

SobolevFusion: 3D Reconstruction of Scenes undergoing Free Non-rigid Motion

Supplementary Material

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In this document we first provide implementation details on how to create separable 1D Sobolev filters. Afterwards we present a study over parameters for our non-rigid model.

1. Calculating Sobolev Kernels

Here we explain how to obtain the three separable 1D filters starting with Eq. (12) of the paper:

$$(Id - \lambda\Delta)S = v. \quad (12)$$

Let the size of the 3D Sobolev filter we are interested in be $s \times s \times s$. Then the terms in the above equation are as follows:

- Id is the $s^3 \times s^3$ identity matrix.
- Δ is the 7-point stencil finite difference Laplacian matrix describing neighbouring voxels, resulting in the occupancy shown in Figure 1.
- v is a one-hot s^3 -element vector with 1 at the middle index $\lfloor \frac{s^3}{2} \rfloor$ (assuming indexing starting at 0). It corresponds to a discretized Dirac impulse of size $s \times s \times s$ voxels.
- S is the s^3 -element solution of the linear system that we are looking for. By restructuring it into a $s \times s \times s$ volume, we obtain the sought 3D Sobolev filter.

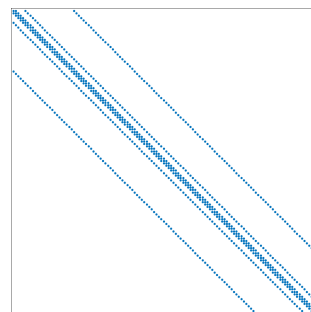


Figure 1. Occupancy of the $s^3 \times s^3$ matrix Δ .

In order to obtain the corresponding 1D filters, we make an approximation using the higher-order SVD decomposition of the tensor S . It yields three $s \times s$ U-matrices with equal elements. We take the first singular vector from each of this matrices, obtaining the approximated 1D filters S_x , S_y and S_z . Note that they have equal entries, but we use the subscript to indicate the spatial direction in which they are applied.

This procedure needs to be done only once for selected neighbourhood size s and Sobolev parameter λ , and then the 1D filter entries are stored. The separable convolutions are then applied over the energy derivative in each gradient descent step.

2. Parameter Study

For our parameter study we tested combinations of the following values: $s \in \{3, 7, 15\}$, $\lambda \in \{0.05, 0.1, 0.2, 0.4\}$ and $w_{reg} \in \{0, 0.1, 0.2, 0.4\}$. The outputs are displayed in Figures 2, 3, 4.

As Figure 2 shows, a Sobolev kernel size of $s = 3$ is not sufficient, while the larger ones deliver similar results. To ensure acceptable runtimes, we choose the smaller value of $s = 7$ as default.

As stated in the paper, a larger λ ensures a stronger influence of the Sobolev filter and a faster convergence rate. Therefore we recommend a value of 0.1 or 0.2, even though the other values also give good results.

Finally, the results show that motion smoothness is required to overcome noise, but even a small contribution is enough. Thus we set $w_{reg} = 0.2$ for our experiments.

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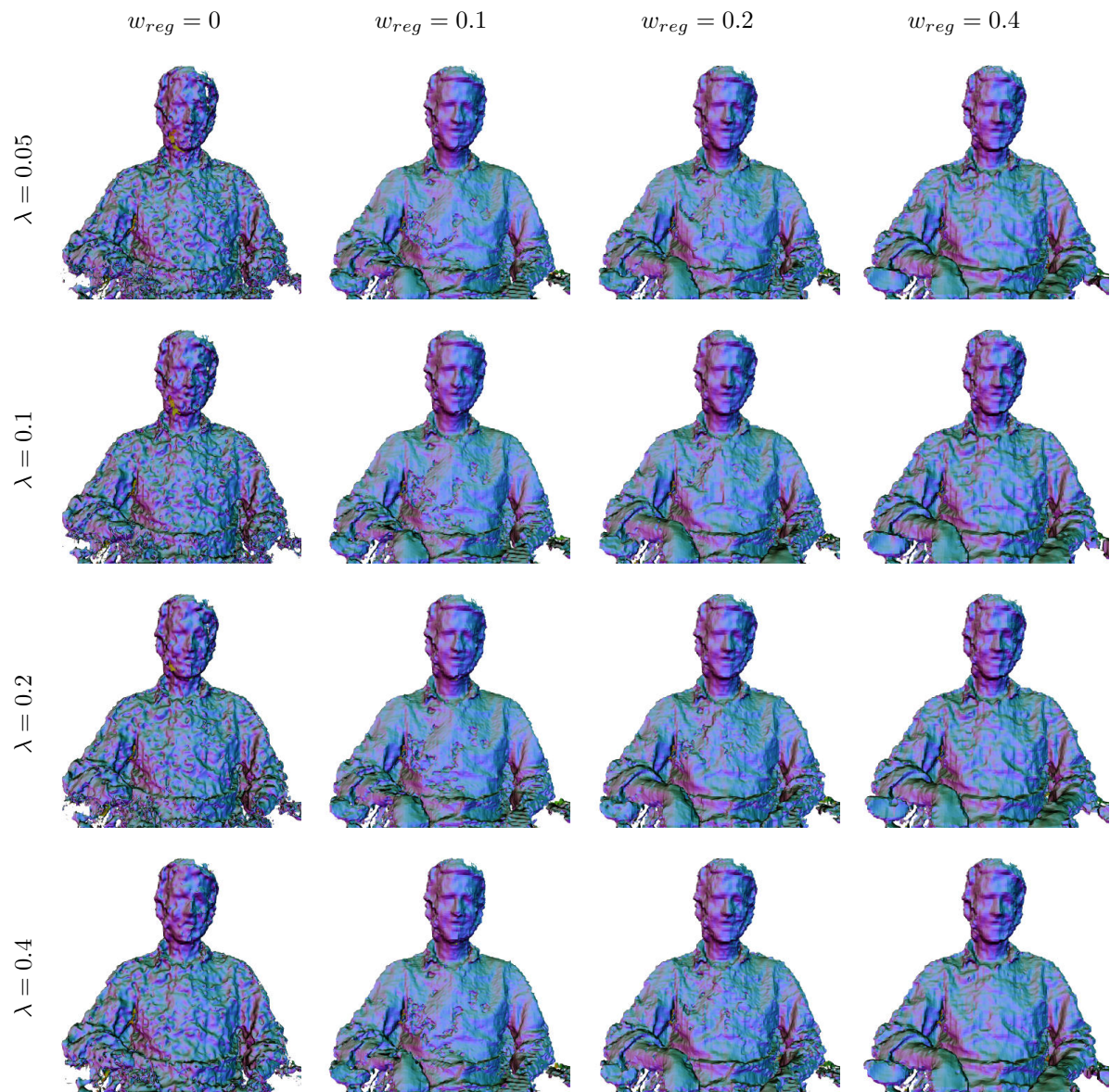


Figure 2. Neighbourhood size $s = 3$.

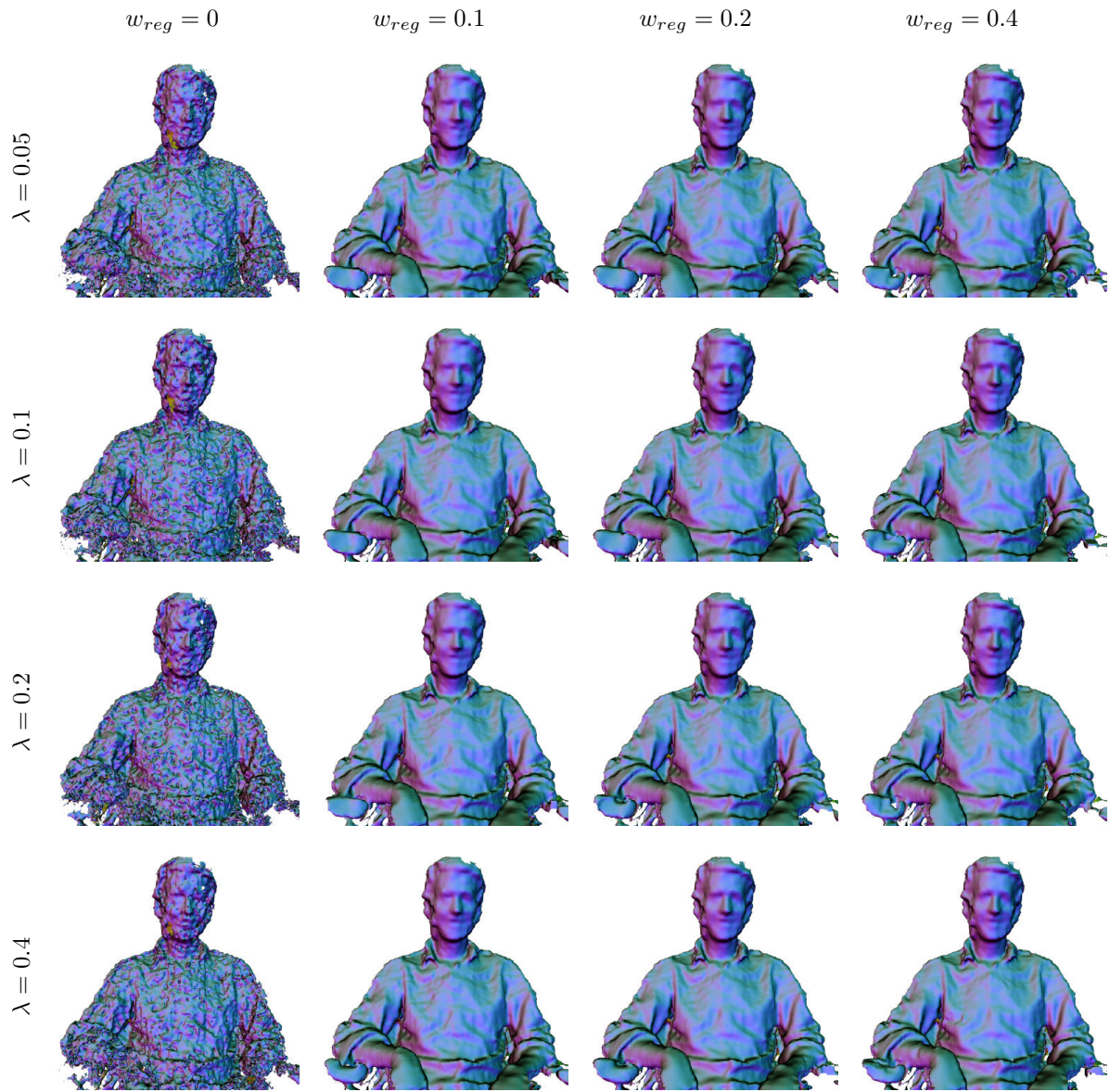


Figure 3. Neighbourhood size $s = 7$.

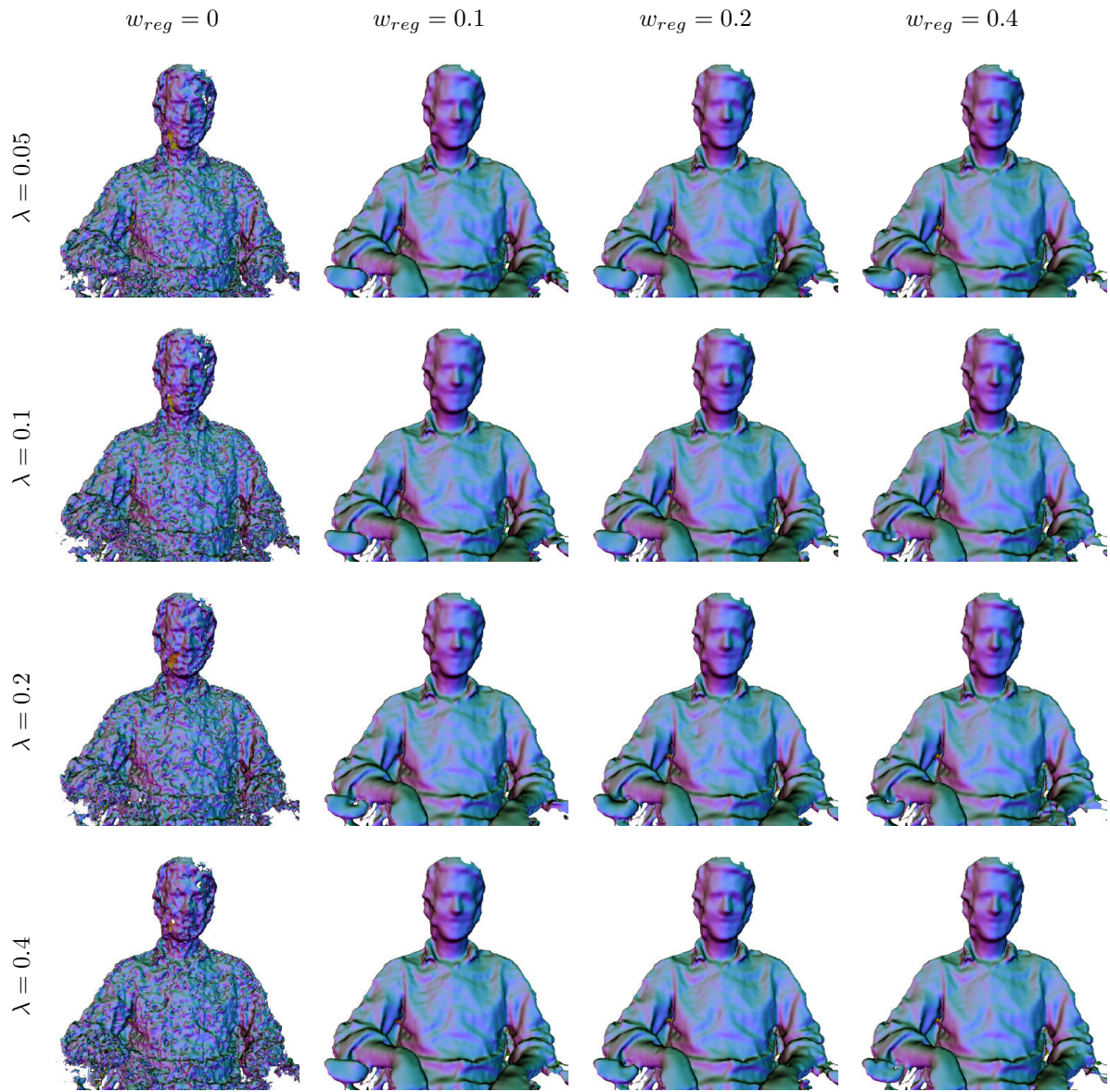


Figure 4. Neighbourhood size $s = 15$.