L-Tangent Norm: a Low Computational Cost Criterion for Choosing Regularization Weights and its use for Range Surface Reconstruction

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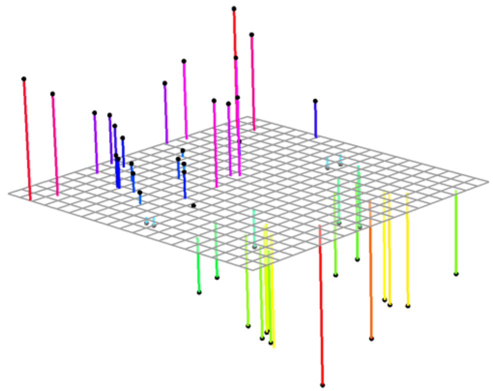
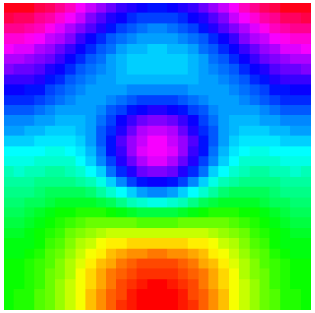
Presentation Outline

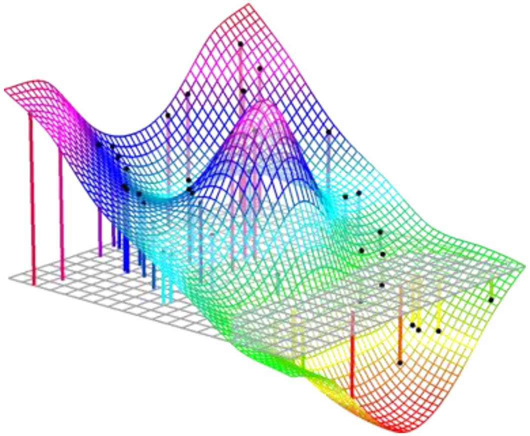
1. Introduction
2. Previous work
3. The L-Tangent Norm
4. Experimental results
5. Conclusion

Surface reconstruction

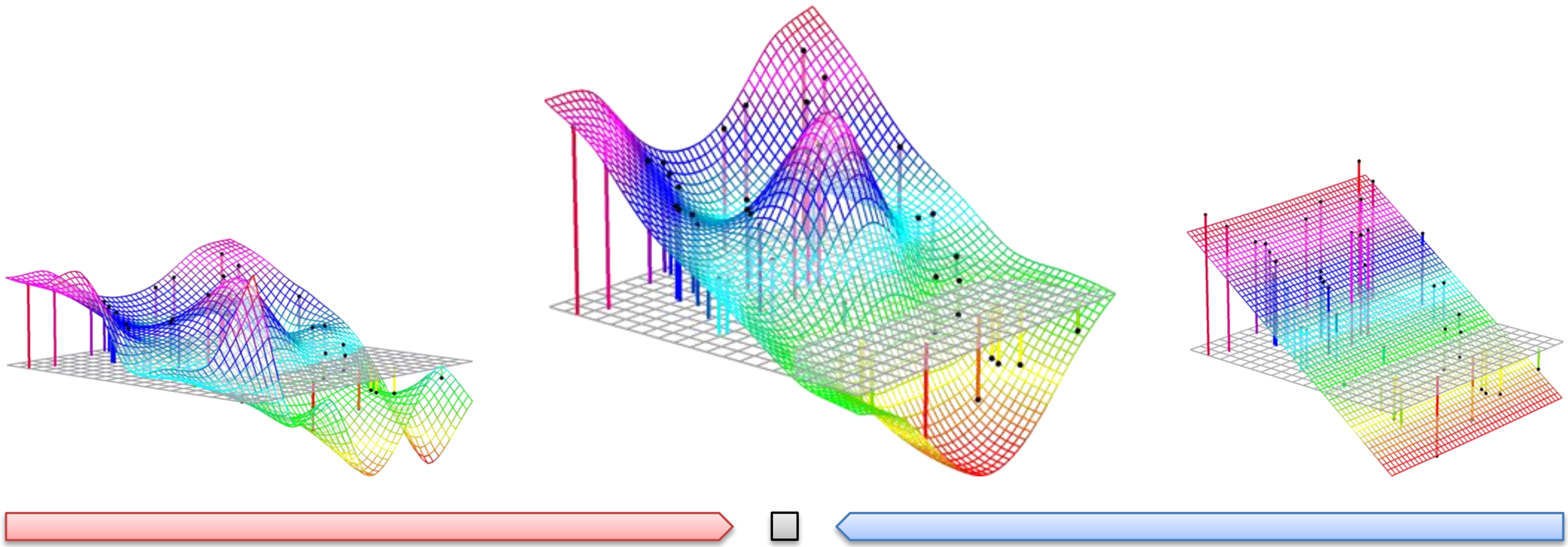




Approximating a set of range (2.5D) data points by a smooth surface



The regularization problem

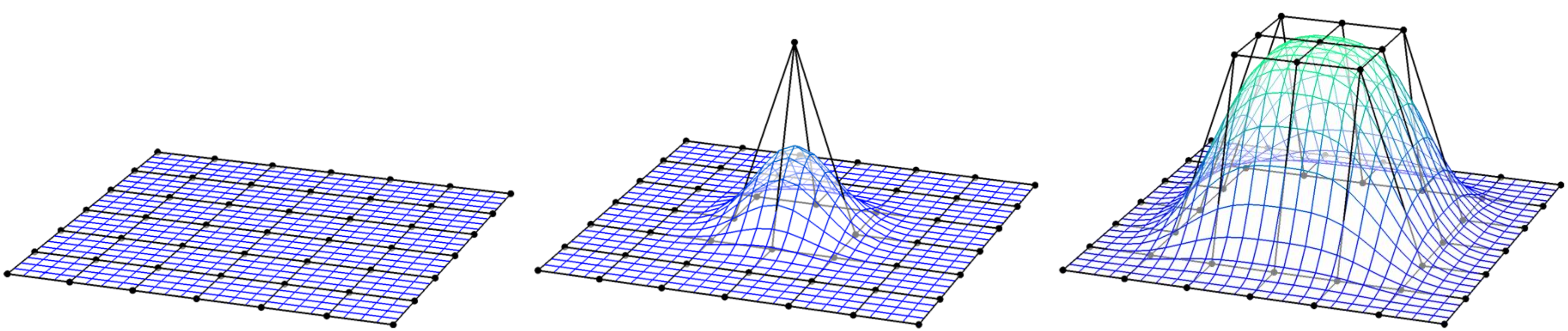


Find a good **compromise** between
overfitting and **underfitting**

Surface model

- Parametric surface model linear in its control points $P_{i,j}$
- In this talk: 1D curves and 2.5D B-spline surfaces

$$f(x, y; \mathbf{p}) = \sum_i \sum_j P_{i,j} B_i(x) B_j(y)$$



A minimization problem

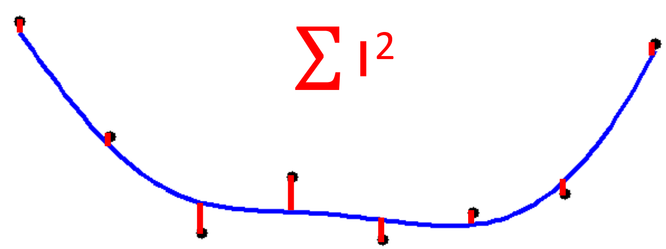
$$\mathbf{p}^* = \arg\min_{\mathbf{p}\in\mathbb{R}^h} \mathcal{E}_d(\mathbf{p}) + \frac{\lambda}{1-\lambda}\mathcal{E}_r(\mathbf{p})$$

Data term

Error between the surface and the data points

Mean squared residual

$$\mathcal{E}_d(p) = \frac{1}{n}\sum_{i=1}^{n}(f(x_i, y_i; \mathbf{p}) - z_i)^2$$



A minimization problem

$$\mathbf{p}^* = \arg\min_{\mathbf{p}\in\mathbb{R}^h} \mathcal{E}_d(\mathbf{p}) + \frac{\lambda}{1-\lambda}\mathcal{E}_r(\mathbf{p})$$

Smoothness term

Measures the surface smoothness

Bending energy

$$\mathcal{E}_r(\mathbf{p}) = \iint_\Omega \sum_{d=0}^{2} \binom{d}{2} \left(\frac{\partial^2 f}{\partial x^{2-d} \partial y^d}(x, y; p) \right)^2 \mathrm{d}x\mathrm{d}y$$

A Linear Least Squares problem

$$\mathbf{p}^* = \arg\min_{\mathbf{p}\in\mathbb{R}^h} \|M\mathbf{p} - \mathbf{y}\|_2^2 \quad \Leftrightarrow \quad \mathbf{p}^* = M^\dagger \mathbf{y}$$

$$M$$

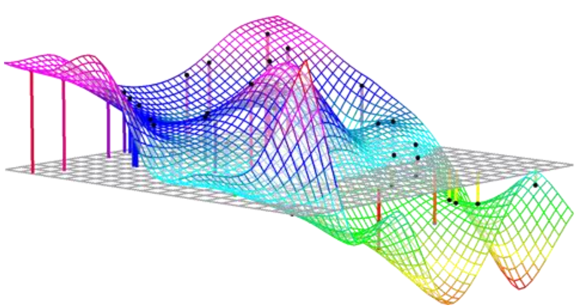
$$\mathcal{E}_d(\mathbf{p}) \qquad \mathcal{E}_r(\mathbf{p}) \qquad \lambda$$

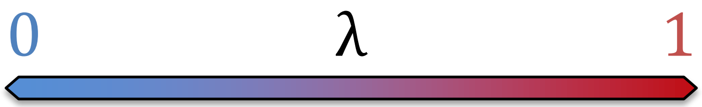
A minimization problem

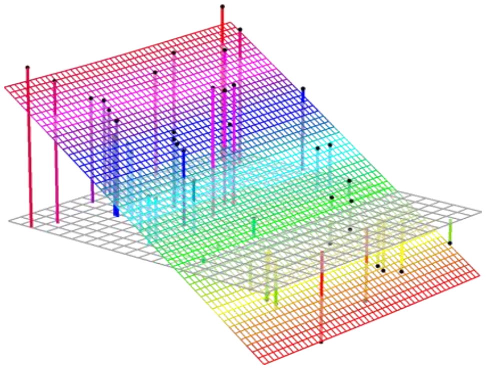
$$\mathbf{p}^* = \arg\min_{\mathbf{p}\in\mathbb{R}^h} \mathcal{E}_d(\mathbf{p}) + \frac{\lambda}{1-\lambda}\mathcal{E}_r(\mathbf{p})$$

Regularization parameter

Controls the trade-off between the closeness of the surface to the data and its smoothness



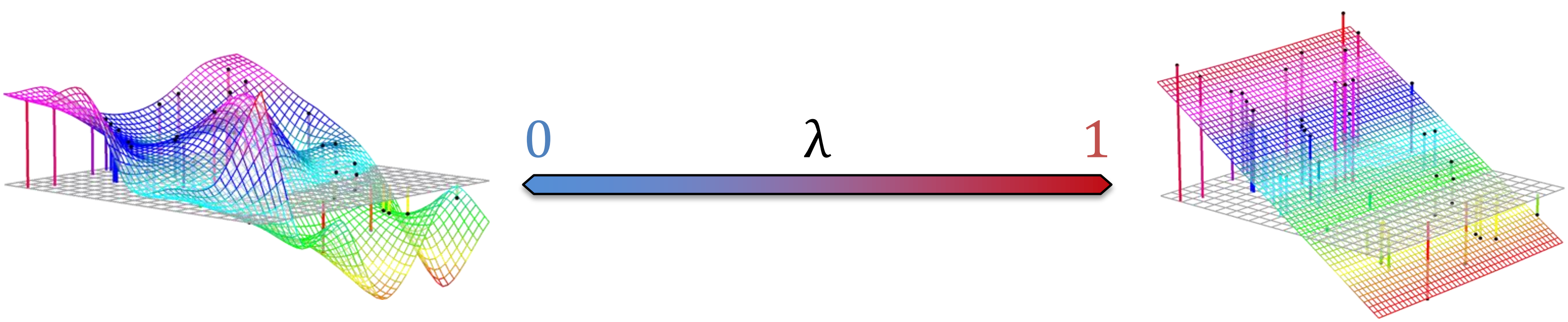




A minimization problem

$$\mathbf{p}^* = \arg\min_{\mathbf{p}\in\mathbb{R}^h} \mathcal{E}_d(\mathbf{p}) + \frac{\lambda}{1-\lambda}\mathcal{E}_r(\mathbf{p})$$

Goal: automatically select the regularization parameter



Outline

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Cross-Validation

- Introduced by [Wahba 1979]

Cross-Validation

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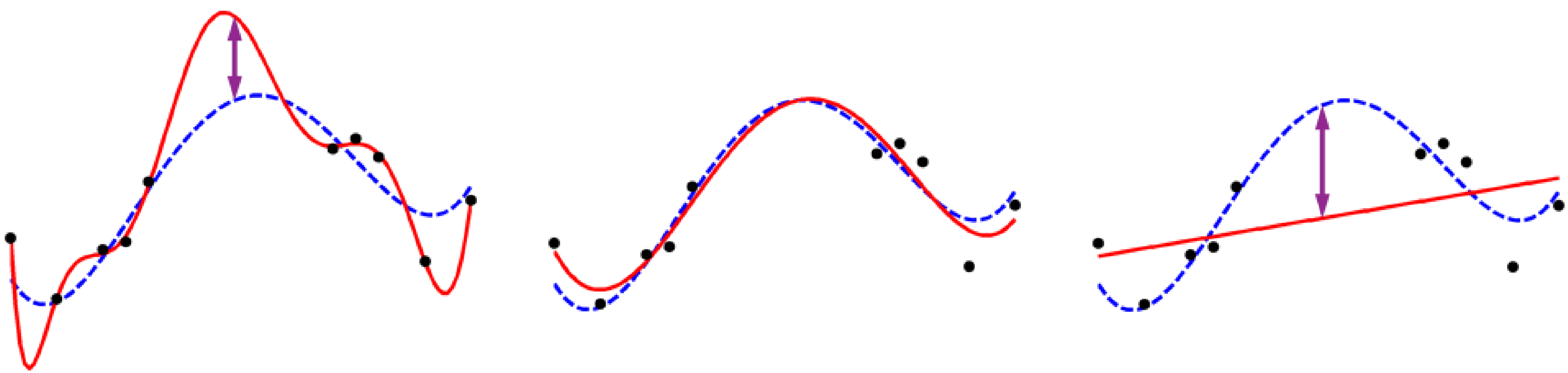
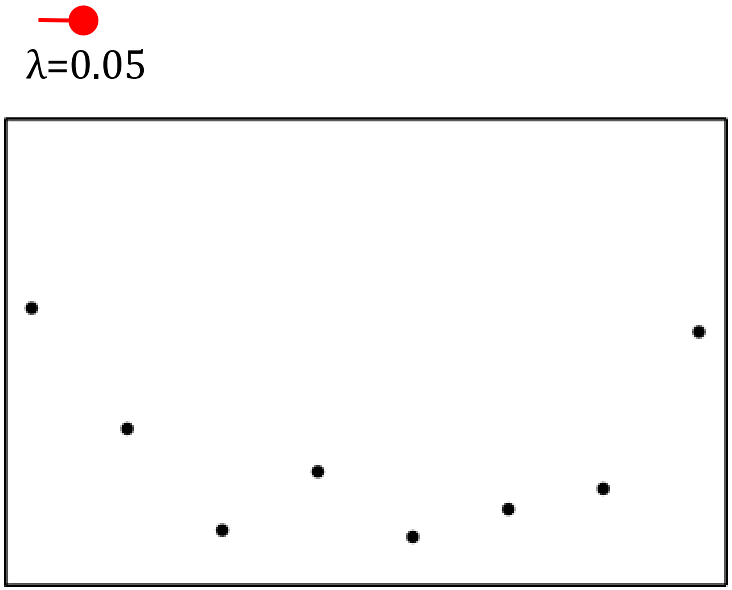
- A well reconstructed surface is one that *generalizes*

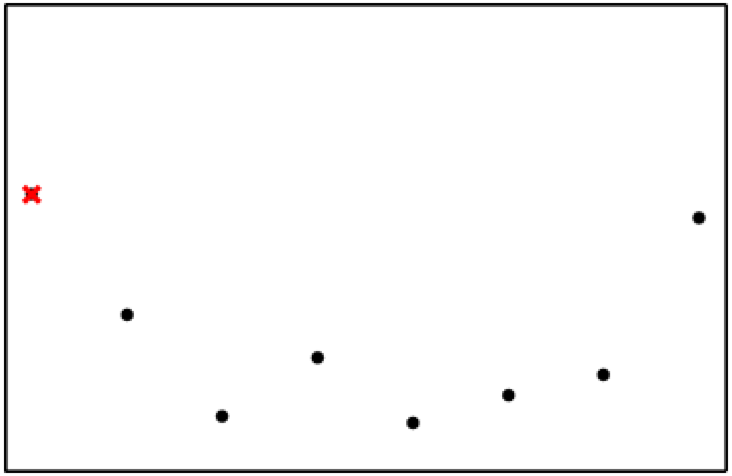


Cross-Validation



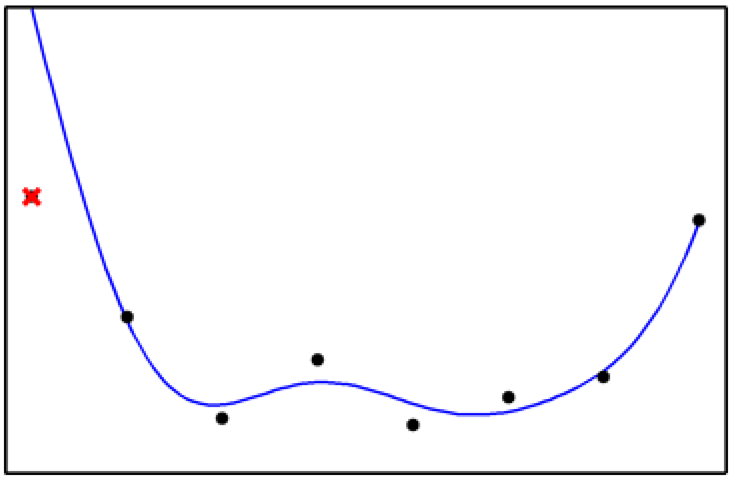
Cross-Validation

$\lambda=0.05$



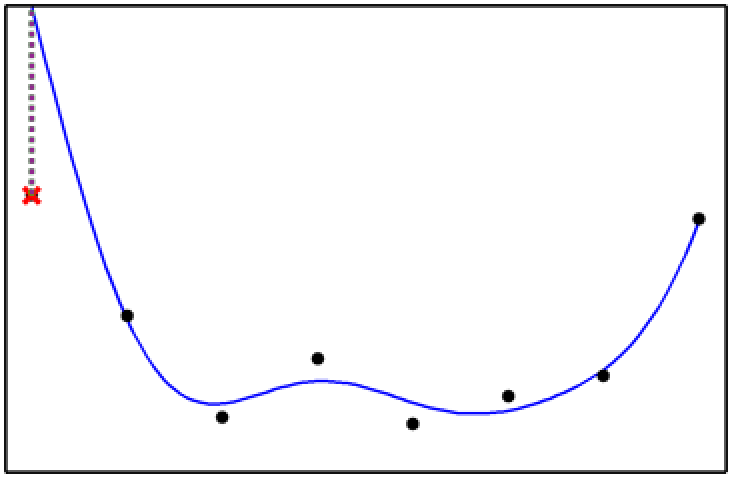
Cross-Validation

λ=0.05



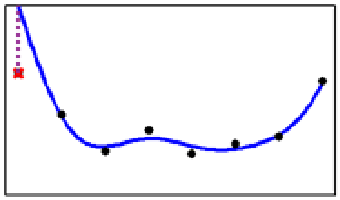
Cross-Validation

λ=0.05



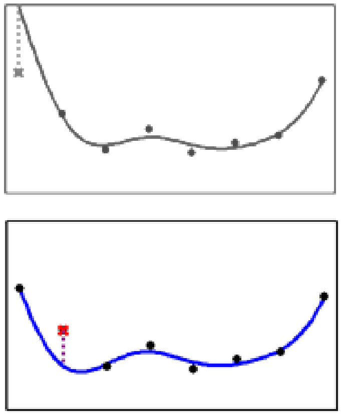
Cross-Validation

$\lambda=0.05$



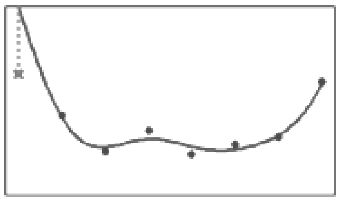
Cross-Validation

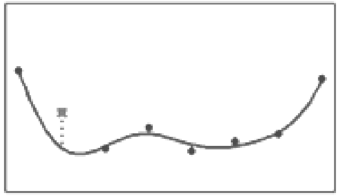
$\lambda=0.05$

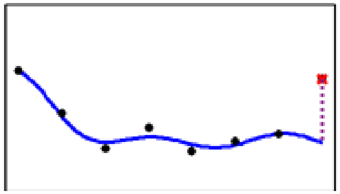


Cross-Validation

$\lambda=0.05$

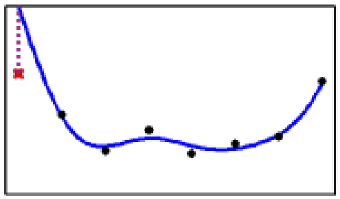


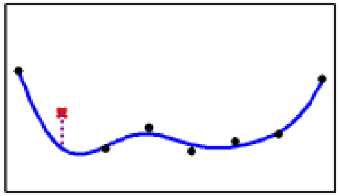


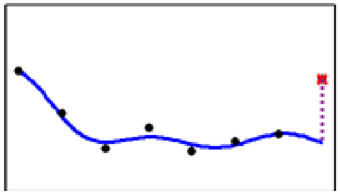


Cross-Validation

$\lambda=0.05$

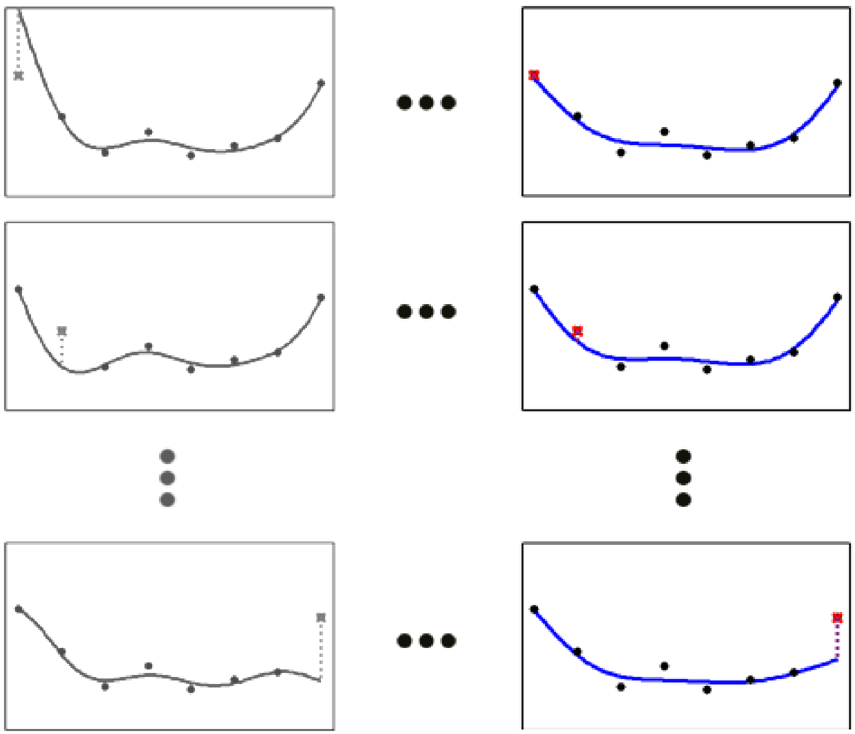


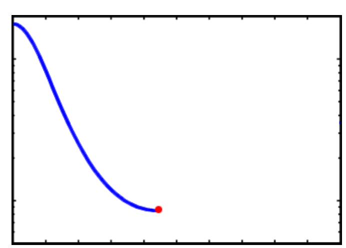




Cross-Validation

$\lambda=0.05$ $\lambda=0.45$





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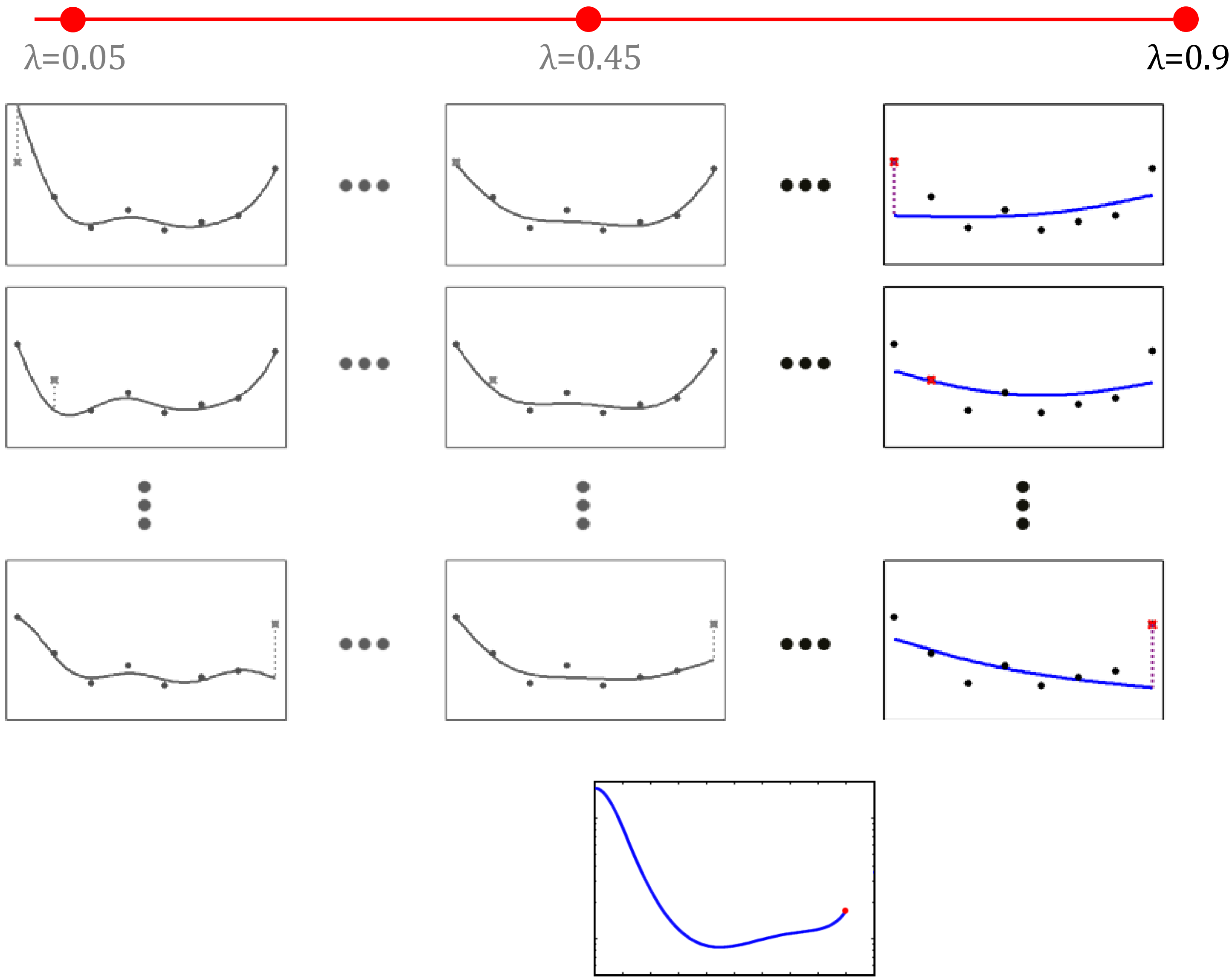

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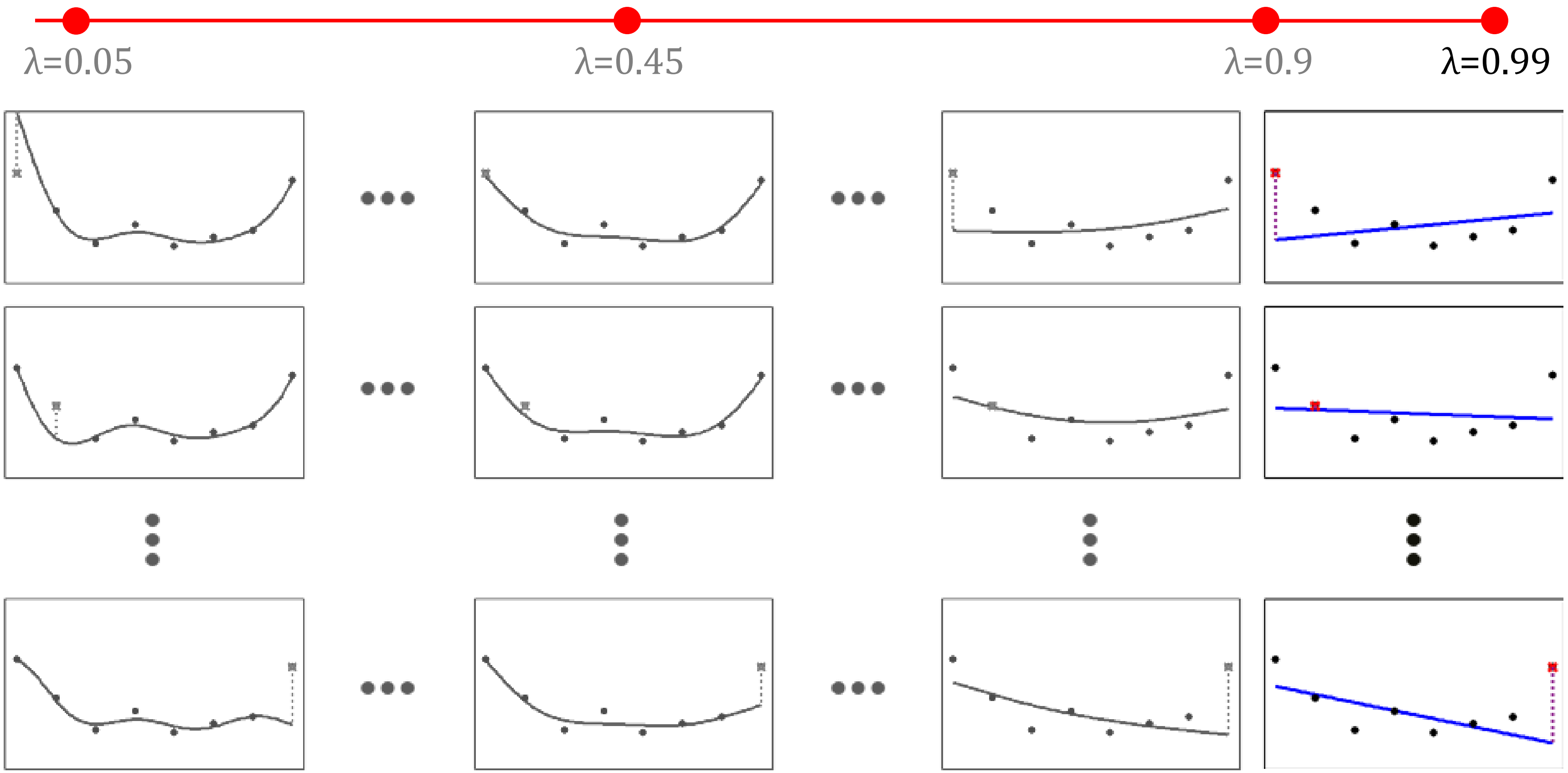

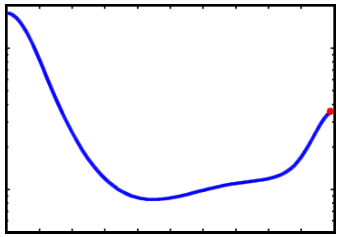

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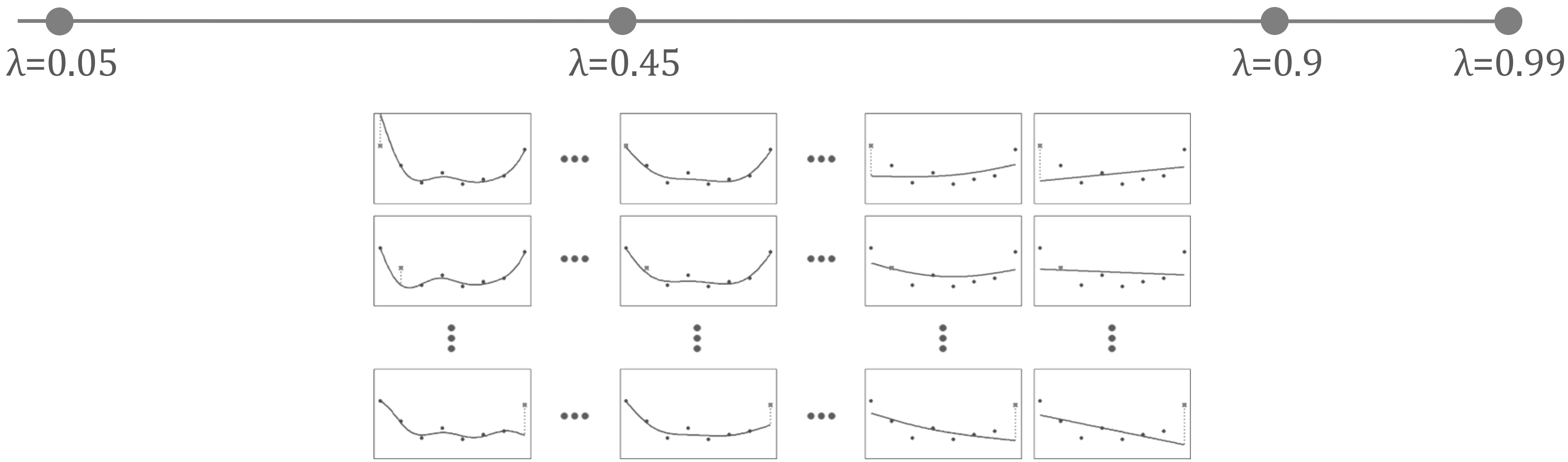
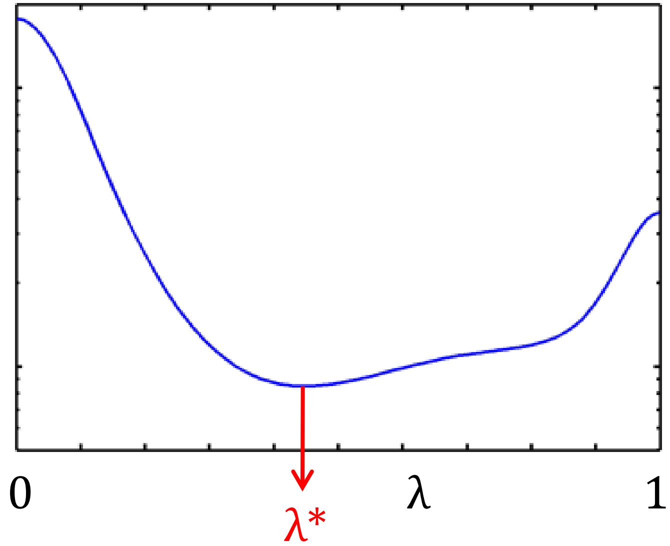
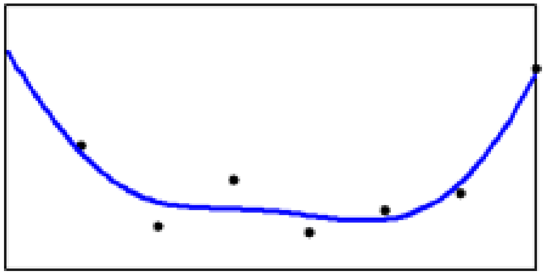






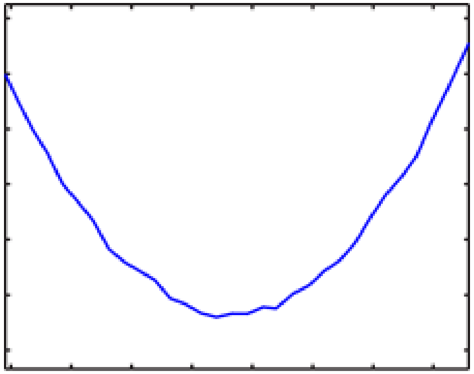
Problems with Cross-Validation

+

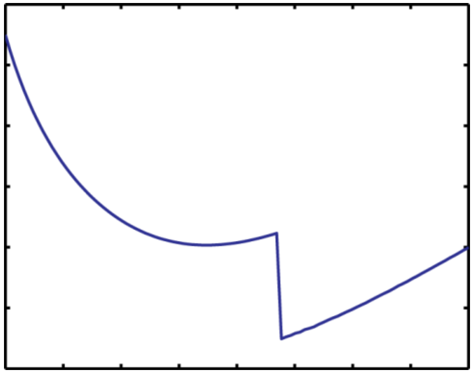
- Closed form expression

−

- Computation time
- Numerical instability
- Pathological cases



λ



λ

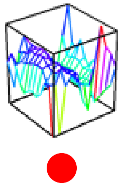
The L-Curve

- Introduced by [Lawson and Hanson, 1974]

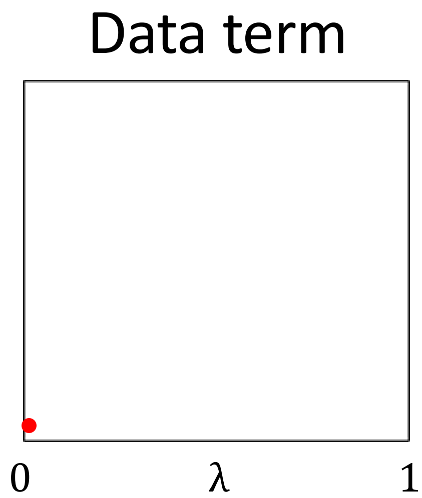
The L-Curve

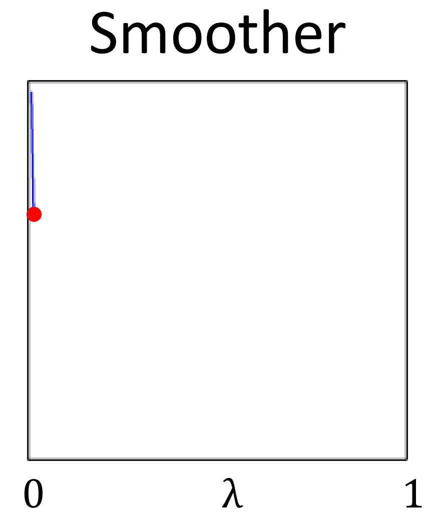
- Introduced by [Lawson and Hanson, 1974]
- Search a trade-off between underfitting and overfitting
  - The data term shouldn't be too large
  - The regularization term shouldn't be too large

The L-Curve

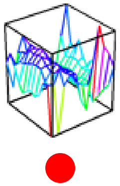


λ=0.01

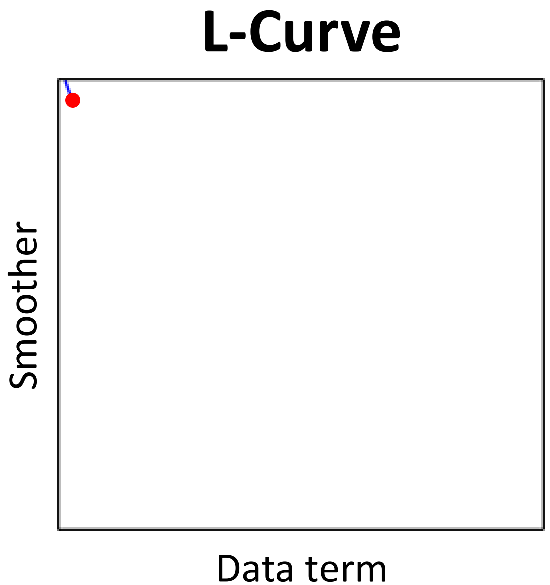


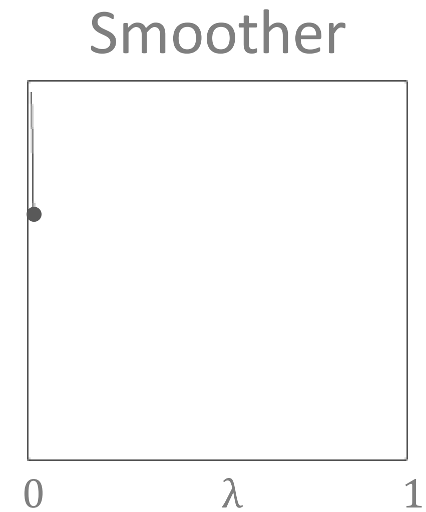


The L-Curve



λ=0.01





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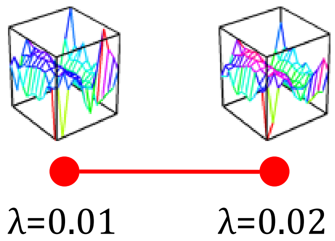
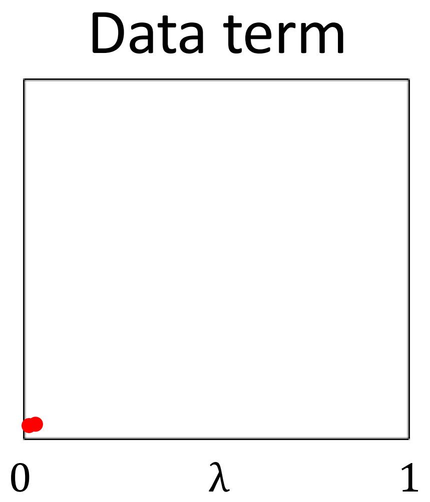
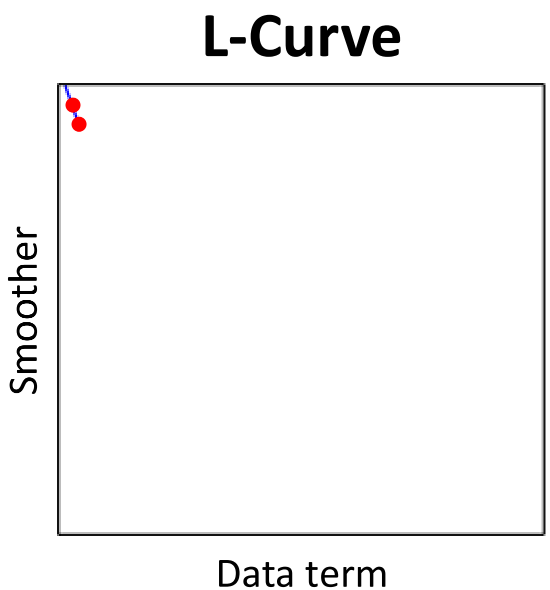
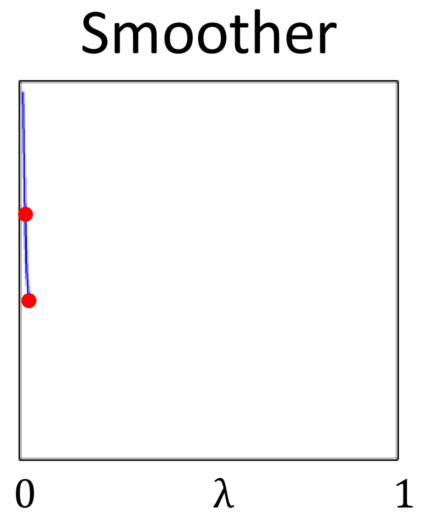
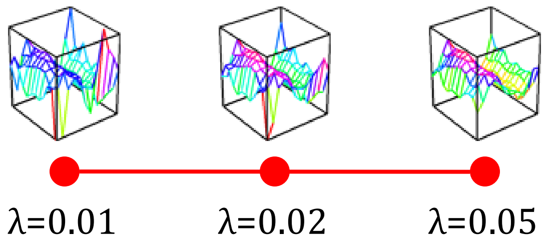

λ=0.01
λ=0.02


Data term
0
λ
1

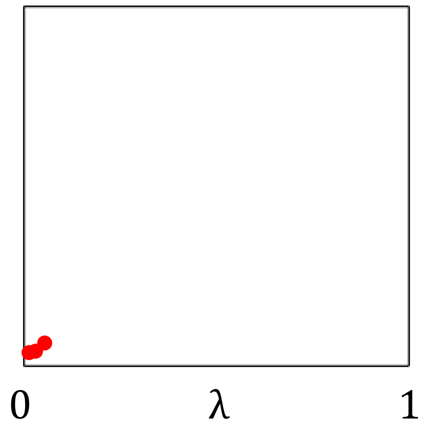

L-Curve
Smoother
Data term


Smoother
0
λ
1

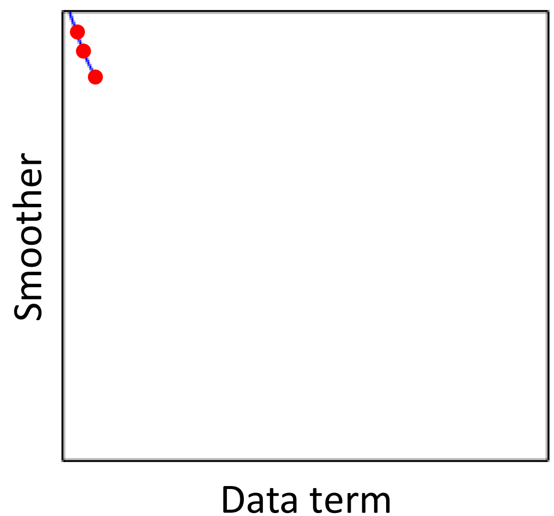
The L-Curve



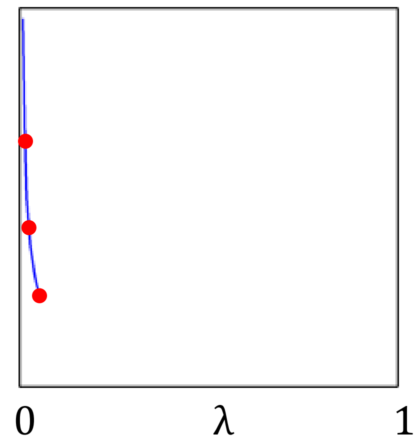
Data term



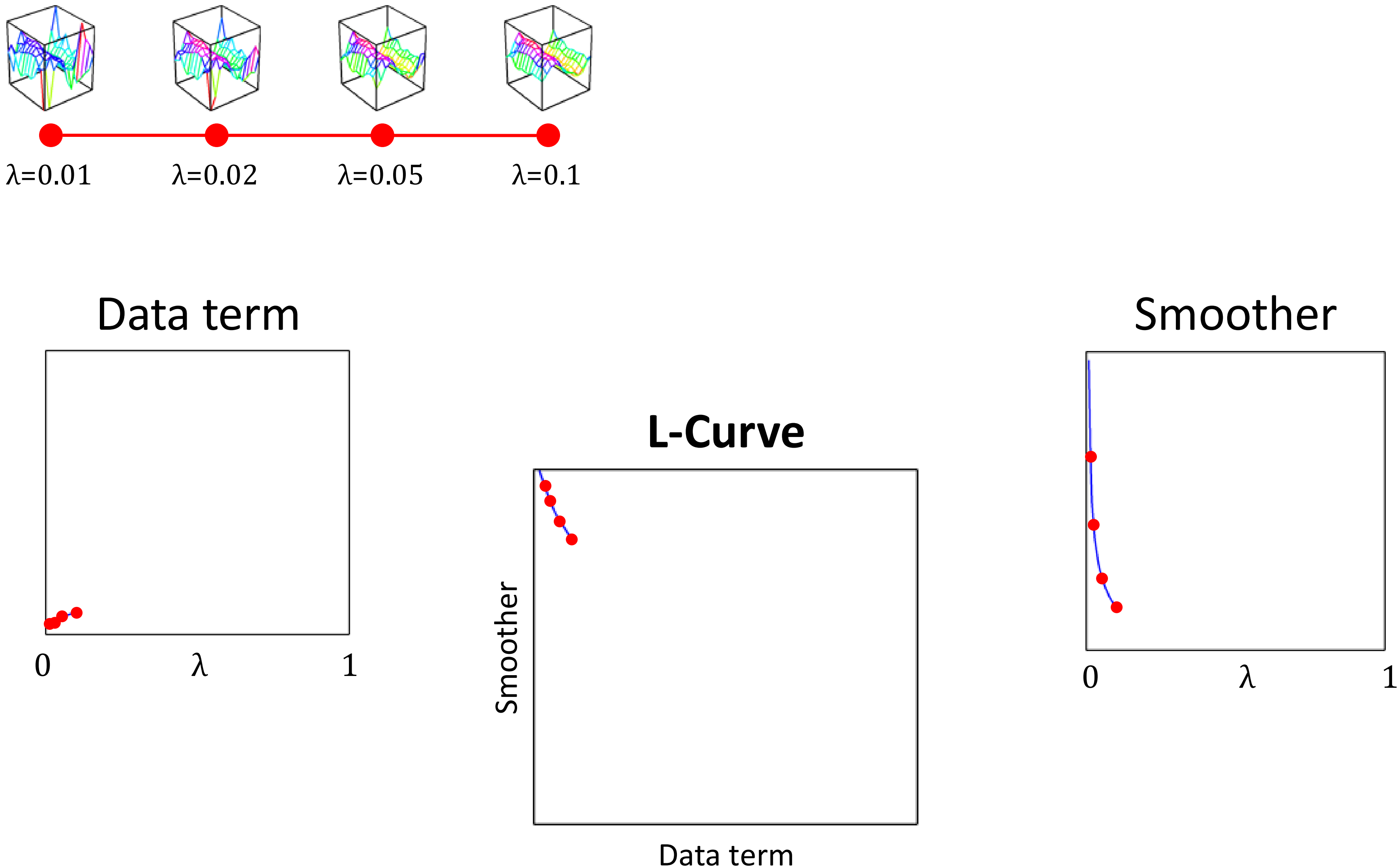
L-Curve



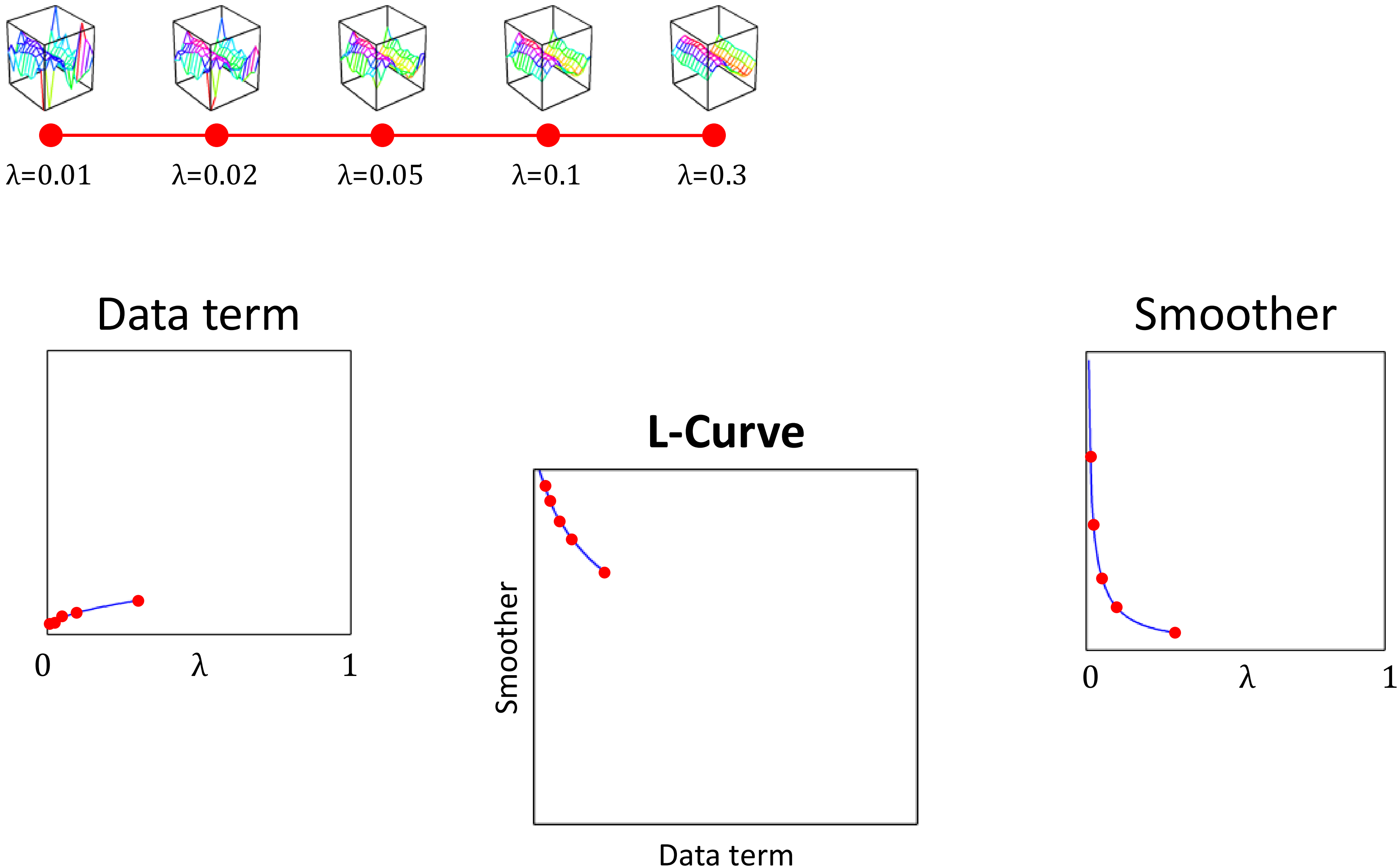
Smoother



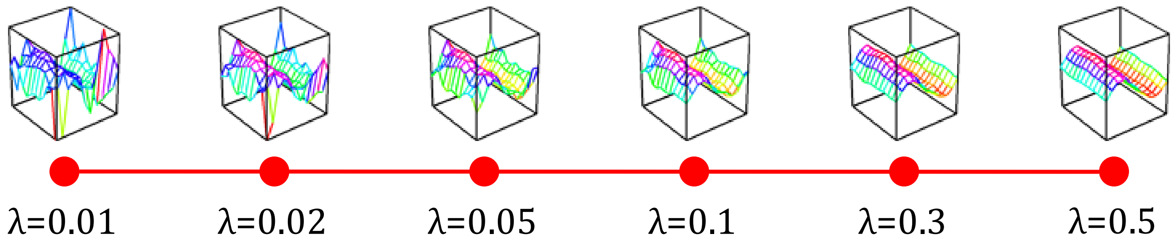
The L-Curve


λ=0.01
λ=0.02
λ=0.05
λ=0.1
Data term
0
λ
1
L-Curve
Smoother
Data term
Smoother
0
λ
1

The L-Curve


λ=0.01
λ=0.02
λ=0.05
λ=0.1
λ=0.3
Data term
0
λ
1
L-Curve
Smoother
Data term
Smoother
0
λ
1

The L-Curve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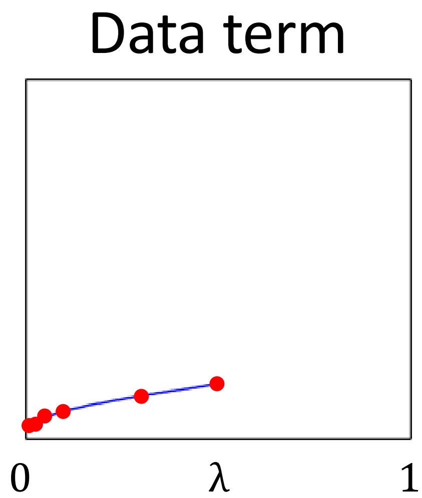
λ=0.02
λ=0.05
λ=0.1
λ=0.3
λ=0.5


Data term
0
λ
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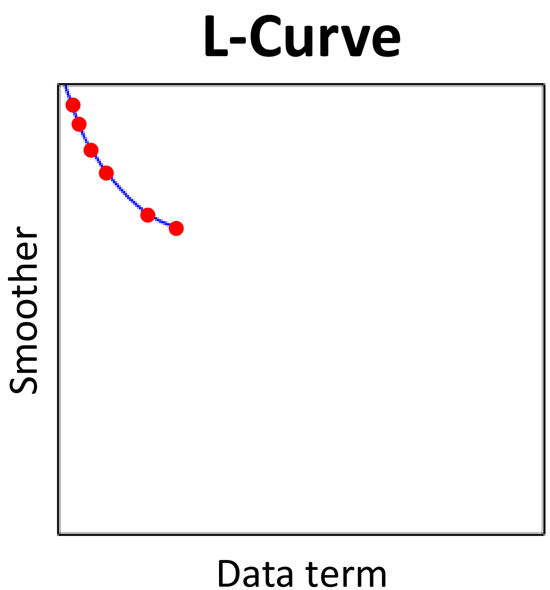
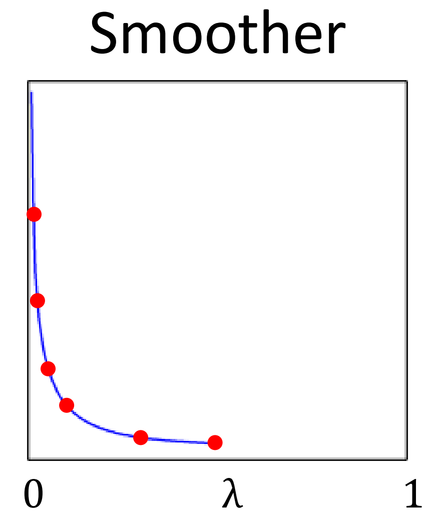
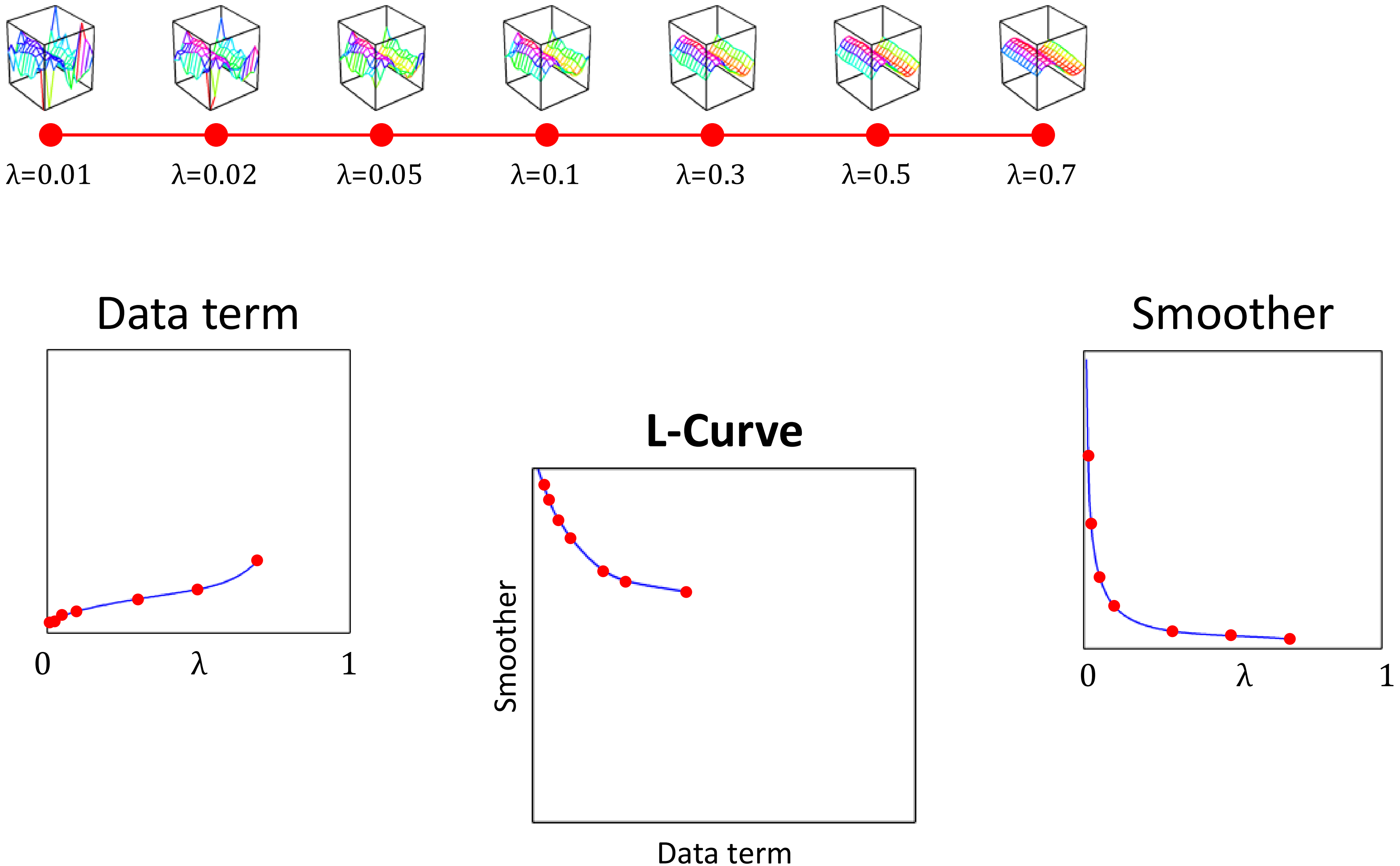
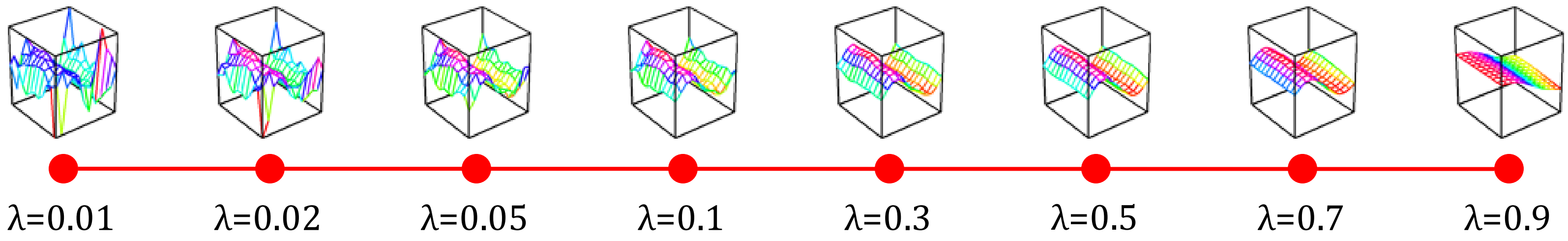

L-Curve
Smoother
Data term


Smoother
0
λ
1

The L-Curve


λ=0.01
λ=0.02
λ=0.05
λ=0.1
λ=0.3
λ=0.5
λ=0.7
Data term
0
λ
1
L-Curve
Smoother
Data term
Smoother
0
λ
1

The L-Curve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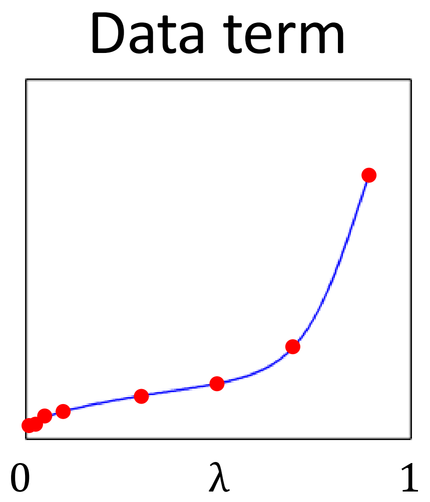
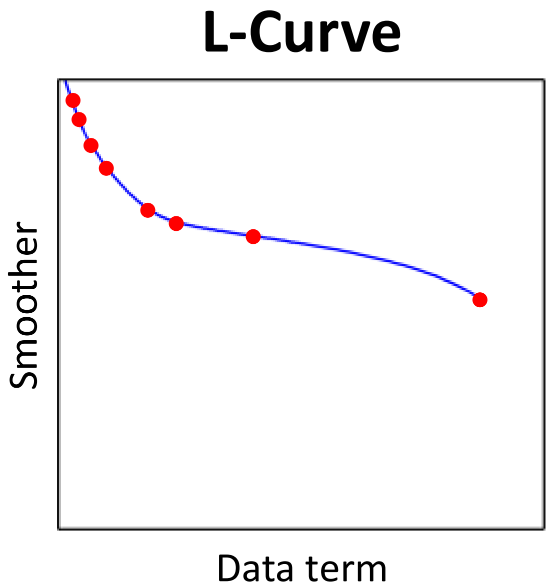
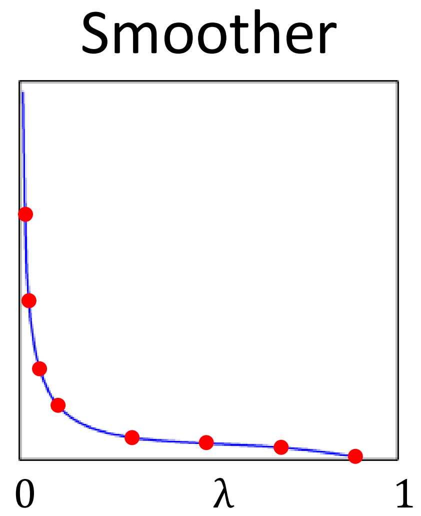
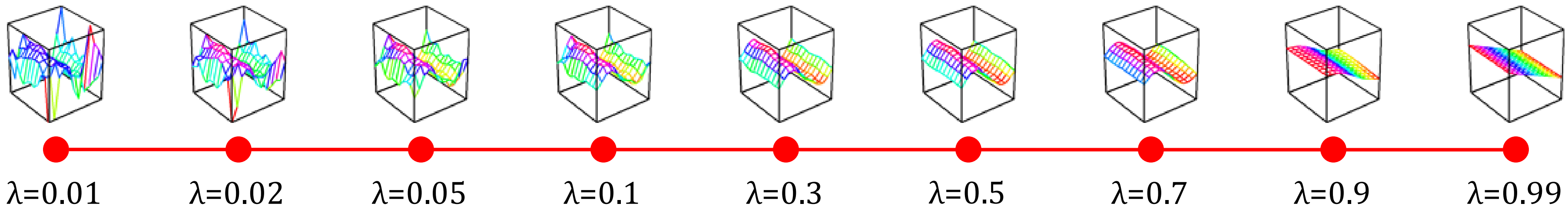
λ=0.02
λ=0.05
λ=0.1
λ=0.3
λ=0.5
λ=0.7
λ=0.9

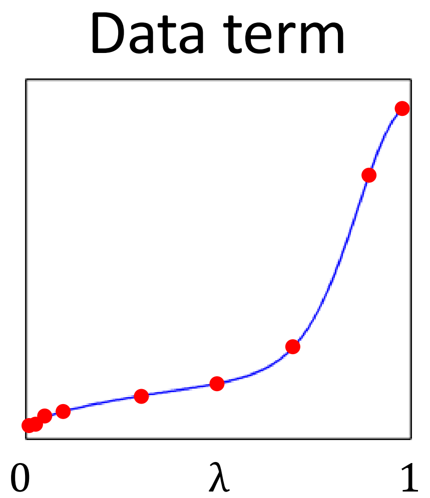

Data term
0
λ
1

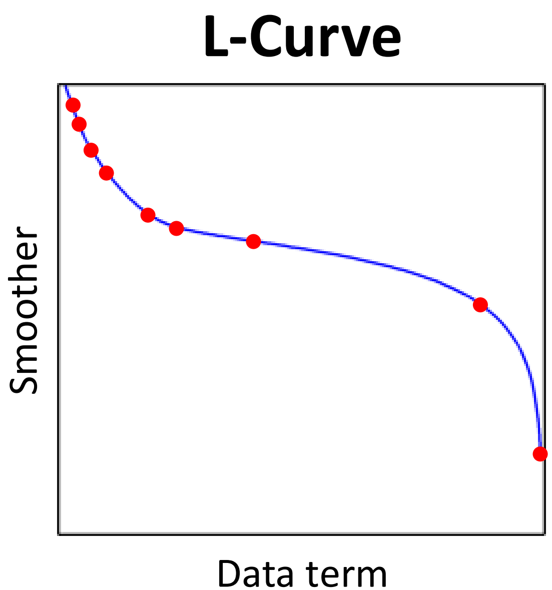

L-Curve
Smoother
Data term

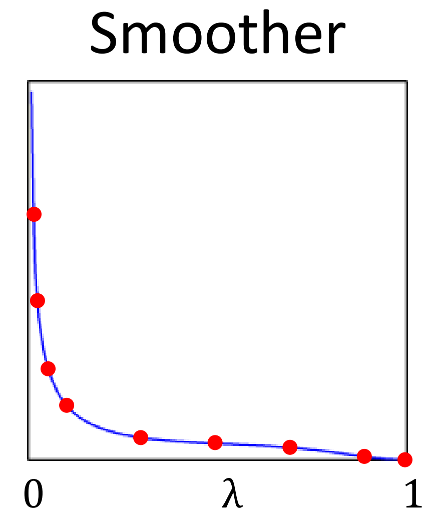

Smoother
0
λ
1

The L-Curve


λ=0.01
λ=0.02
λ=0.05
λ=0.1
λ=0.3
λ=0.5
λ=0.7
λ=0.9
λ=0.99


Data term
0
λ
1

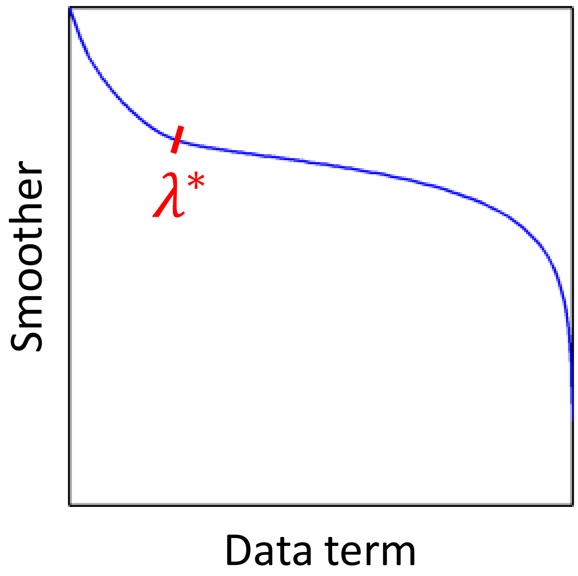

L-Curve
Smoother
Data term


Smoother
0
λ
1

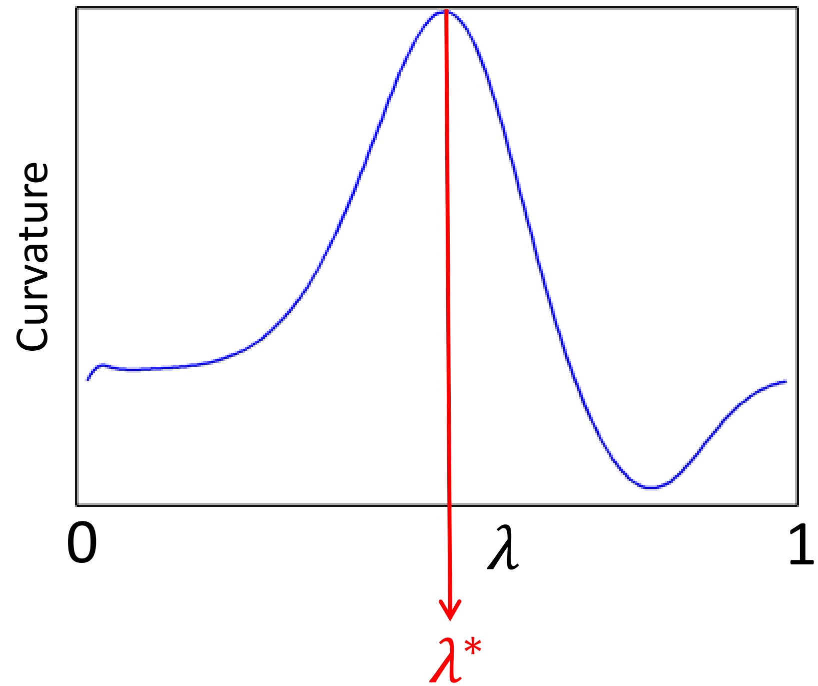
The L-Curve

The regularization parameter is chosen as the *corner* of the L-Curve (the maximal curvature)

L-Curve



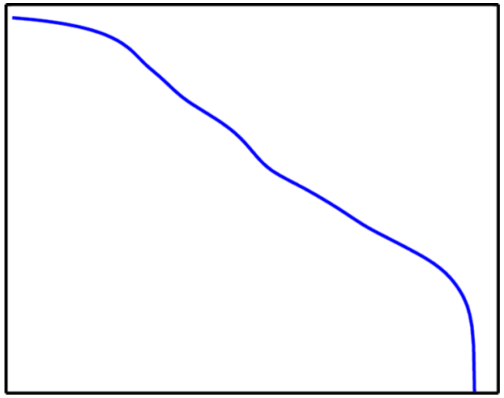
Curvature of the L-Curve

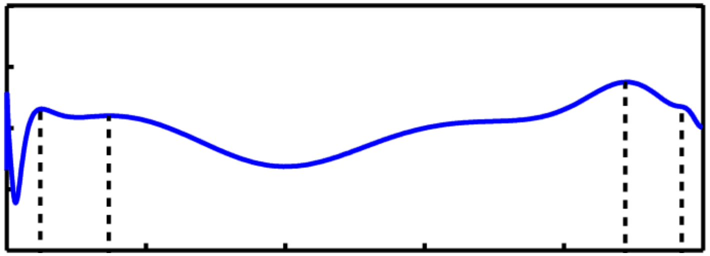


The L-Curve

- Computations are fast

- There usually are several maxima

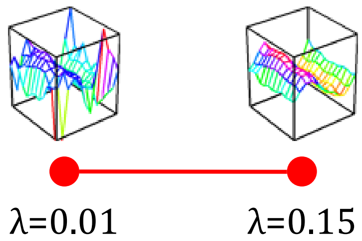


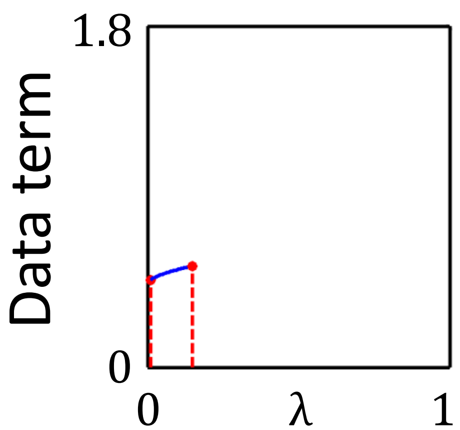


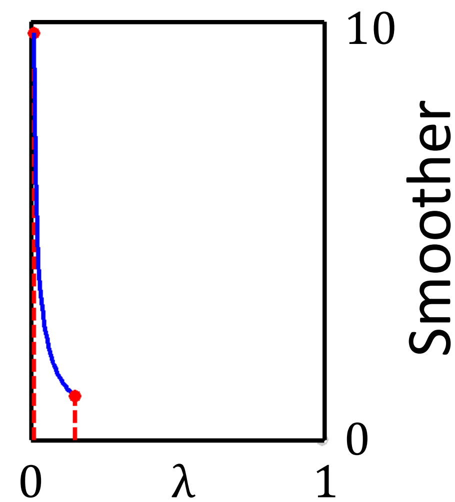
Outline

The L-Tangent Norm

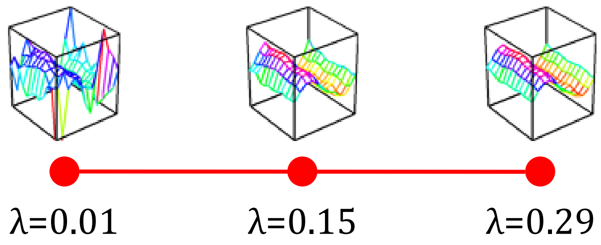
The L-Tangent Norm

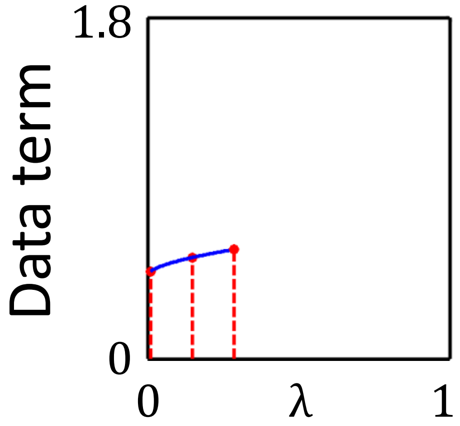


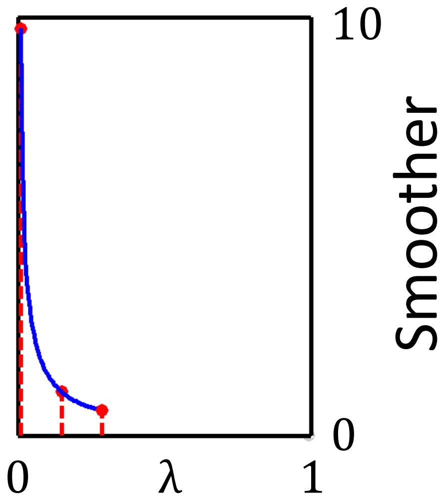




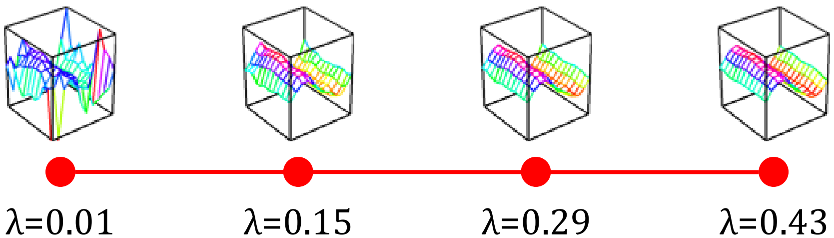
The L-Tangent Norm

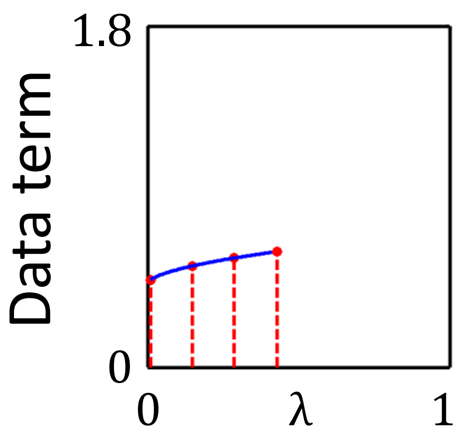


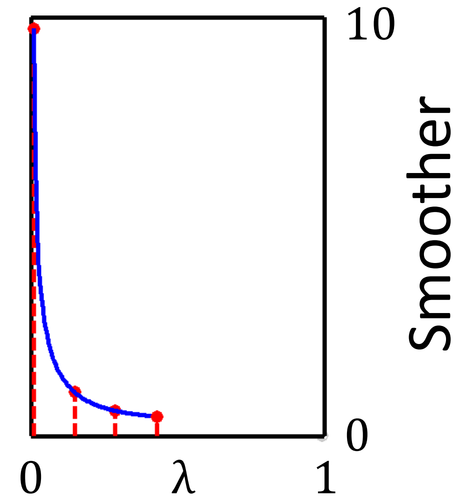




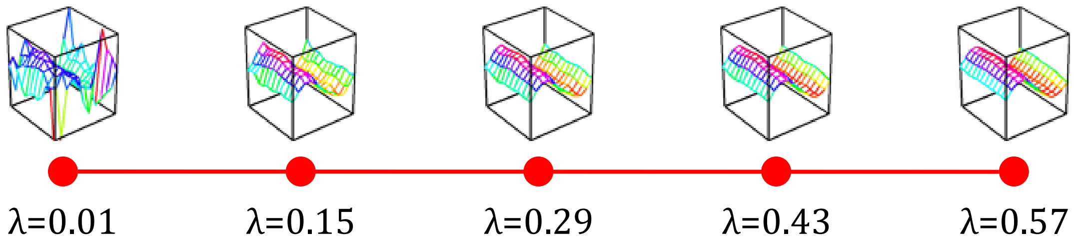
The L-Tangent Norm

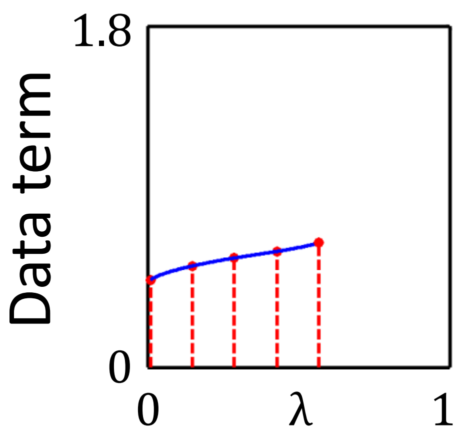


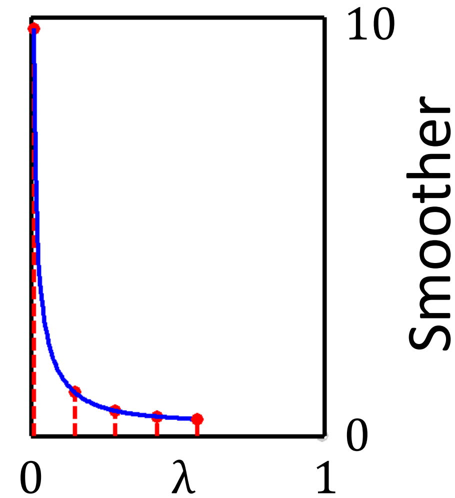




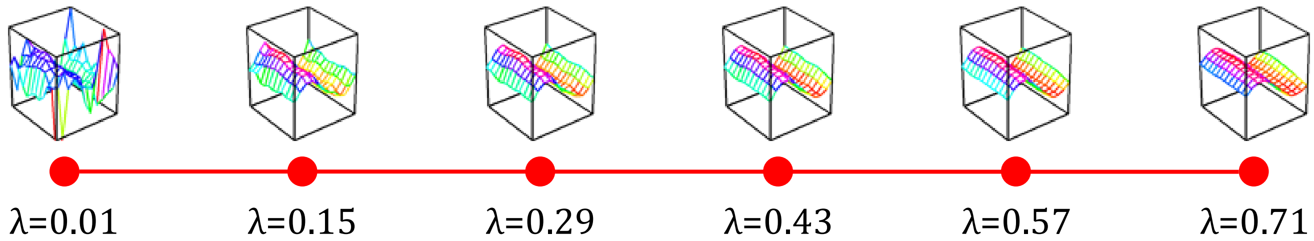
The L-Tangent Norm

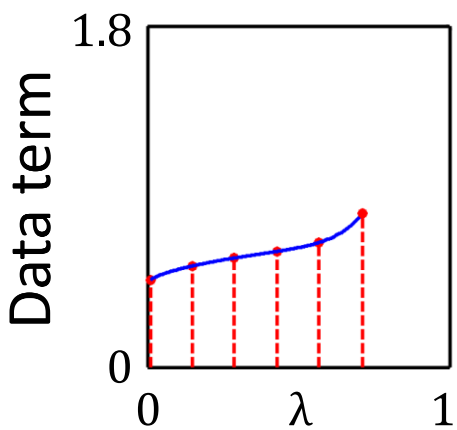


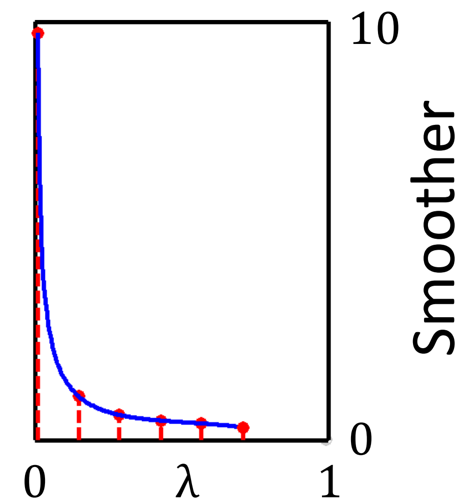




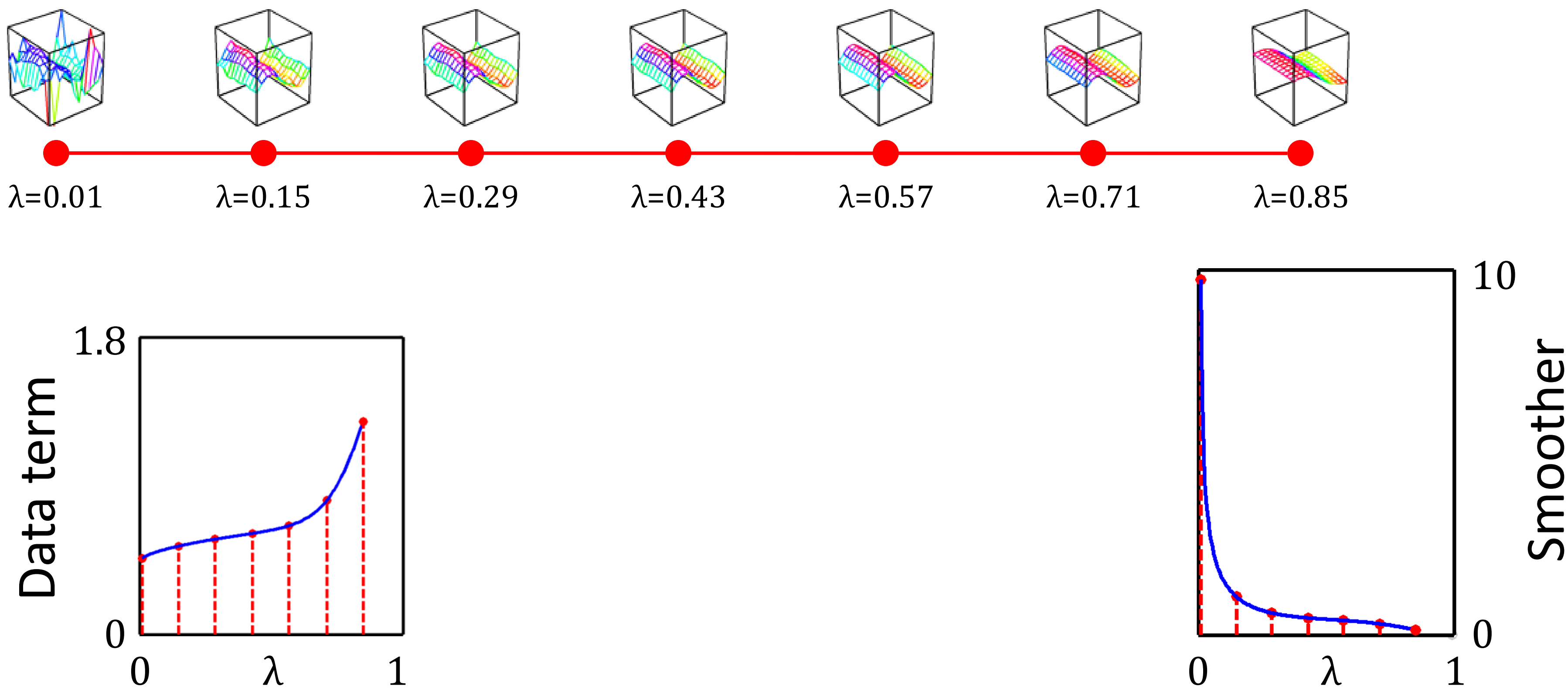
The L-Tangent Norm







The L-Tangent Norm



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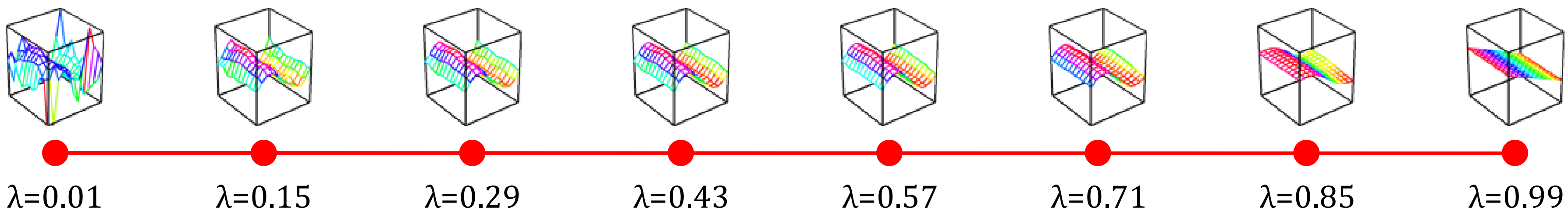
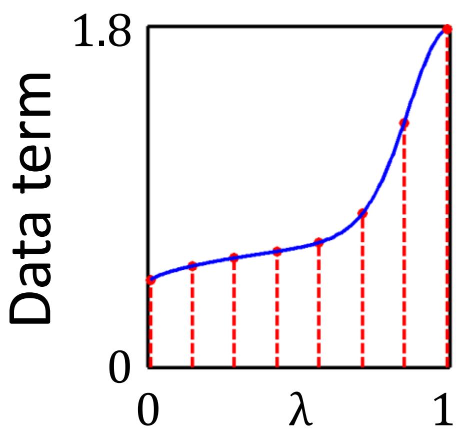
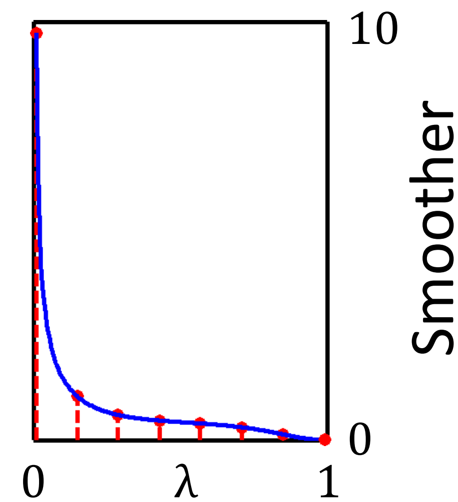
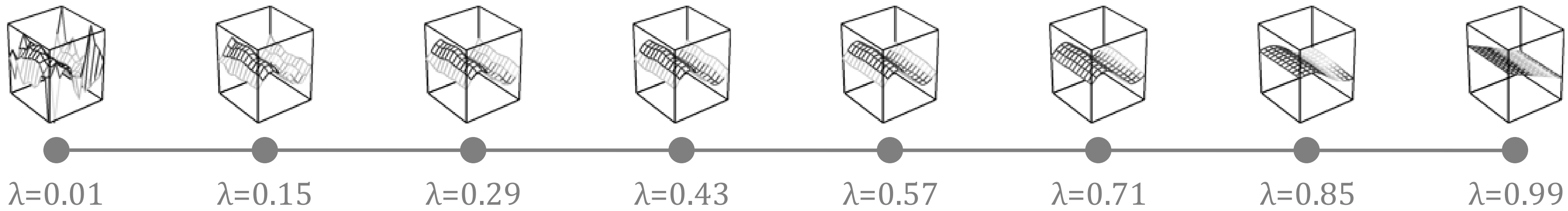
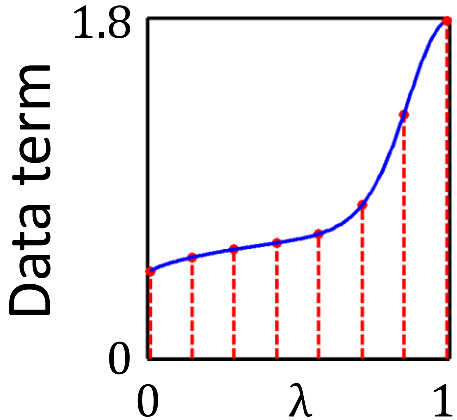


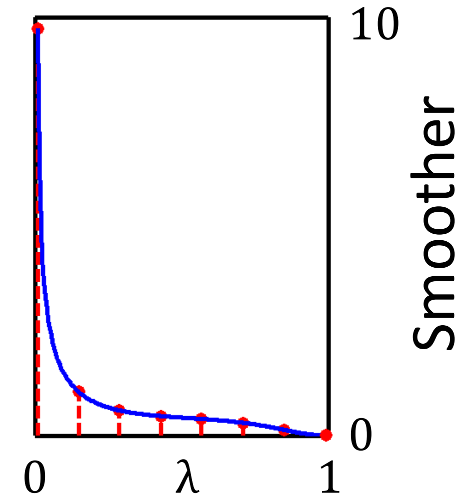


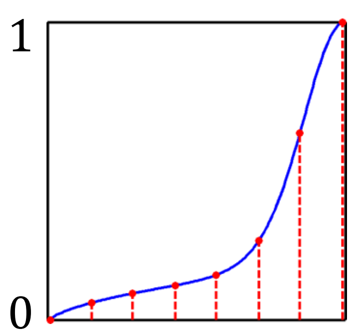


The L-Tangent Norm

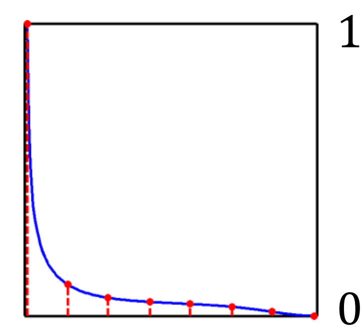




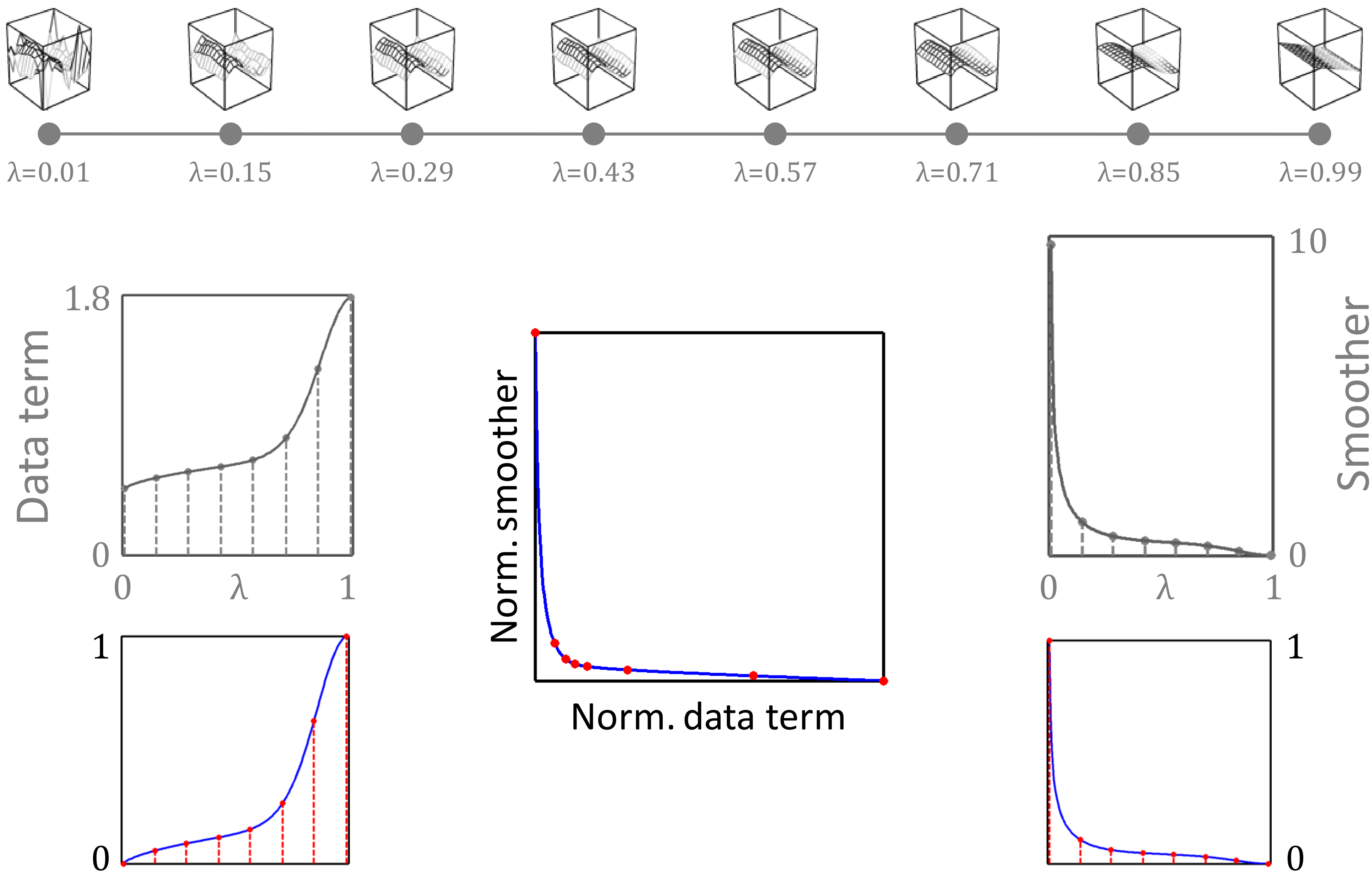




Normalization



The L-Tangent Norm



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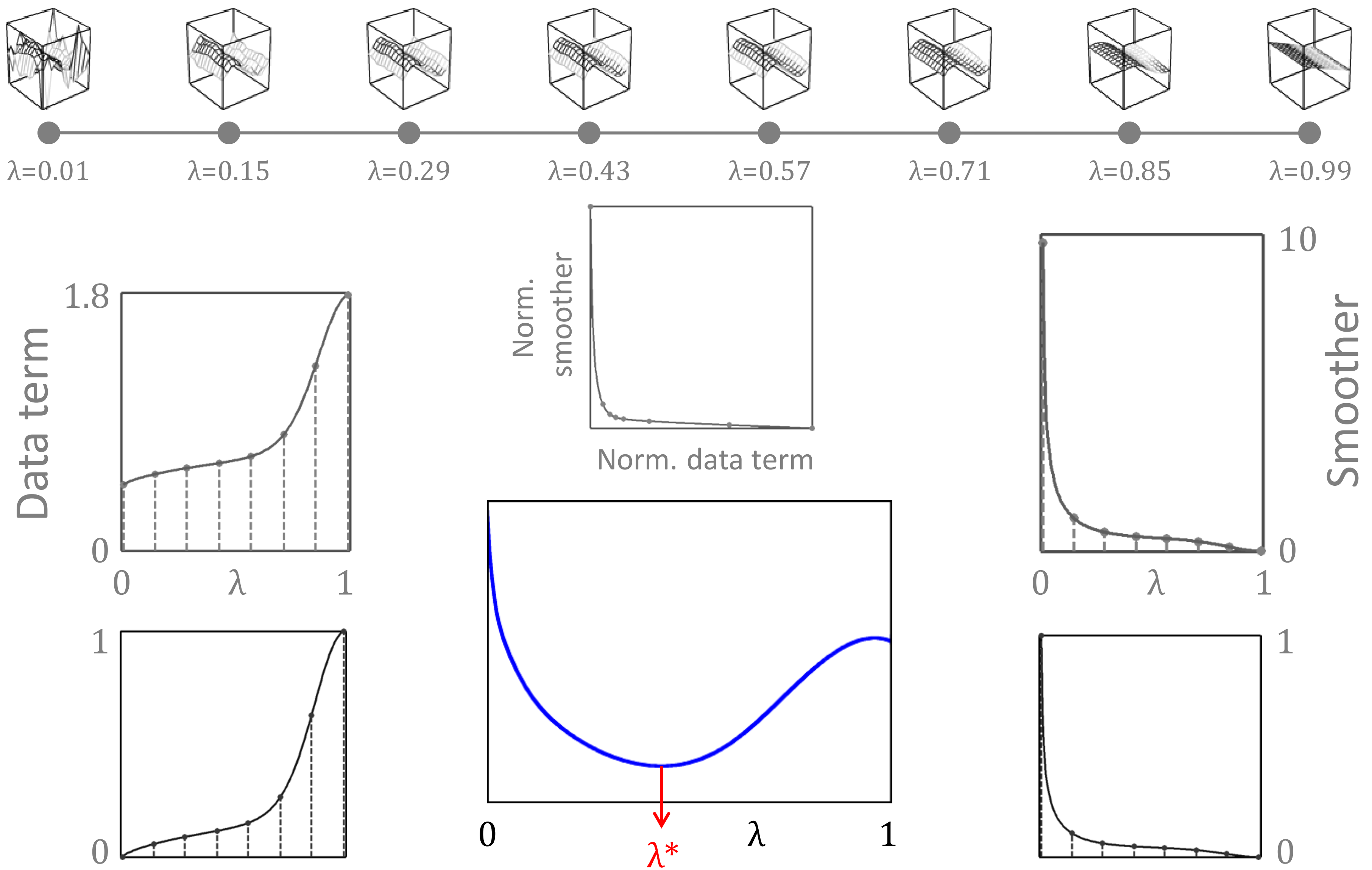

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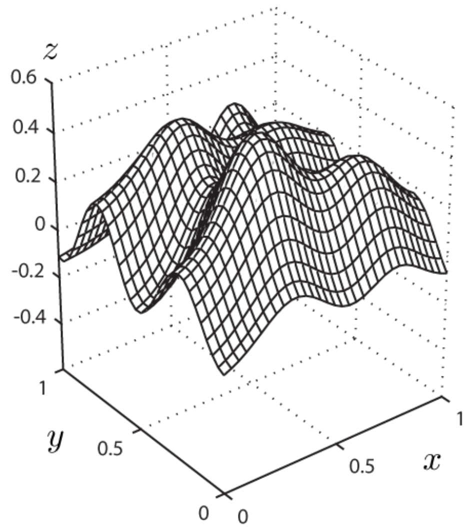

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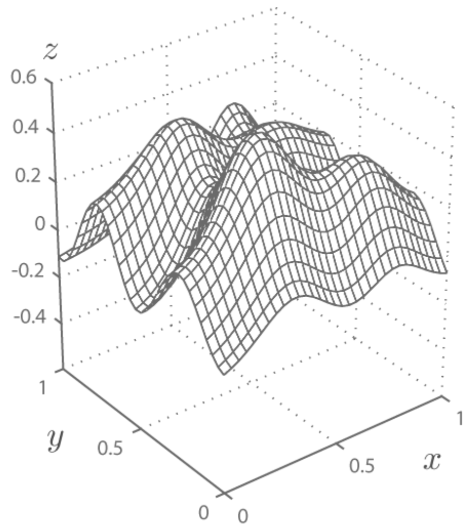
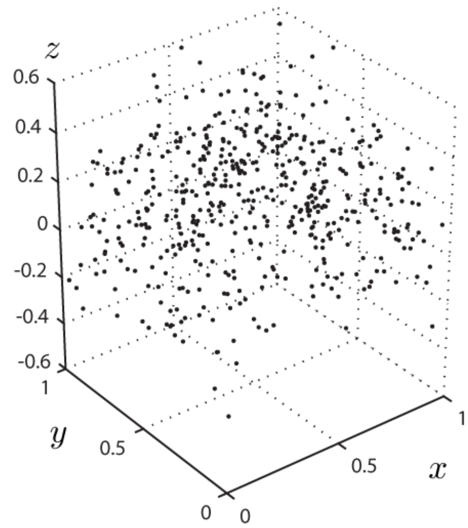
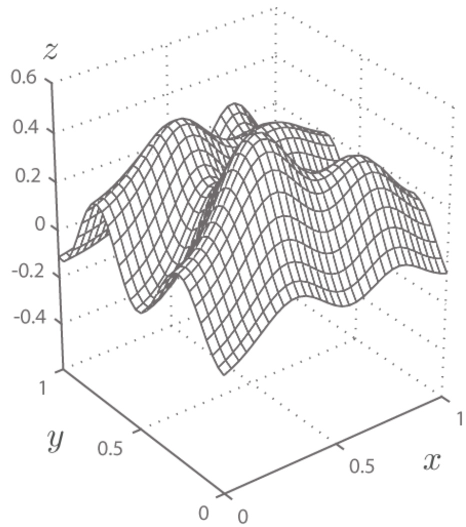
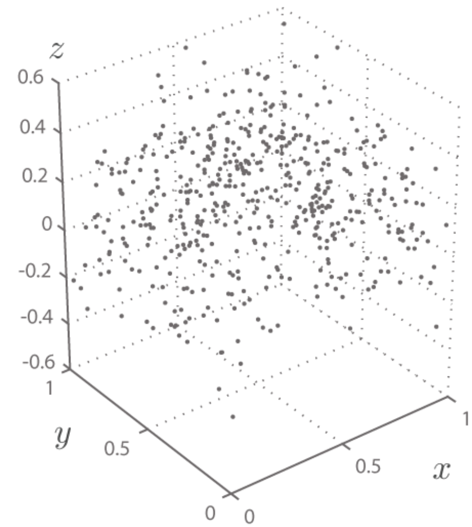
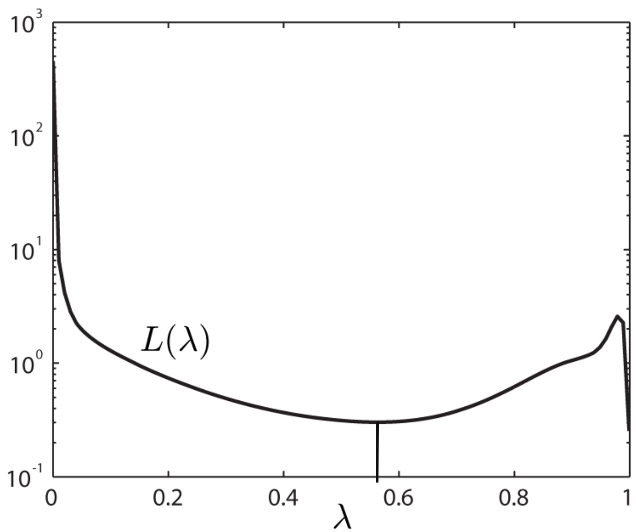




The L-Tangent Norm







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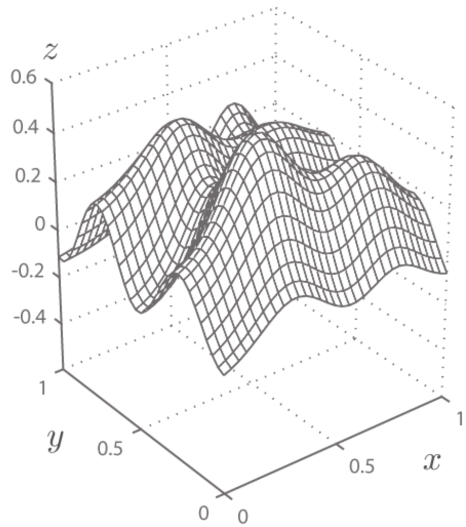
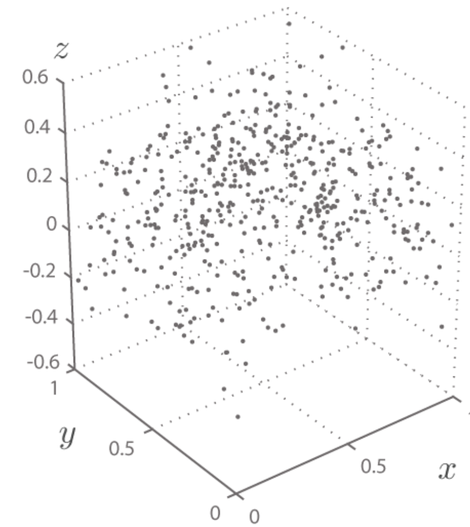
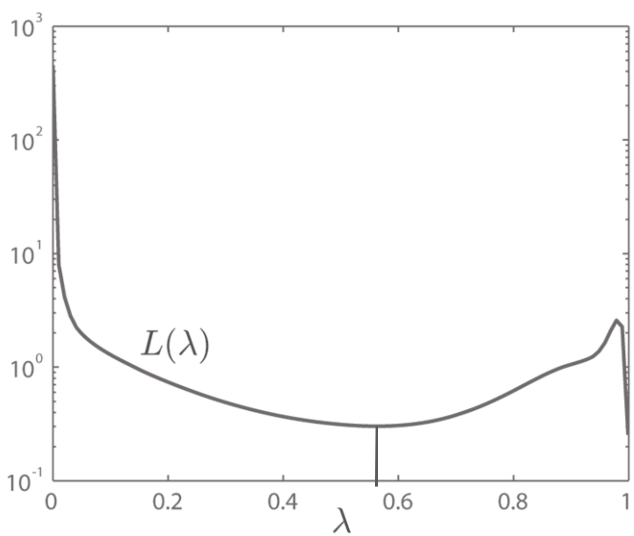
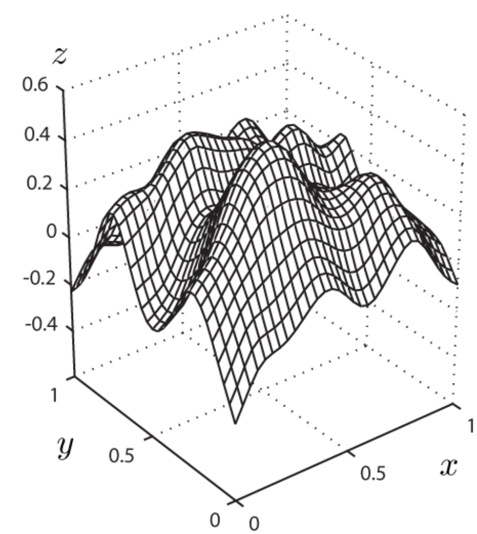






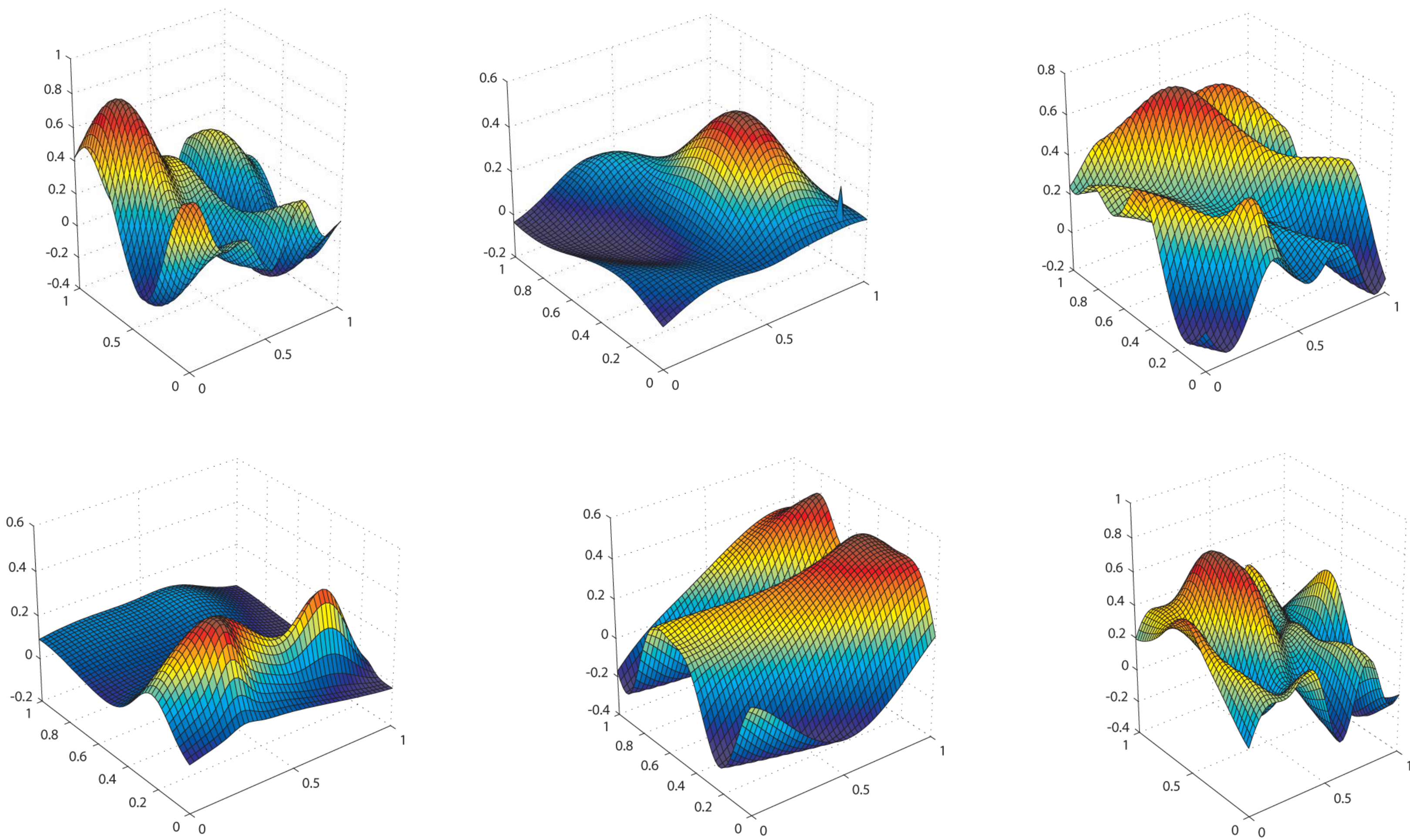


Outline

1. Introduction
2. Previous work
3. The L-Tangent Norm
4. **Experimental results**
5. Conclusion

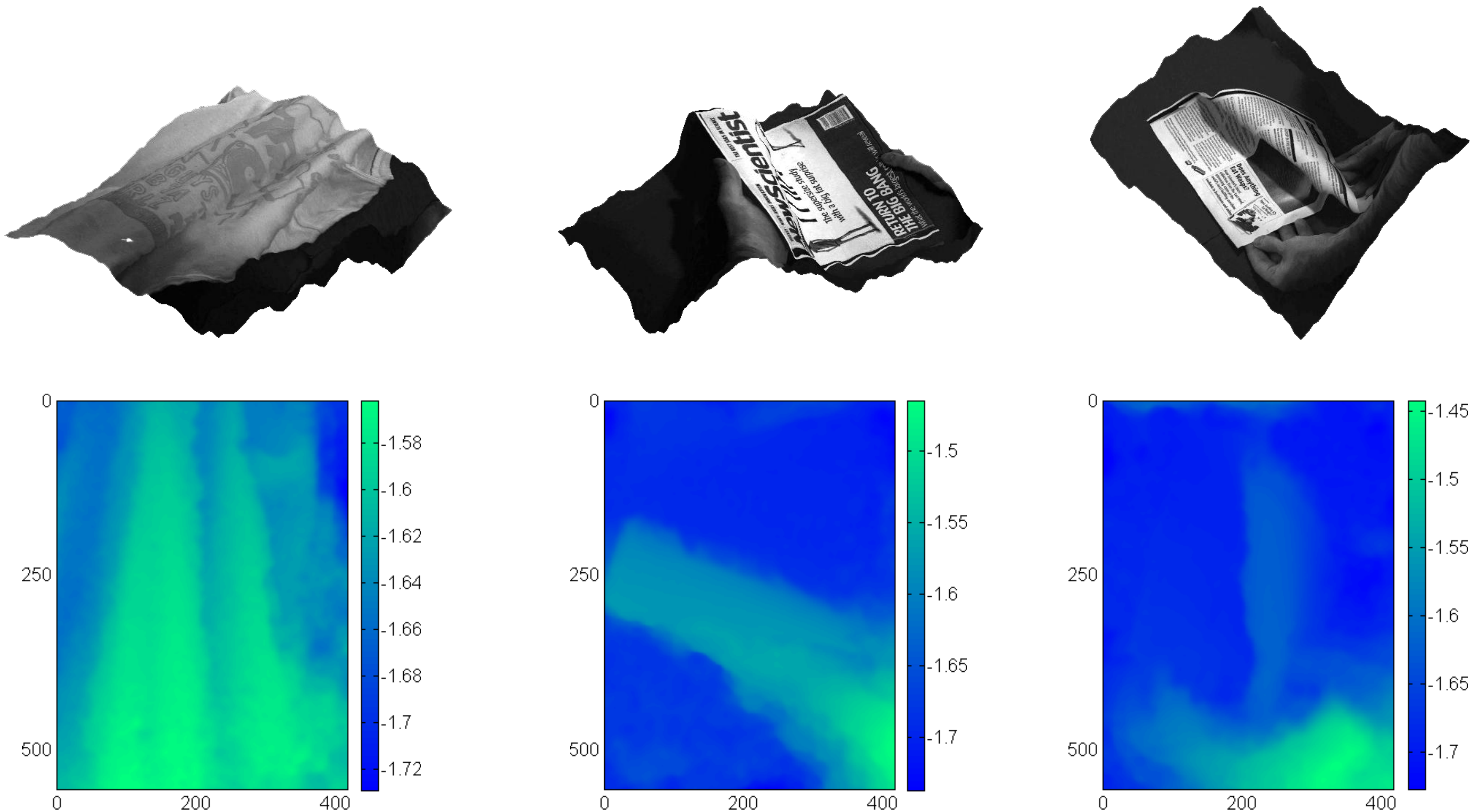
Experimental results

Synthetic data



Experimental results

Real range images

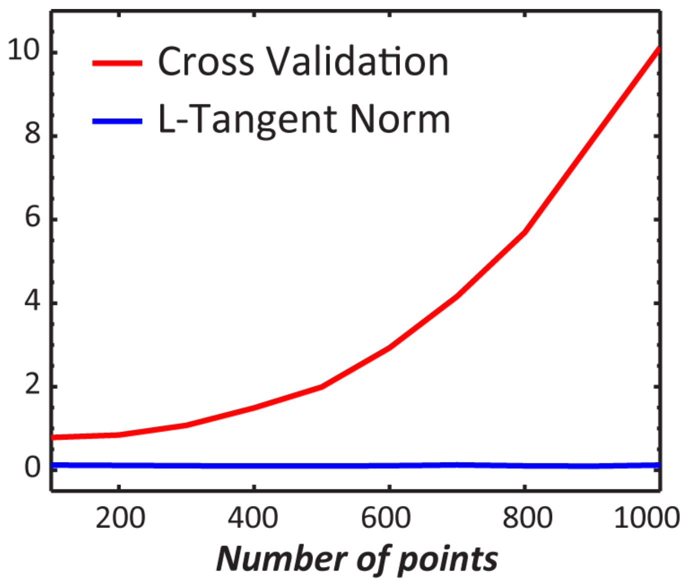


Experimental results

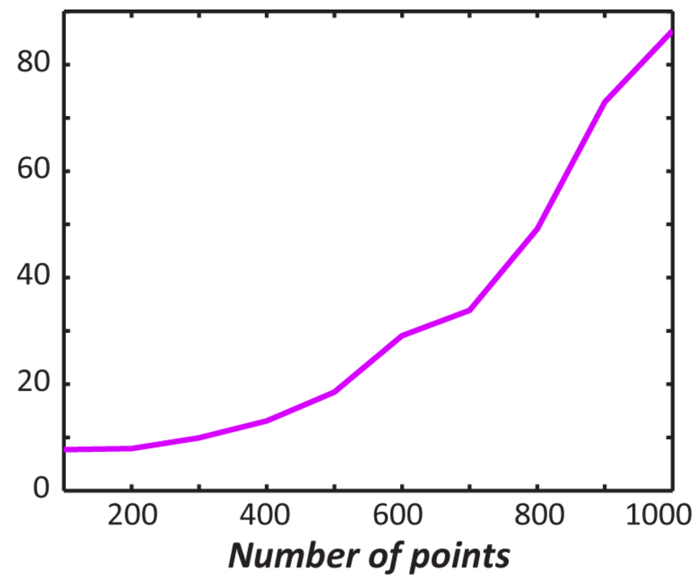
Evaluation of the criterion for a single λ

- Cross Validation vs. L-Tangent Norm
- Synthetic data

Computation time (seconds)



Ratio $\dfrac{\textbf{Cross Validation}}{\textbf{L-Tangent Norm}}$

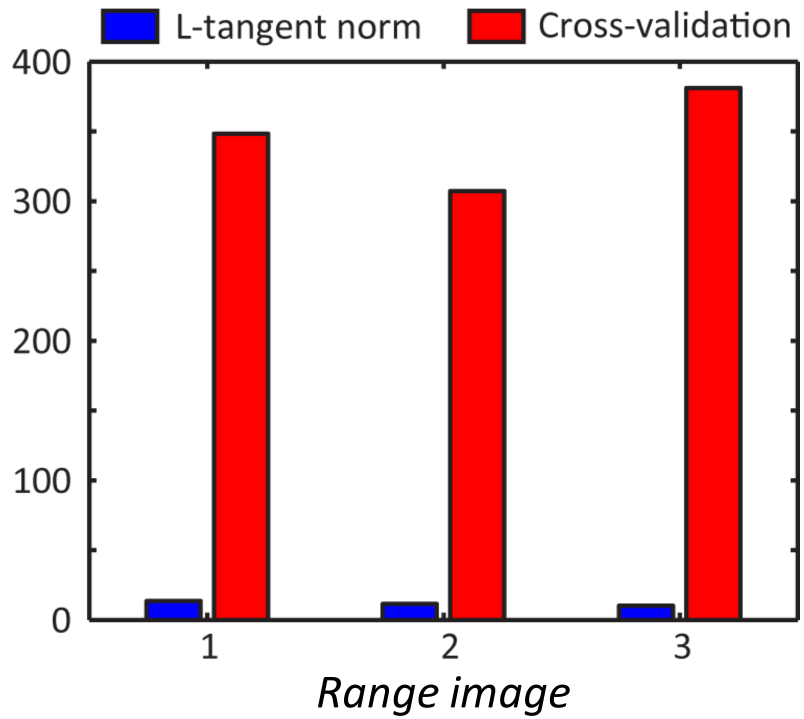


Experimental results

Total time of the surface reconstruction

- Cross Validation vs. L-Tangent Norm
  - Real range images

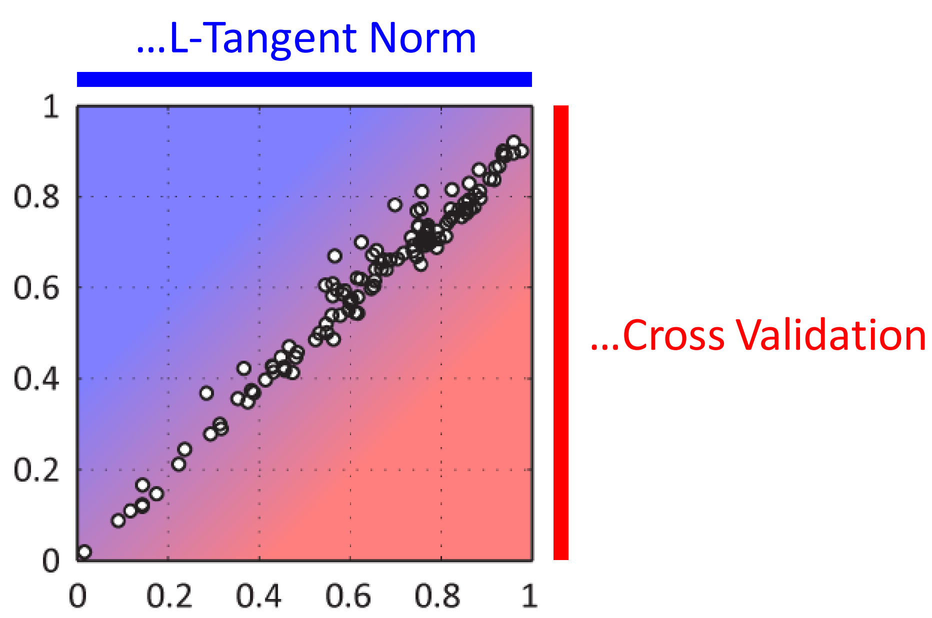
Computation time (seconds)



Experimental results

Comparison of the regularization parameters

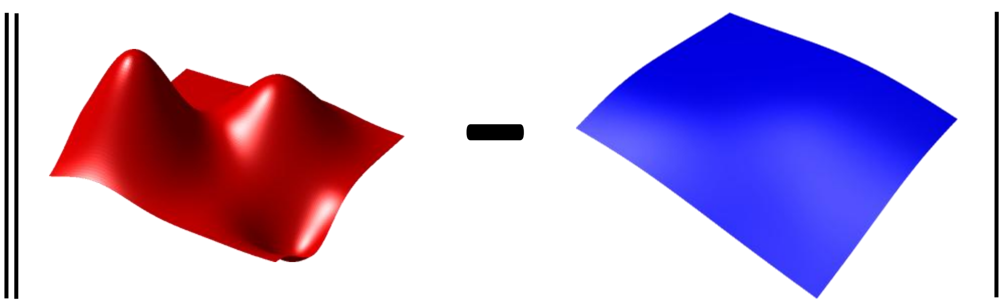
Regularization parameters obtained with…



Experimental results

Quality of the reconstructed surfaces

- 200 randomly generated surfaces (ground truth)
- Comparison *ground truth* vs. *reconstruction*



= Integral Relative Error (IRE)

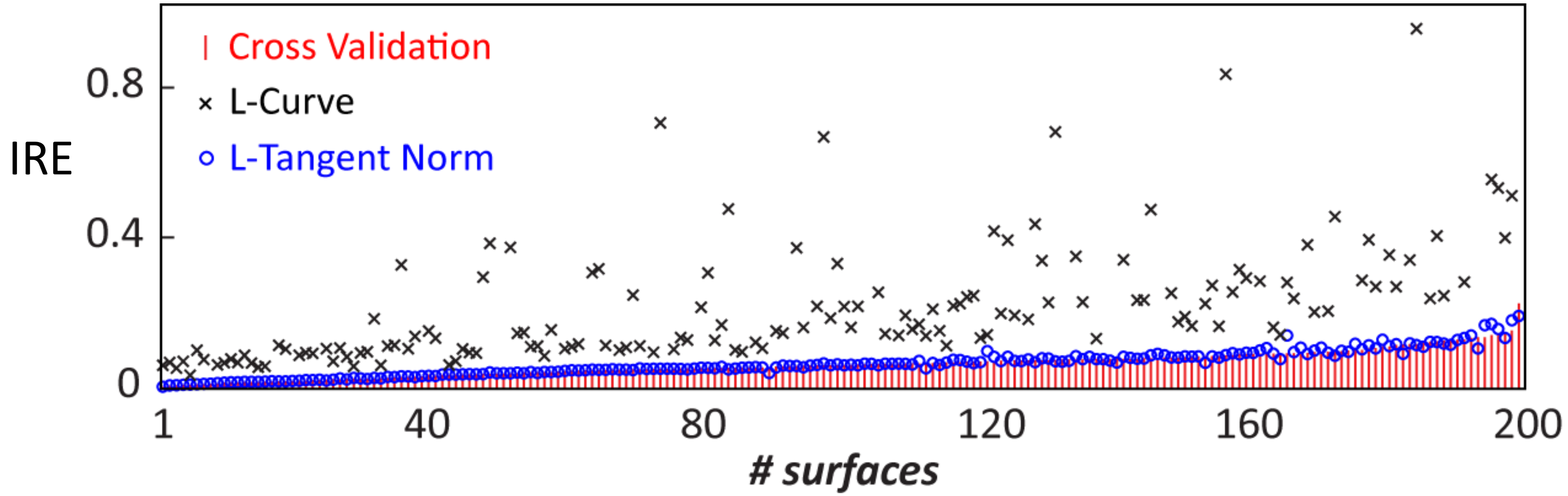
Generated surface (*ground truth*)

Reconstructed surface

Experimental results

Quality of the reconstructed surfaces

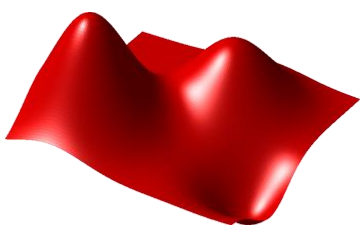
- 200 randomly generated surfaces (ground truth)
- Comparison *ground truth* vs. *reconstruction*



Experimental results

Quality of the reconstructed surfaces

- Real range images



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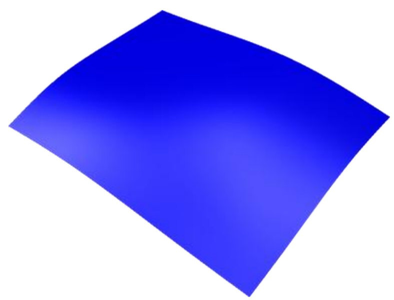

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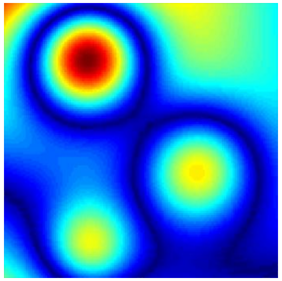


Generated surface (*ground truth*)

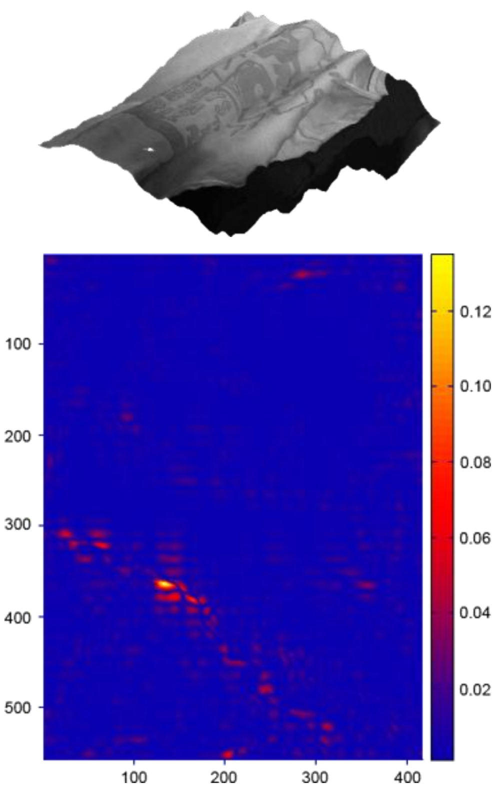
Reconstructed surface

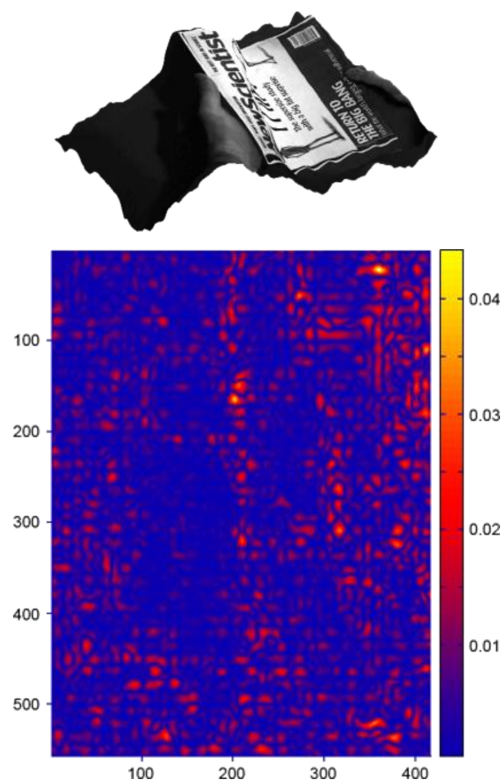
Error Map

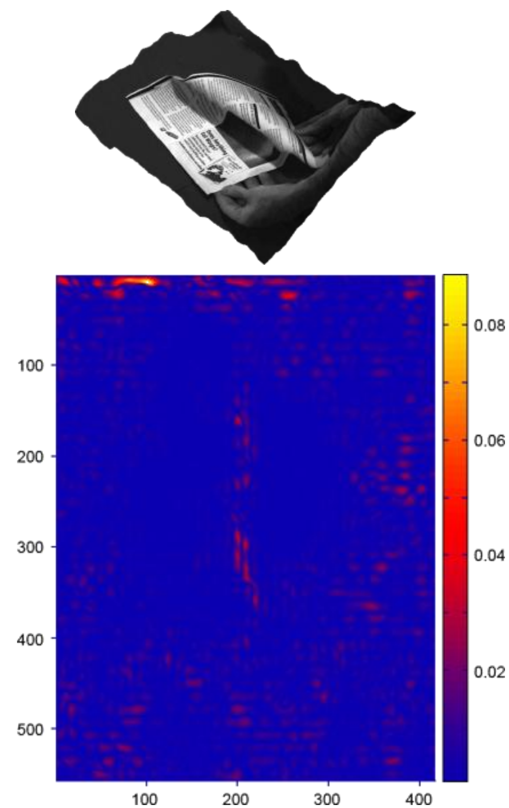
Experimental results

Quality of the reconstructed surfaces

- Real range images
- Compact representation: 40x40 control points







Outline

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Conclusion

- Novel approach to choose the regularization parameter
- Low computational cost, easy optimization process
- Approximation of the Cross-Validation
- Automatically provides a compact and analytical form of large datasets

What's next?

- Prove that the LTN is an approximation of the CV
- Multiple regularization terms

Thanks for your attention!