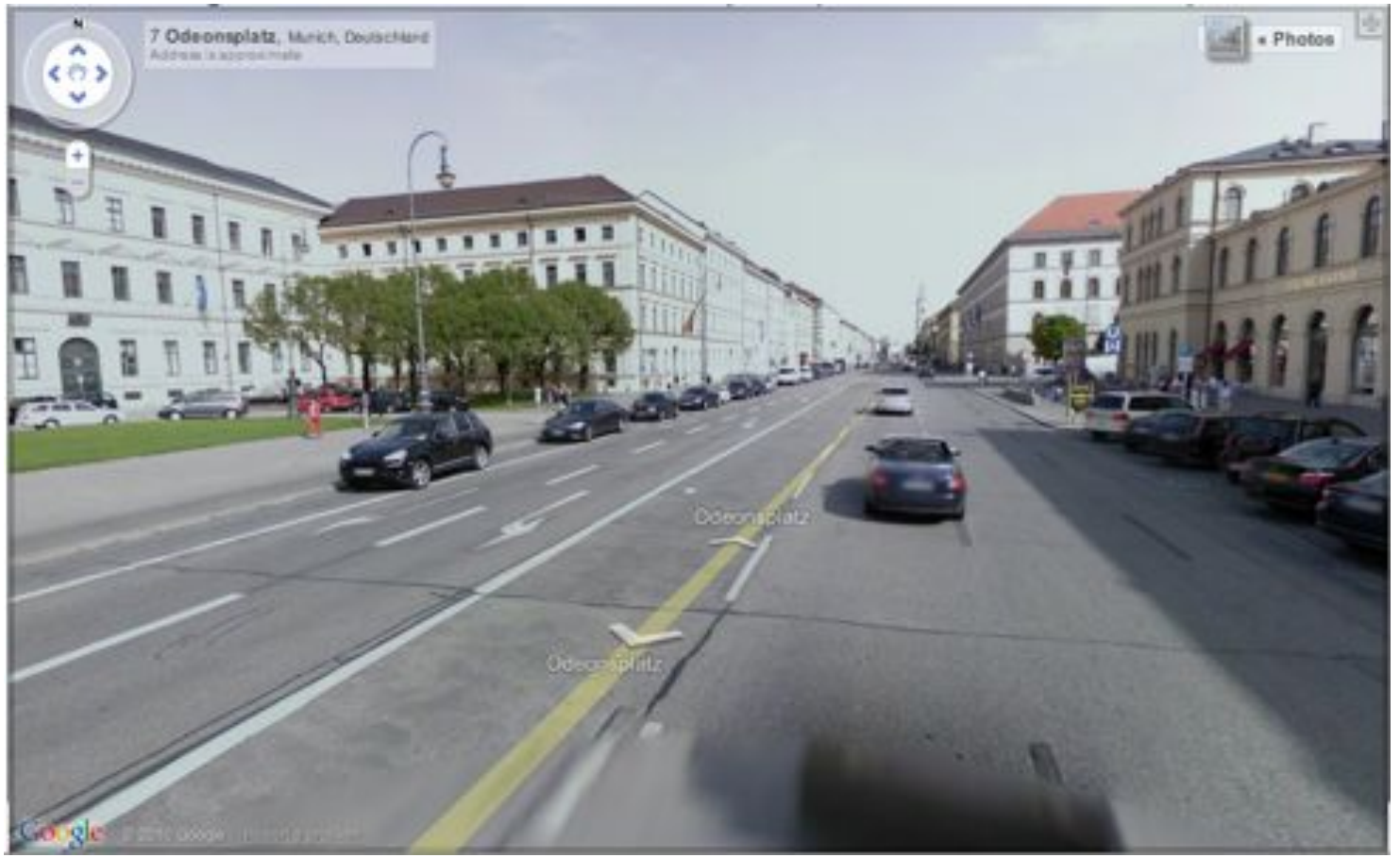
- 2d3 (www.2d3.com)
  - Boujou (camera tracking)
  - SteadyMove (video stabilizer)
  - ...
- Realviz (bought by Autodesk)
  - MatchMover (camera tracking)
  - Stitcher (image mosaicing)
  - ImageModeler (computer-aided 3D models from images)
  - ...
- Google Street view
- Microsoft Fototurism

Autostitch



http://people.cs.ubc.ca/~mbrown/autostitch/autostitch.html

Street View



http://www.talkmunich.com/services/munich_map.php

Bundler

Bundler: Structure from Motion for Unordered Image Collections

Photo Tourism

Photosynth

http://photosynth.net/

Outline