

Predicting and Estimating the Accuracy of n-Ocular Optical Tracking Systems

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The Setup

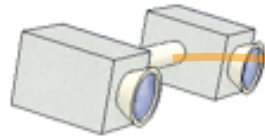


Image plane error (IPE):

Gaussian error on the image plane (2DoF)



Target Registration Error (TRE):

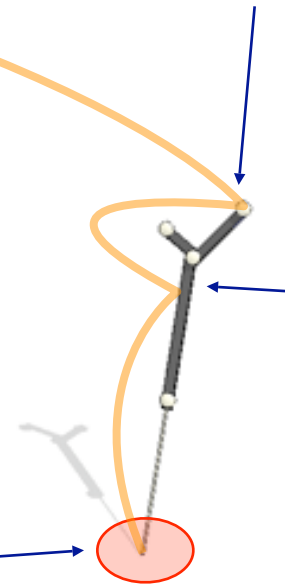
How accurate can we locate the point of interest (3DoF)?

Fiducial Location Error (FLE):

How good can we determine the location of the fiducials (3DoF)

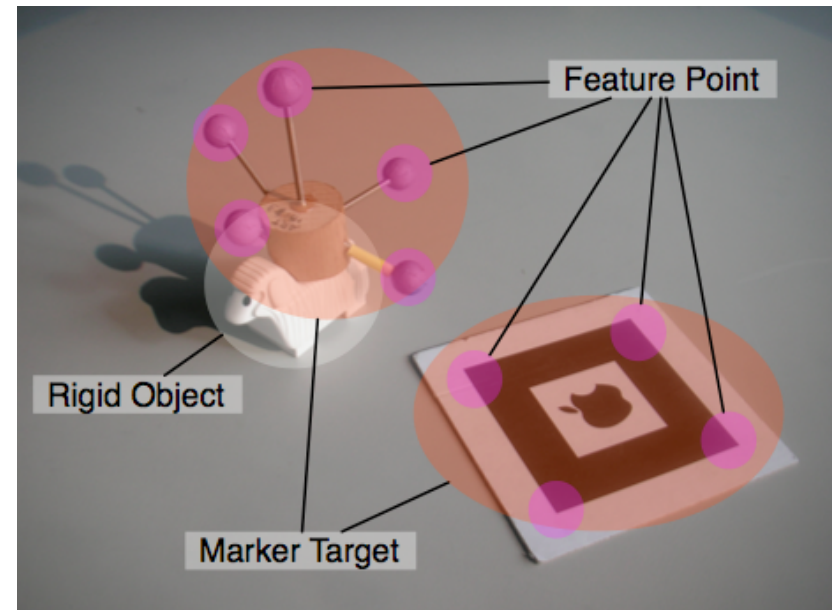
Marker Target Error (MTE):

How good can the target be detected (6DoF)?



Optical Tracking Systems

- Cameras
 - Monocular
 - Stereo, n-ocular
- Targets
 - Rigid Objects
- Features:
 - Retroreflective Markers
 - Visible Markers
 - Natural Features



Why Prediction of Accuracy?

- Planning of tracking setups
- Optimizing target design
 - *L. Davis, E. Clarkson, and J. P. Rolland. Predicting accuracy in pose estimation for marker-based tracking.* International Symposium on Mixed and Augmented Reality, 2003.
 - *L. Davis, F. G. Hamza-Lup, and J. P. Rolland. A method for designing marker-based tracking probes.* IEEE and ACM International Symposium on Mixed and Augmented Reality, 2004.
- Online: Accuracy monitoring for safety critical applications
 - Warning when specified accuracy can not be guaranteed
 - Adapting the user interface to current accuracy
 - *E. M. Coelho, B. MacIntyre, and S. Julier. OSGAR: A scenegraph with uncertain transformations.* International Symposium on Mixed and Augmented Reality 2004.

Outline

- Predict fiducial location error (FLE) from image plane error (IPE)
 - FLE is anisotropic in common setups
 - estimate IPE from a set of experiments
- Predict marker target error (MTE) from FLE
 - anisotropic propagation
 - orientation matters
- Predict target registration error (TRE) from MTE

- Occlusions of cameras or fiducials

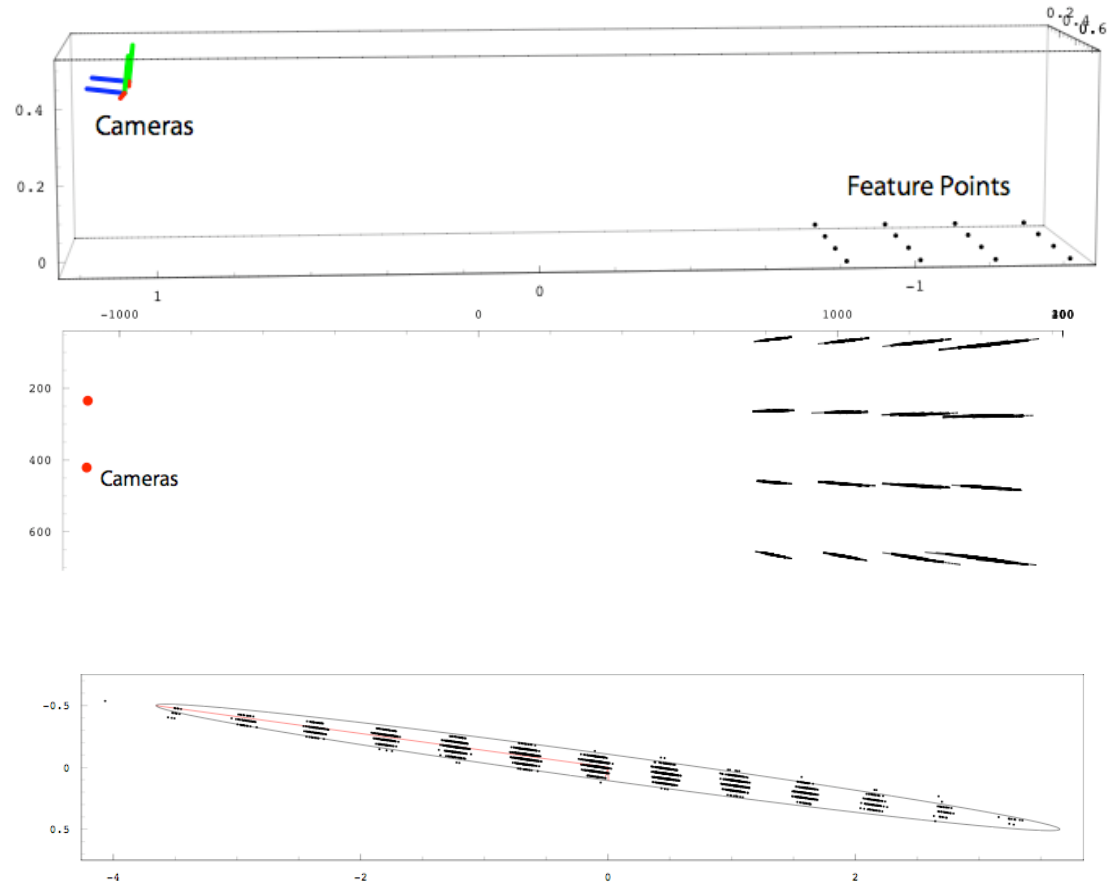
From FLE to MTE: Related Work

□ Experimental evaluations

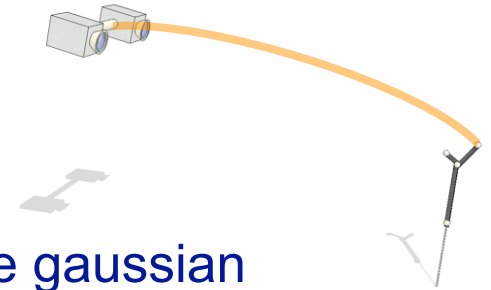
- *R. Khadem, C. C. Yeh, M. Sadeghi-Tehrani, M. R. Bax, J. A. Johnson, J. N. Welch, E. P. Wilkinson, and R. Shahidi. **Comparative tracking error analysis of five different optical tracking systems.** Computer Aided Surgery, 5(2):98–107, 2000.*
- **Optical 3d measuring systems - imaging systems with point-by-point probing.** VDI/VDE guideline 2634/1, 2002.
- *A. Wiles, D. Thompson, and D. Frantz. **Accuracy assessment and interpretation for optical tracking systems.** SPIE Medical Imaging, 5367, 2004.*
- many more ...
- *R. Langley. **Dilution of precision.** GPS World, 10(5):52–59, 1999*
- *B. D. Allen and G. Welch. **A general method for comparing the expected performance of tracking and motion capture systems.** Proc. Virtual Reality Software and Technology 2005*

An experimental setup in our lab

- Small baseline:
20cm
- 4x4 grid
of fiducial points
- Resulting error
magnified
by factor 50
- Assuming unbiased
Gaussian noise
- Artifacts due to
discretization in
tracking algorithms



From IPE to FLE



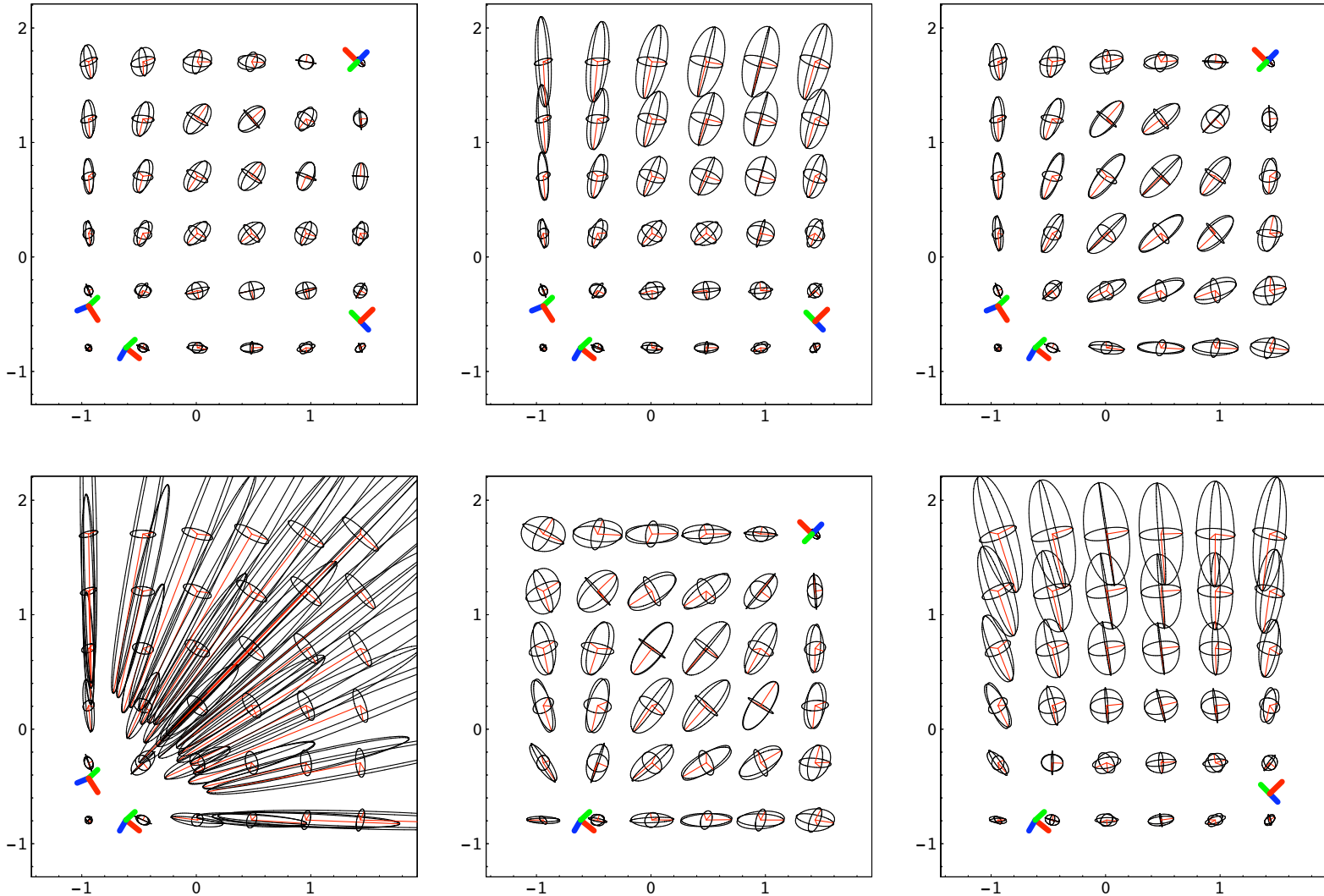
- Assume that all errors can be modelled in some gaussian unbiased noise on the image planes (Image plane error IPE)
- Use error propagation based only on the geometric constellation of the cameras

$$\text{From } p: \begin{array}{l} \vec{u}_1 = \rho_1 \mathbf{K}_1 \mathbf{T}_1 \vec{x} \\ \vdots \\ \vec{u}_n = \rho_n \mathbf{K}_n \mathbf{T}_n \vec{x} \end{array}$$

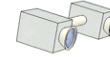
$$\text{compute } \Sigma_x = \left(J_p^T \begin{bmatrix} \Sigma_{u_1} & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \Sigma_{u_n} \end{bmatrix}^{-1} J_p \right)^+ \quad \text{with } J_p = \frac{\delta p}{\delta \vec{x}}$$

- C++ code analytically generated with Mathematica

From IPE to FLE: Different camera setups



Estimating the TRE from the FLE



- *J. M. Fitzpatrick, J. B. West, and C. R. Maurer. Jr.* **Predicting error in rigid-body, point-based registration.** IEEE Transactions on Medical Imaging, 17(5):694–702, 1998.
 - Assumes independent, isotropic unbiased errors

- *W. Hoff and T. Vincent.* **Analysis of head pose accuracy in augmented reality.** In IEEE Transactions on Visualization and Computer Graphics, volume 6(4), pages 319–334. IEEE Computer Society, 2000)
 - Based only on the geometric constellation of the POI and fiducials in the target
 - Assumes unbiased (zero-mean) but not independent or isotropic errors

Estimating the TRE from the FLE (2)

- From FLE to MTE:
applying backward propagation results in

$$\Sigma_{\vec{c}} = \left(M^T \begin{bmatrix} R^T \Sigma_{p_1} R & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & R^T \Sigma_{p_n} R \end{bmatrix}^{-1} M \right)^+$$

with $R^T \Sigma_{p_i} R$ the FLE covariance in target coordinates
and M a function that maps the 6D MTE to the 3D FLEs

- From MTE to TRE:
compute forward propagation $\Sigma_{\vec{p}} = J_f \Sigma_{\vec{c}} J_f^T$



From FLE to MTE

- Marker Target Error (MTE): Error in the centroid of the target
- Use 3D/3D pose estimation algorithm to get $R\vec{q}_k + \vec{t} = p_k + \Delta\vec{p}_k$
- treat c as a random variable representing the error and build

$$J_f(\vec{q}) = \left. \frac{\partial f(\vec{p}, \vec{q})}{\partial \vec{c}} \right|_{\vec{c}=\vec{0}} = \begin{bmatrix} 1 & 0 & 0 & 0 & q_z & -q_y \\ 0 & 1 & 0 & -q_z & 0 & q_x \\ 0 & 0 & 1 & q_y & -q_x & 0 \end{bmatrix}$$

with $f(\vec{p}, \vec{q}) = R\vec{q} + \vec{t} - \vec{p}$

- This maps the 6D MTE to the 3D FLEs

$$\begin{bmatrix} \Delta p_1 \\ \vdots \\ \Delta p_n \end{bmatrix} = \begin{bmatrix} J_f(q_1) \\ \vdots \\ J_f(q_n) \end{bmatrix} \Delta x = M \Delta x$$

W. Hoff and T. Vincent. Analysis of head pose accuracy in augmented reality.

In IEEE Transactions on Visualization and Computer Graphics, volume 6(4), pages 319–334, 2000)

From FLE to MTE (2)

- using the backward propagation formula, we get

- $$\Sigma_{\vec{c}} = \left(M^T \begin{bmatrix} R^T \Sigma_{p_1} R & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & R^T \Sigma_{p_n} R \end{bmatrix}^{-1} M \right)^+$$

- where $R^T \Sigma_{p_i} R$ is the FLE in the target coordinate system
- MTE is given in the target coordinate system
 - to visualize the MTE together with the FLE we would need to backtransform it into world coordinates
 - or visualize the rotated FLE

From MTE to TRE

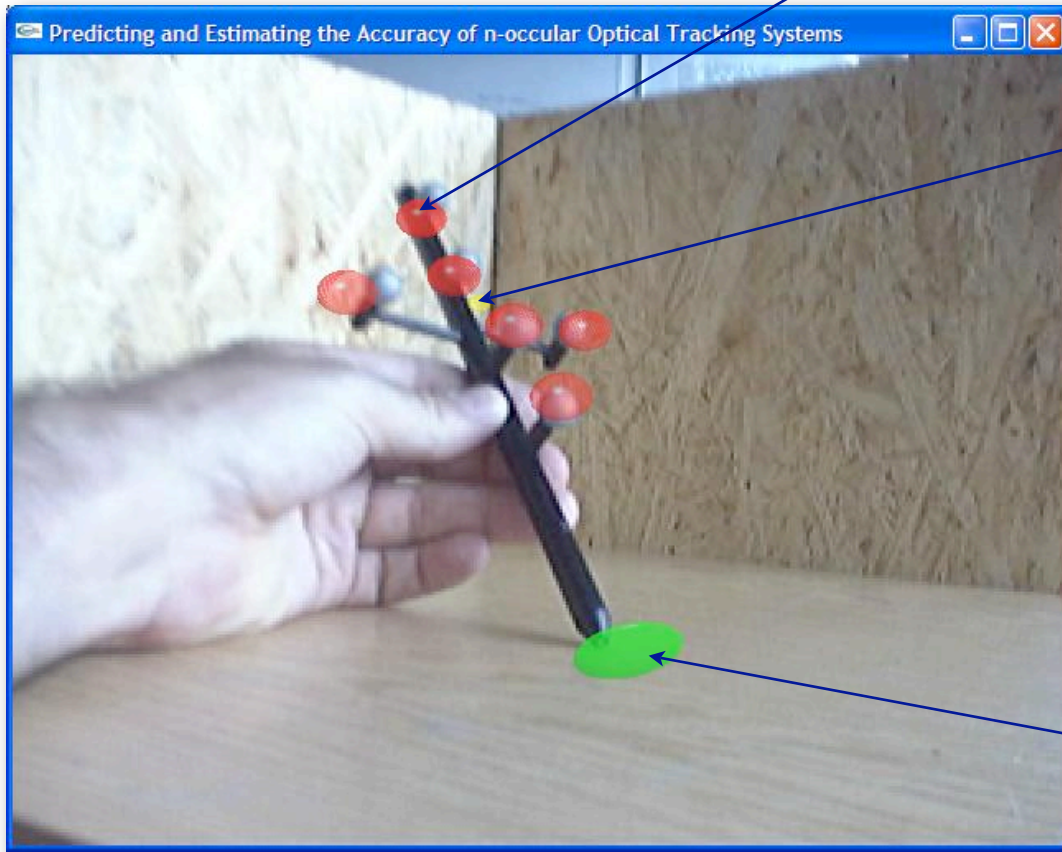
- MTE is given at the centroid, needs forward propagation to get the error at some point of interest

- use again $J_f(q) = \begin{bmatrix} 1 & 0 & 0 & 0 & q_z & -q_y \\ 0 & 1 & 0 & -q_z & 0 & q_x \\ 0 & 0 & 1 & q_y & -q_x & 0 \end{bmatrix}$

- and compute $\Sigma_{\vec{p}} = J_f \Sigma_{\vec{c}} J_f^T$

- resulting error at the point of interest is the sum of the translational part in the MTE and the propagated angular part in the MTE

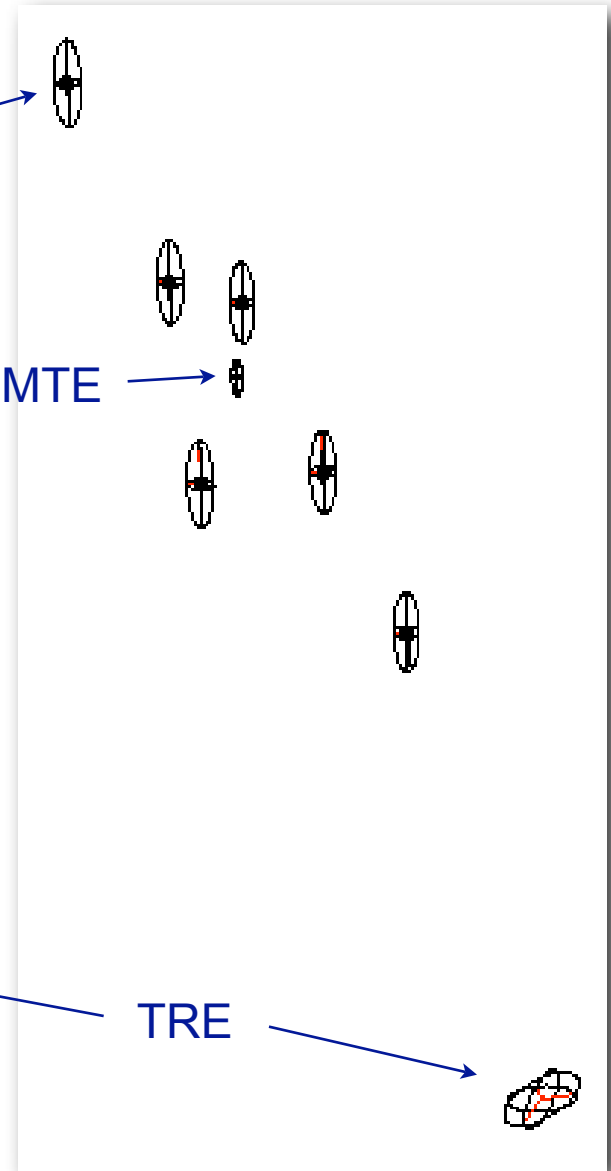
Example Propagation



FLE

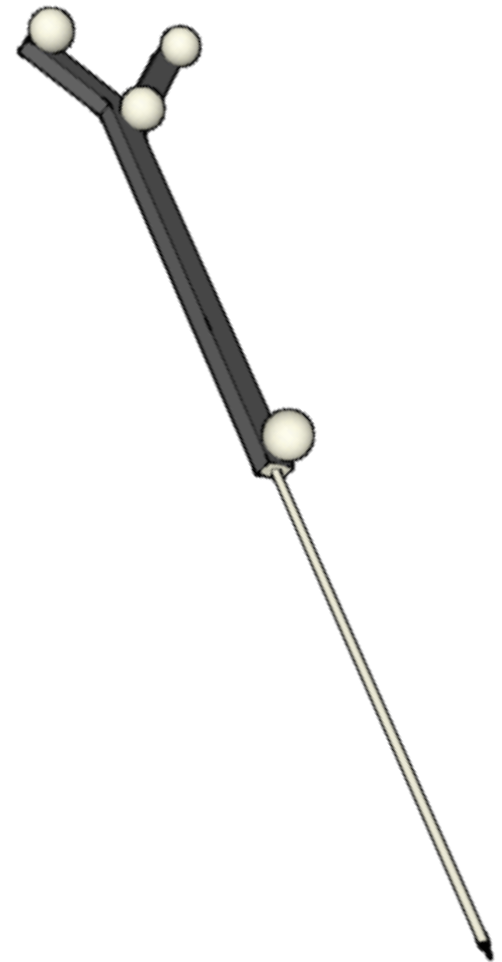
MTE

TRE



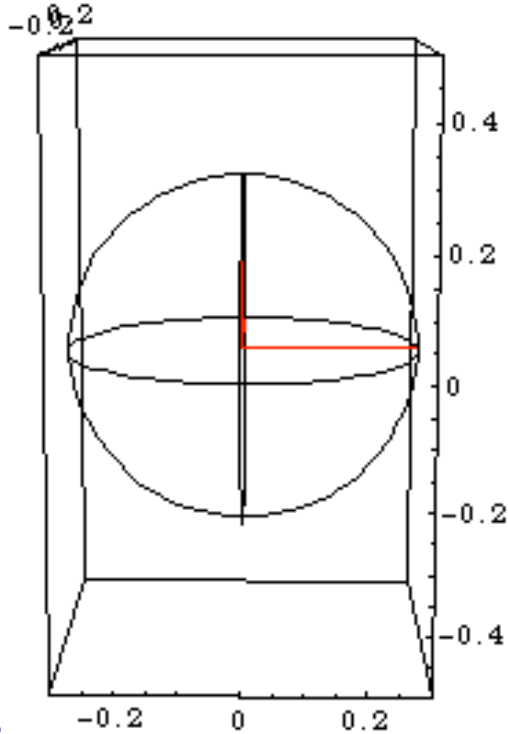
RMS error vs. Covariance Propagation

- Compare RMS error (isotropic, independent) with full covariance propagation
 - *RMS: Fitzpatrick et.al. 1998*
- Example: Pointing device, 4 Marker balls, POI ~15cm from the centroid
- Error propagation from fiducial location error to point of interest
- Compare the results

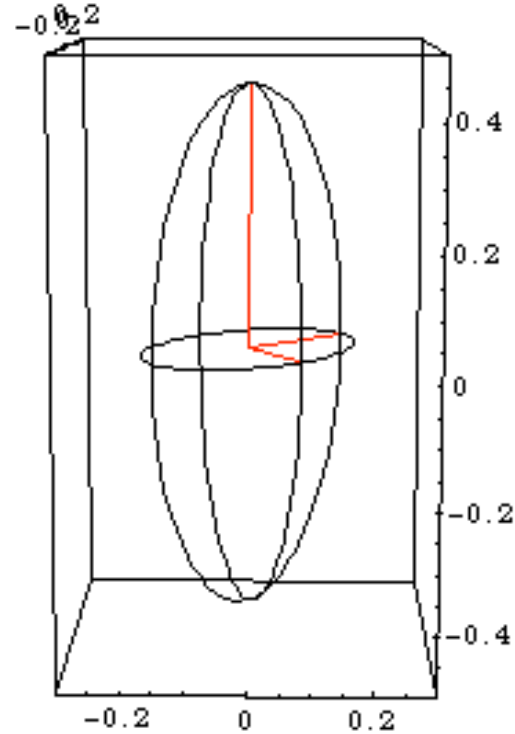


Assumed FLE at the Fiducials

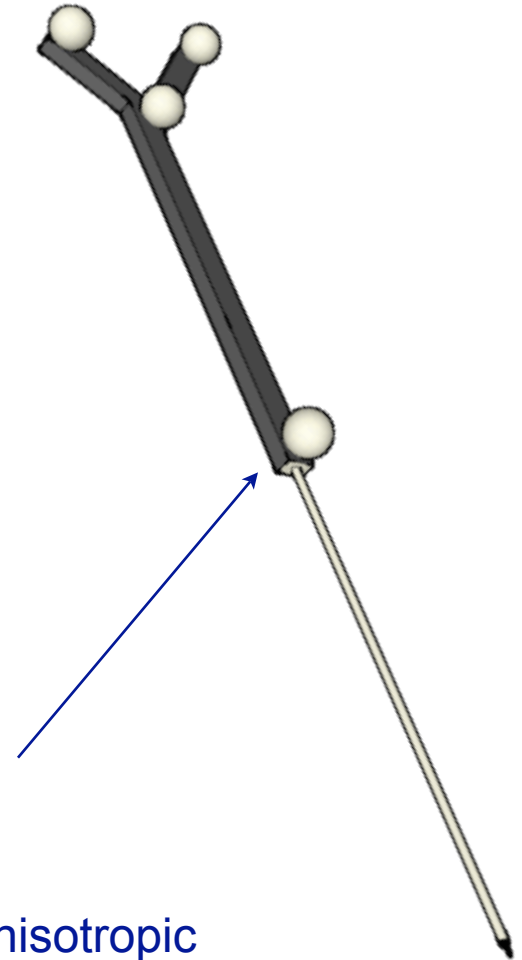
- same RMS error $\sim 0.1259\text{mm}$



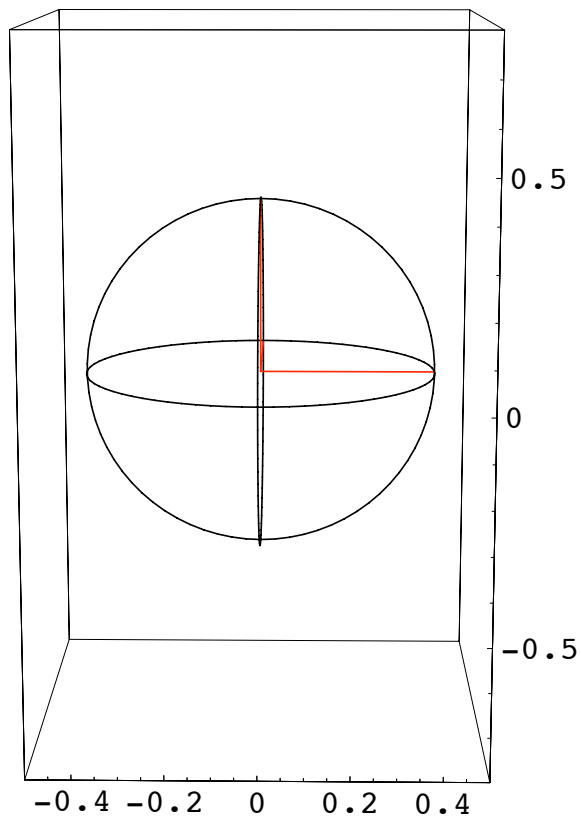
isotropic



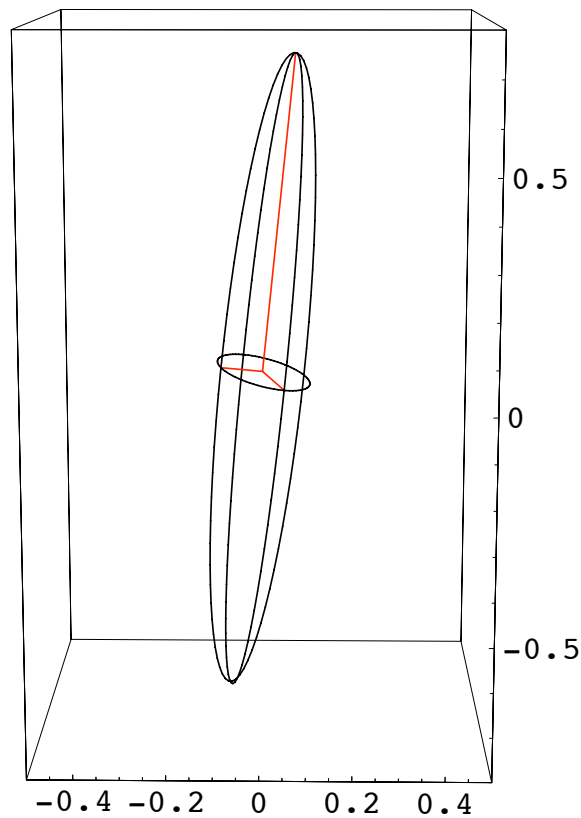
anisotropic



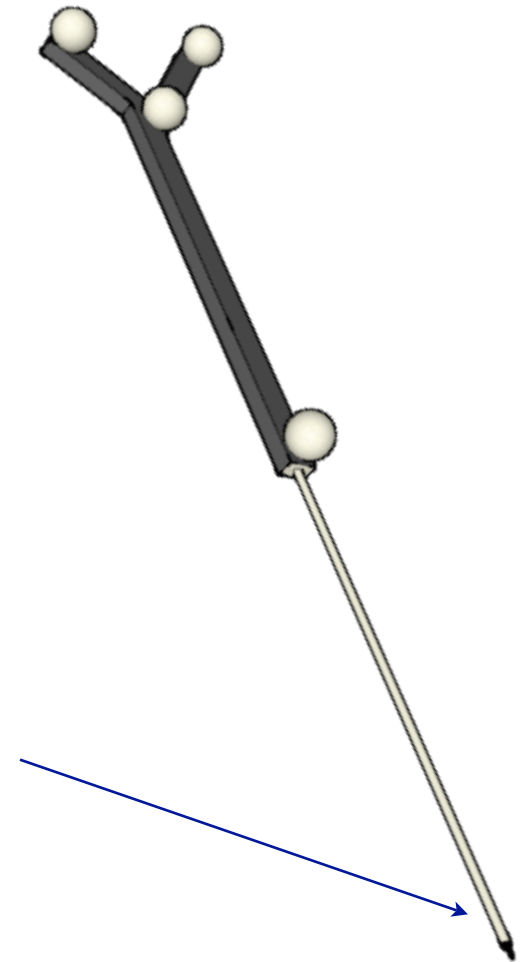
Resulting Error Estimate at Tip



RMS = 0.172 mm
Sqrt(Eigenvalues) =
{0.123, 0.115, 0.036}

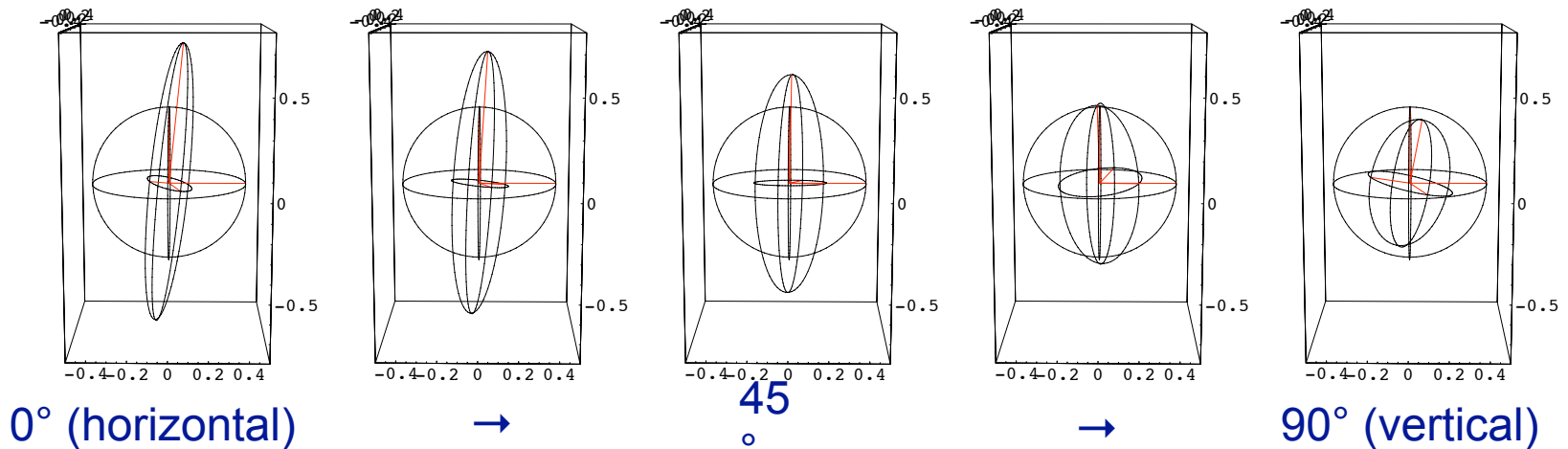


RMS = 0.203 mm
Sqrt(Eigenvalues) =
{0.182, 0.086, 0.024}



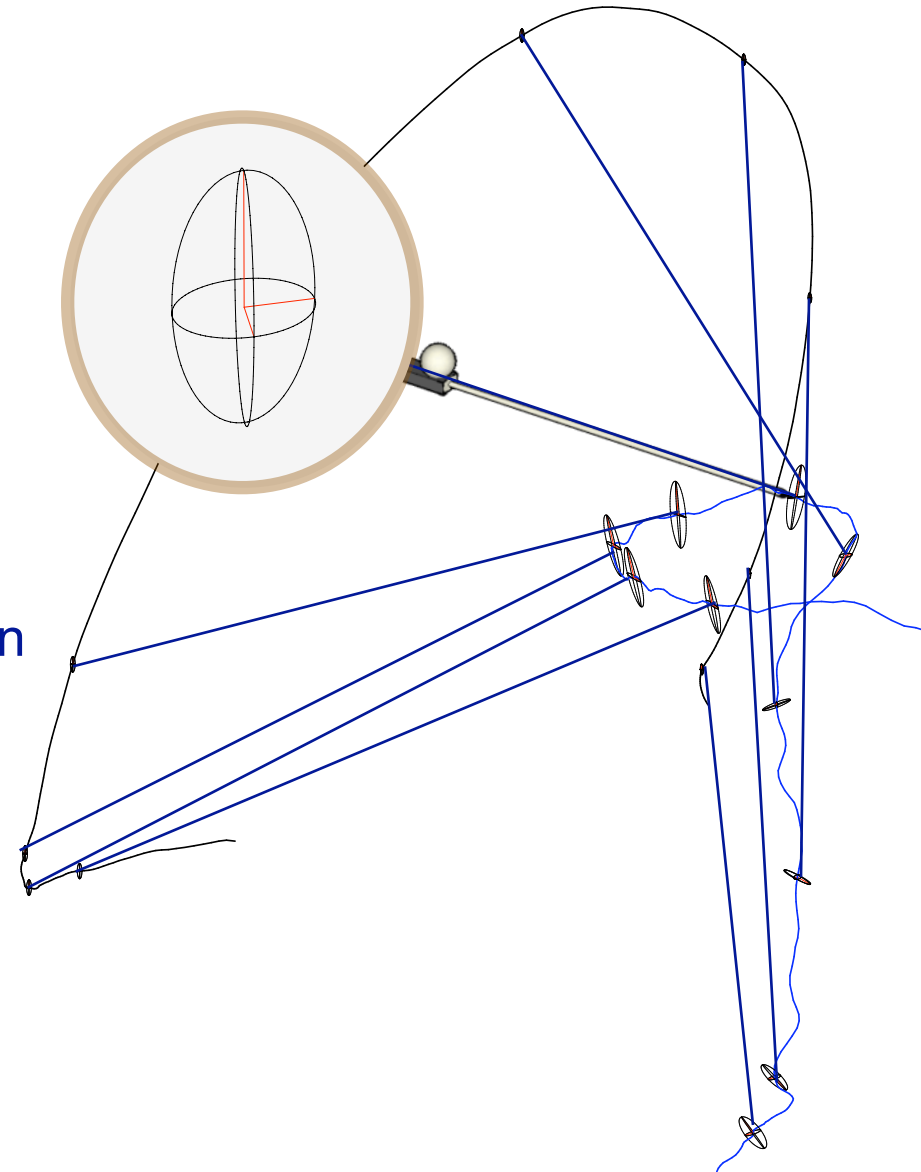
Covariance Propagation vs. RMS estimation

- In some cases TRE is smaller, in other cases TRE is larger



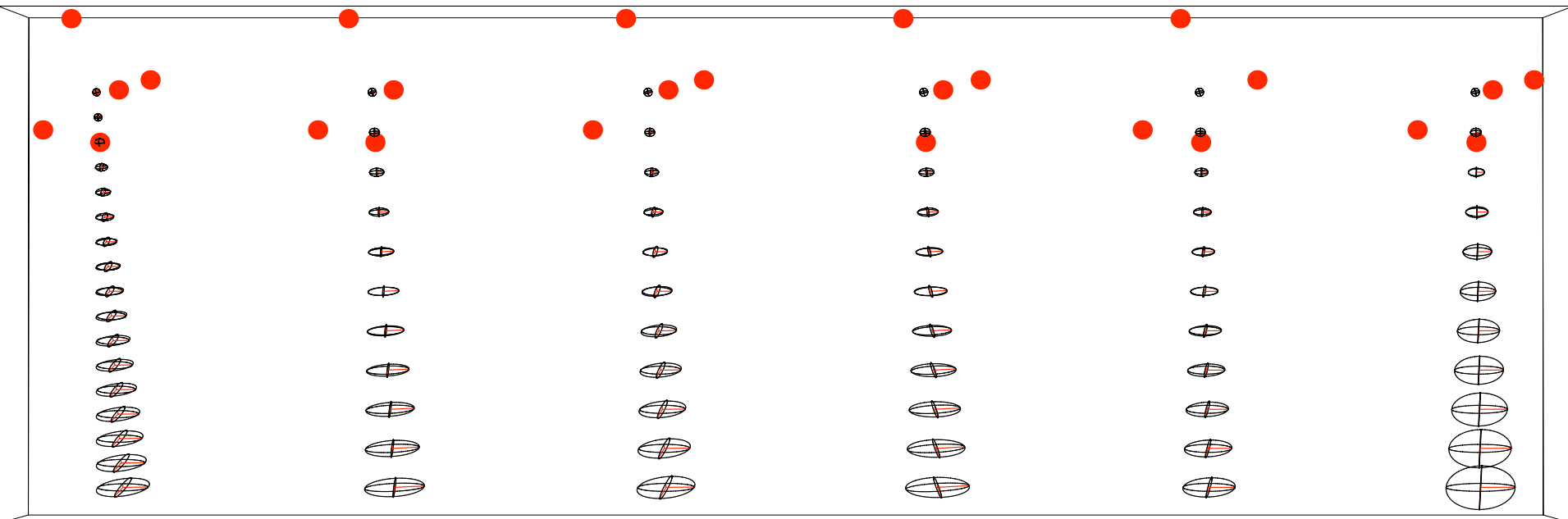
Orientation matters!

- Example path with propagation
 - same FLE covariance over the whole path
 - results in large differences
 - in the example, pointing down is much better than pointing horizontally
- Due to the marker configuration, error propagation is not the same in all directions
- Try not to amplify already large errors!



Occlusion of single fiducials

- different fiducial have different impact
- additionally depending on the FLE



full target

single fiducial occluded

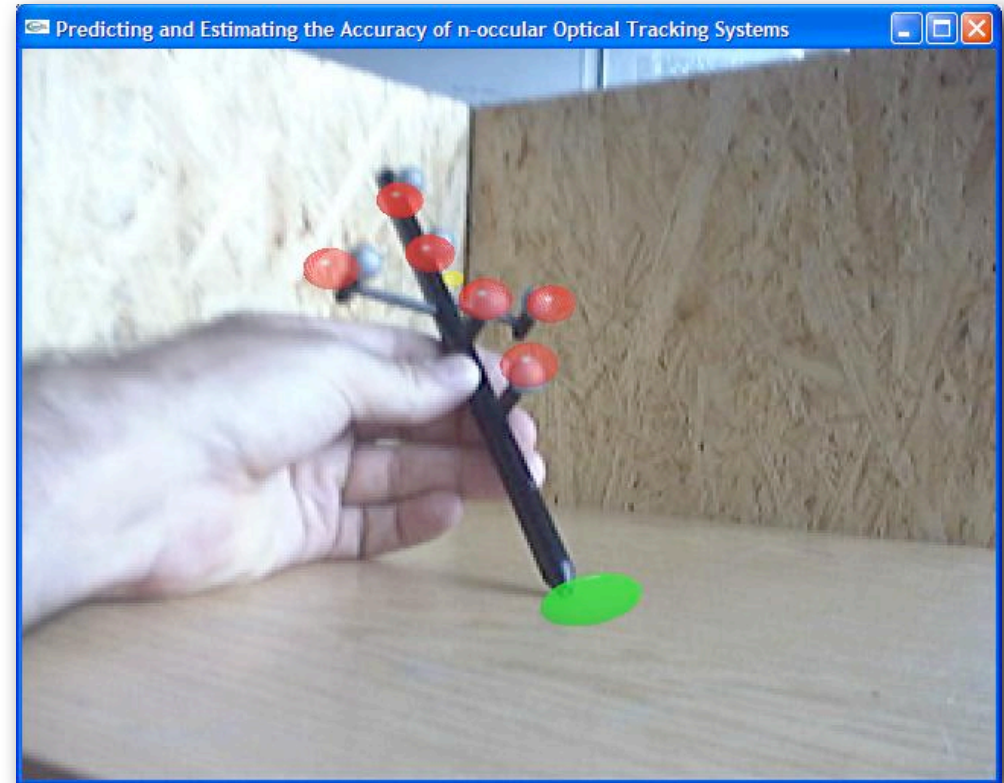
Conclusion

- Significant anisotropic behavior in
 - FLE Errors
 - Error Propagation
- RMS error estimates are not enough
- For common tools and targets, the worst part is often in the direction we are interested in...

- **Proposed method estimates the final target registration error (TRE) from the geometric setup of cameras, marker, fiducials and point of interest**

Questions?

- Come and see the demo



- Special thanks to
 - my co-authors
 - my students Xinxing Feng and Daniel Muhra for their help in programming the demo applications
 - my colleagues Pierre Georgel, Martin Groher, Marco Feuerstein, and Hauke Heibel for their help with OpenCV

