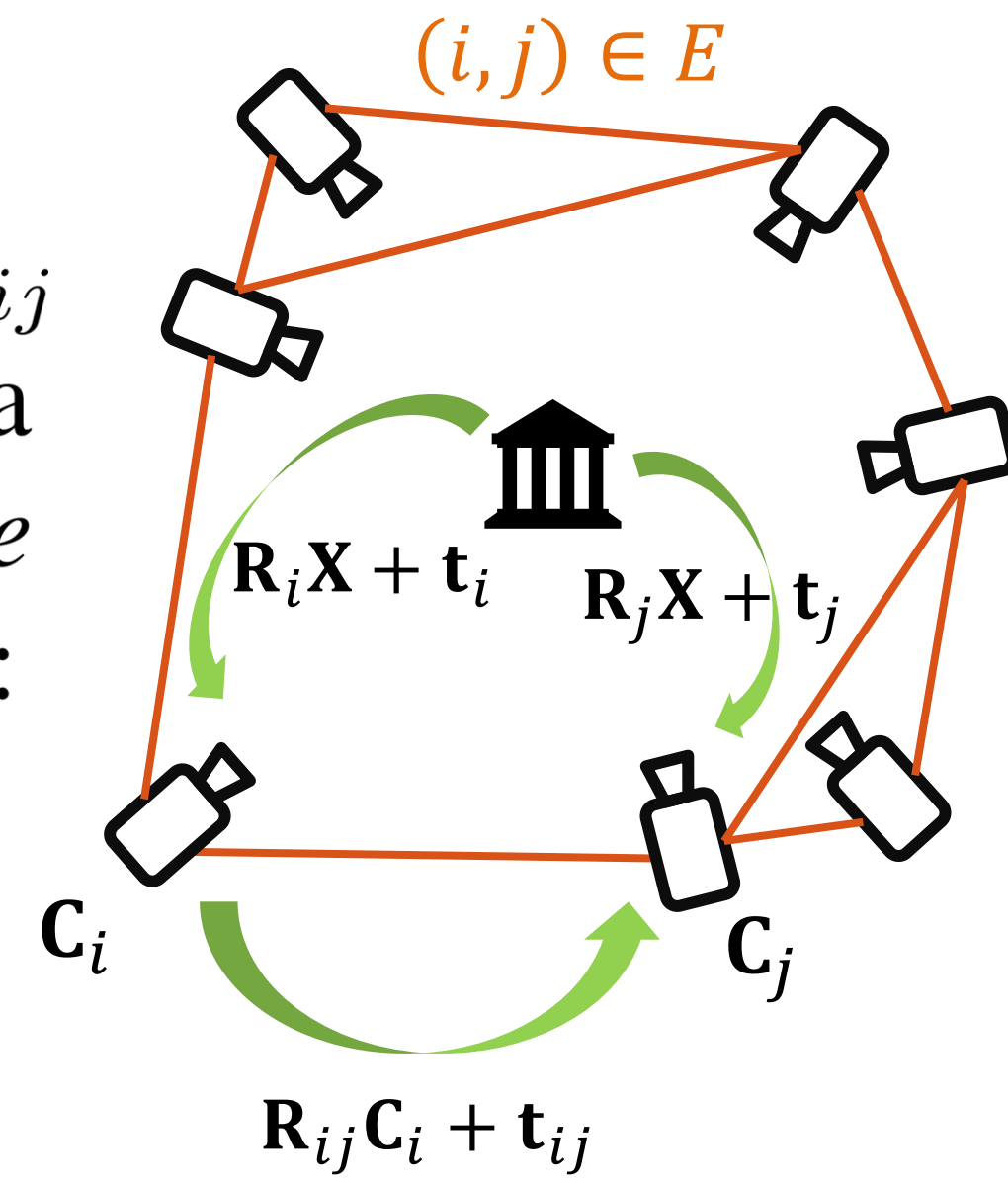


## Introduction

### Multiple Motion Averaging

Given the *relative* pose estimates  $\{\mathbf{M}_{ij} \in SE(3)\}$ , find the *absolute* camera poses  $\{\mathbf{M}_i \in SE(3)\}$  s.t. the *cycle consistency constraint* is satisfied [1]:

$$\mathbf{M}_{ij} \approx \mathbf{M}_j \mathbf{M}_i^{-1}, \forall i \neq j$$



### Contributions

- With observations  $\mathcal{D} \equiv \{\mathbf{q}_{ij}, \mathbf{t}_{ij}\}_{(i,j) \in E}$  and latent variables (unknown)  $\mathbf{Q} \equiv \{\mathbf{q}_i\}_{i=1}^n$ ,  $\mathbf{T} \equiv \{\mathbf{t}_i\}_{i=1}^n$  and  $\mathbf{x} = \{\mathbf{Q}, \mathbf{T}\}$ , compute MAP estimate:

$$(\mathbf{Q}^*, \mathbf{T}^*) = \arg \max_{\mathbf{Q}, \mathbf{T}} p(\mathbf{Q}, \mathbf{T} | \mathcal{D}) =$$

$$\arg \max_{\mathbf{Q}, \mathbf{T}} \left( \sum_{(i,j) \in E} \{ \log p(\mathbf{q}_{ij} | \mathbf{Q}, \mathbf{T}) + \log p(\mathbf{t}_{ij} | \mathbf{Q}, \mathbf{T}) \} \right)$$

Novel probabilistic model

$$+ \sum_i \log p(\mathbf{q}_i) + \sum_i \log p(\mathbf{t}_i)$$

- Obtain the full posterior distribution (via MCMC):

$$p(\mathbf{Q}, \mathbf{T} | \mathcal{D}) \propto p(\mathcal{D} | \mathbf{Q}, \mathbf{T}) \times p(\mathbf{Q}) \times p(\mathbf{T})$$

- with theoretical guarantees:

Samples close to the global minimum

$$|\mathbb{E} \hat{U}_N - U^*| = \mathcal{O}\left(\frac{\beta}{Nh} + \frac{h}{\beta} + \frac{1}{\beta}\right)$$

- controlled by a single inverse temperature parameter  $\beta$ .

$\beta = 1$  : SG-GMC [2]  
 $\beta \rightarrow \infty$  : Riemannian-GD + Momentum

$\mathbf{q}$  : quaternion  
 $\mathbf{t}$  : translation  
 $\mathbf{x}^*$  : global optimum  
 $U^* \triangleq U(\mathbf{x}^*)$   
 $\hat{U}_N$  : sample average  
 $N$  : iterations  
 $h$  : step-size

## Quaternions & Bingham Distributions

An antipodally symmetric probability distribution lying on  $\mathbb{S}^{d-1}$  with PDF  $\mathcal{B} : \mathbb{S}^{d-1} \rightarrow \mathbb{R}$ :

$$\mathcal{B}(\mathbf{x}; \Lambda, \mathbf{V}) = \frac{1}{F} \exp(\mathbf{x}^T \mathbf{V} \Lambda \mathbf{V}^T \mathbf{x}) = \frac{1}{F} \exp\left(\sum_{i=1}^d \lambda_i (\mathbf{v}_i^T \mathbf{x})^2\right)$$

$$d_{\text{riemann}} = \|\log(\mathbf{R}_1 \mathbf{R}_2^T)\| = 2 \arccos(|\mathbf{q}_1 \bar{\mathbf{q}}_2|) = d_{\text{bingham}}(\mathbf{q}_1, \mathbf{q}_2)$$



## Proposed Model

$$\mathbf{q}_i \sim p(\mathbf{q}_i), \quad \mathbf{t}_i \sim p(\mathbf{t}_i), \quad \mathbf{q}_{ij} | \cdot \sim p(\mathbf{q}_{ij} | \mathbf{q}_i, \mathbf{q}_j), \quad \mathbf{t}_{ij} | \cdot \sim p(\mathbf{t}_{ij} | \mathbf{q}_i, \mathbf{q}_j, \mathbf{t}_i, \mathbf{t}_j)$$

$$\mathbf{q}_{ij} | \mathbf{q}_i, \mathbf{q}_j \sim \mathcal{B}(\Lambda, \mathbf{V}(\mathbf{q}_j \bar{\mathbf{q}}_i)), \quad \mathbf{t}_{ij} | \mathbf{q}_i, \mathbf{q}_j, \mathbf{t}_i, \mathbf{t}_j \sim \mathcal{N}(\boldsymbol{\mu}_{ij}, \sigma^2 \mathbf{I})$$

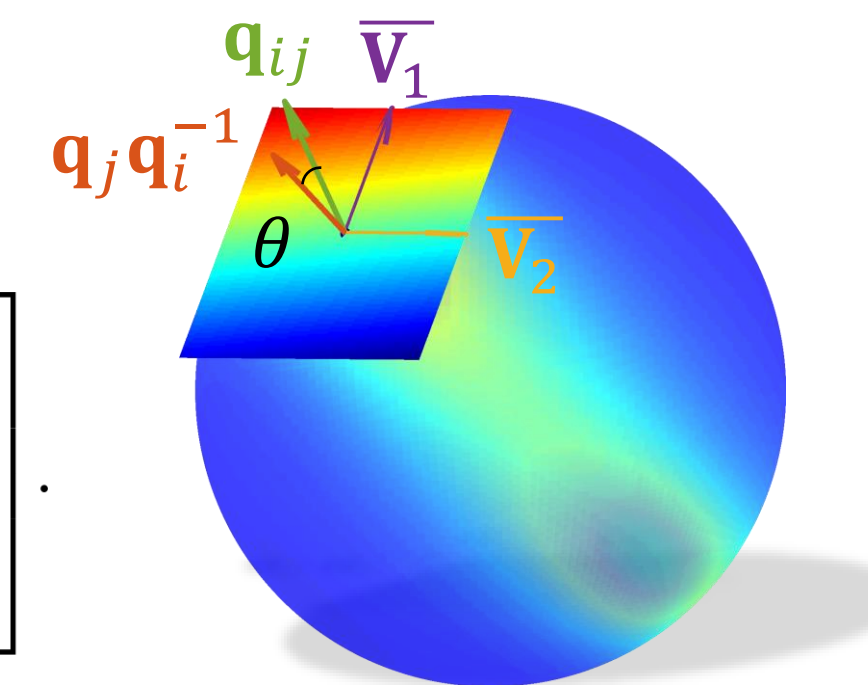
mode  $\equiv$  most likely relative pose:

$$\arg \max_{\mathbf{q}_{ij}} \{p(\mathbf{q}_{ij} | \mathbf{q}_i, \mathbf{q}_j) = \mathcal{B}(\Lambda, \mathbf{V}(\mathbf{q}_j \bar{\mathbf{q}}_i))\} = \mathbf{q}_j \bar{\mathbf{q}}_i$$

$S^3$  is a parallelizable manifold:

$$\mathbf{V}(\mathbf{q}) \triangleq \begin{bmatrix} q_1 & -q_2 & -q_3 & q_4 \\ q_2 & q_1 & q_4 & q_3 \\ q_3 & -q_4 & q_1 & -q_2 \\ q_4 & q_3 & -q_2 & -q_1 \end{bmatrix}$$

$$\boldsymbol{\mu}_{ij} \triangleq \mathbf{t}_j - (\mathbf{q}_j \bar{\mathbf{q}}_i) \mathbf{t}_i (\mathbf{q}_i \bar{\mathbf{q}}_j)$$



## Inference: Tempered Geodesic MCMC (TG-MCMC)

### Proposed SDE

$$d\tilde{\mathbf{x}}_t = \mathbf{G}(\tilde{\mathbf{x}}_t)^{-1} \mathbf{p}_t dt$$

$$d\mathbf{p}_t = -\left(\nabla_{\tilde{\mathbf{x}}} U_\lambda(\tilde{\mathbf{x}}_t) + \frac{1}{2} \nabla_{\tilde{\mathbf{x}}} \log |\mathbf{G}| + c \mathbf{p}_t + \frac{1}{2} \nabla_{\tilde{\mathbf{x}}} (\mathbf{p}_t^T \mathbf{G}^{-1} \mathbf{p}_t) dt + \sqrt{(2c/\beta) \mathbf{M}^T \mathbf{M}} dW_t\right)$$

Hamiltonian Paths converge to a measure with density:  $e^{-\beta U(\mathbf{x})}$

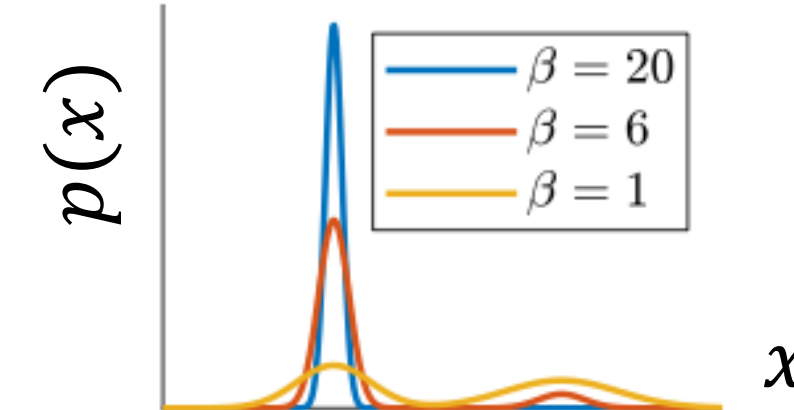
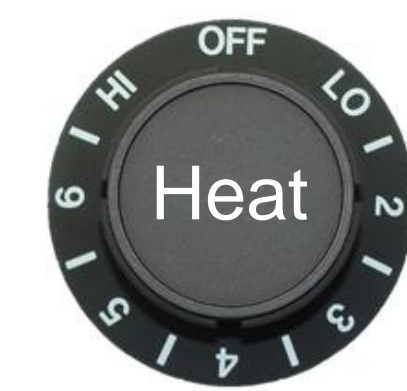
$\mathbf{G}$  : Riemannian metric,  $c$  : friction  
 $\mathbf{p}$  : momentum,  $dW_t$  : Brownian motion

### Split SDE

$$\text{A: } \begin{cases} d\tilde{\mathbf{x}}_t = \mathbf{G}^{-1} \mathbf{p}_t dt \\ d\mathbf{p}_t = \frac{1}{2} \nabla (\mathbf{p}_t^T \mathbf{G}^{-1} \mathbf{p}_t) dt \end{cases}$$

$$\text{B: } \begin{cases} d\tilde{\mathbf{x}}_t = 0 \\ d\mathbf{p}_t = -c \mathbf{p}_t dt \end{cases}$$

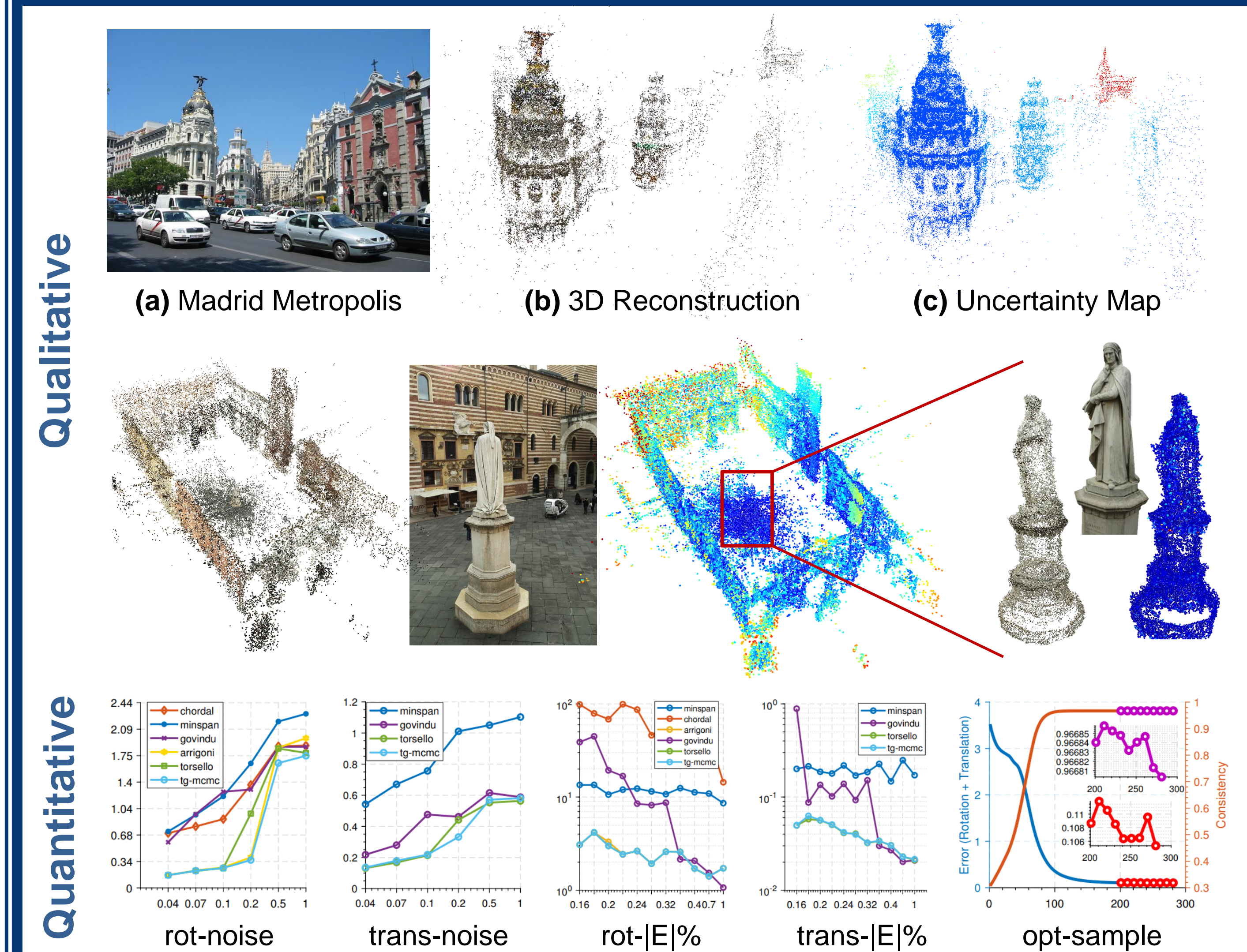
$$\text{O: } \begin{cases} d\tilde{\mathbf{x}}_t = 0 \\ d\mathbf{p}_t = -\left(\nabla U_\lambda(\tilde{\mathbf{x}}_t) + \frac{1}{2} \nabla \log |\mathbf{G}| + \sqrt{\frac{2c}{\beta}} \mathbf{M}^T \mathbf{M} dW_t\right) dt \end{cases}$$



- input:  $\mathbf{x}_0 = \{\mathbf{q}_1, \dots, \mathbf{q}_n, \mathbf{t}_1, \dots, \mathbf{t}_n\}$ ,  $\mathbf{v} = \{\mathbf{v}_1^q, \dots, \mathbf{v}_n^q, \mathbf{v}_1^t, \dots, \mathbf{v}_n^t\}$ ,  $\beta, c, h$
- for  $i = 1, \dots, N$  do
- Compute the gradient  $\nabla_{\mathbf{x}} U(\mathbf{x}_i)$   
    // Update the rotation components
- for  $j = 1, \dots, n$  do
- Run the B, O, A steps (in this order) on  $\mathbf{q}_j, \mathbf{v}_j^q$
- // Update the translation components
- for  $j = 1, \dots, n$  do
- Run the B, O, A steps (in this order) on  $\mathbf{t}_j, \mathbf{v}_j^t$

Results in a simple algorithm

## Evaluations



### References

- Govindu, Venu Madhav. "Combining two-view constraints for motion estimation." *CVPR 2001*.
- Liu, Chang, Jun Zhu, and Yang Song. "Stochastic gradient geodesic MCMC methods." *NIPS 2016*.
- Arrigoni, Federica, Andrea Fusiello, and Beatrice Rossi. "Camera motion from group synchronization." *3DV 2016*.
- Torsello, A., Emanuele R., and Andrea A.. "Multiview registration via graph diffusion of dual quaternions." *CVPR 2011*.

