Motion Interpolation & Sensor Fusion

Final presentation

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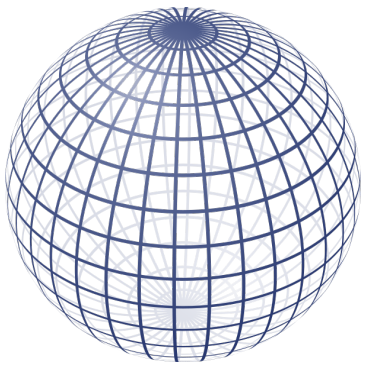
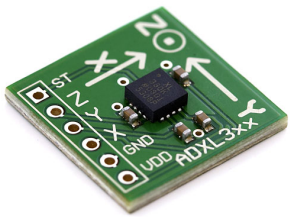
Introduction

- Track 3D Objects with
  - Optical Tracking Systems, e.g. cameras
  - embedded system or inertial measurement units (IMU), e.g. accelerometers





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[1] svgrepo

Introduction

- Motion Interpolation

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- Motion Interpolation
  - Smoothness

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  - Accuracy

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- Motion Interpolation
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  - Accuracy
- **Sensor Fusion**

OTS (e.g. cameras)	IMU (e.g. accelerometers)
Good tracking of translation	Almost impossible to track translation
Bad tracking of rotation	Good tracking of rotation

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  - $\Rightarrow$ combine to make the best motion interpolation

Describing Rotations

Quaternions

- General Form

$$q_1 1 + q_2 i + q_3 j + q_4 k = (q_1, q_2, q_3, q_4)^T$$

$$i^2 = j^2 = k^2 = ijk = -1$$

- Only represent rotations.
- Coordinate system independency.
- No gimbal lock.
- Simple interpolation.

Describing Rotations

Unit Quaternions

- Set H_1: $\| q \| = 1$.
- H_1 constitutes a hypersphere in quaternion space.
- The set is closed under multiplication.

Rotations

- $SO(3)$: space of three-dimensional rotations.
- Rotation about the axis $v = (v_1, v_2, v_3) \in \mathbb{R}^3$, angle θ

$$q = [\cos(\theta/2), \sin(\theta/2)v]$$

- For each rotation there are 2 unit quaternions: q and $-q$ (antipodal). H_1 is a "double-covering" of SO_3.

Visualizing Quaternions

Explain here the process to convert quaternions trajectory on the unit sphere !

Basic Interpolation Methods

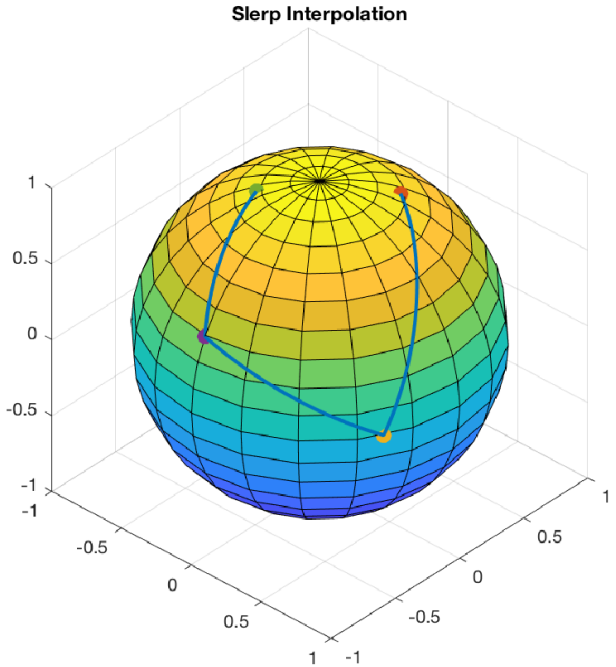
Let $q_0, q_1, \overline{q_0} \in \mathbb{H}$ and $h \in [0, 1]$:

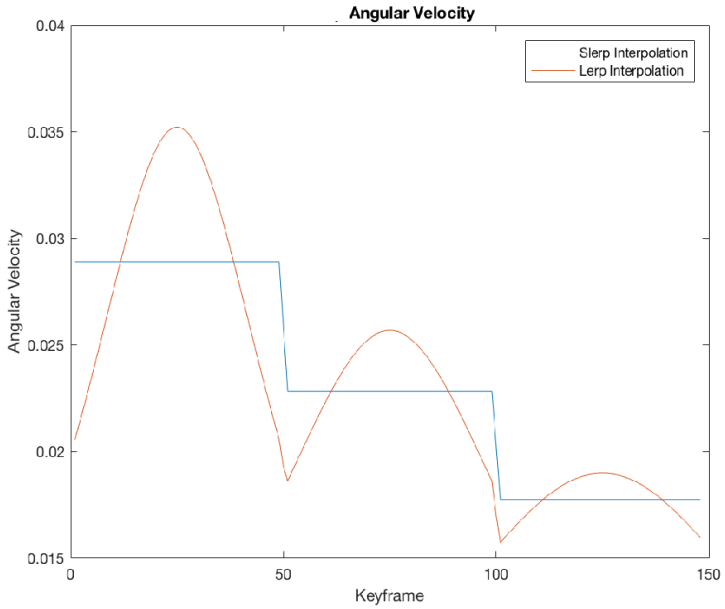
- Liner Quaternion interpolation

$$Lerp(q_0, q_1, h) = q_0(1 - h) + q_1 h.$$

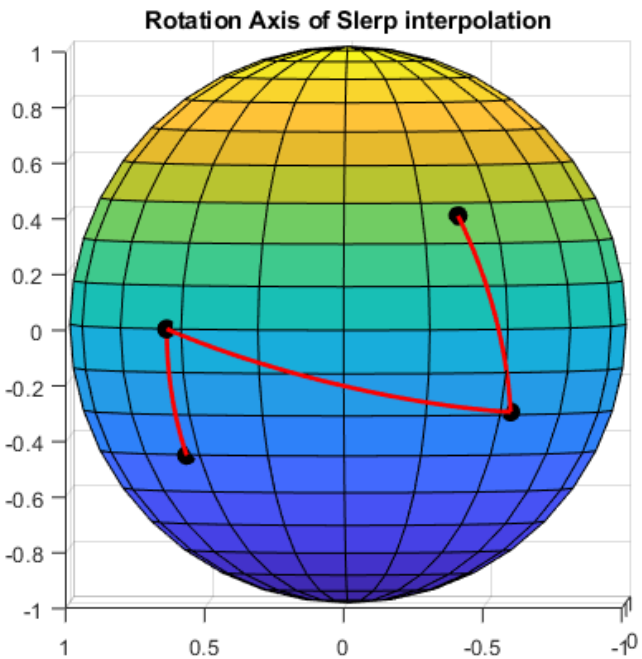
- Spherical Linear Quaternion interpolation

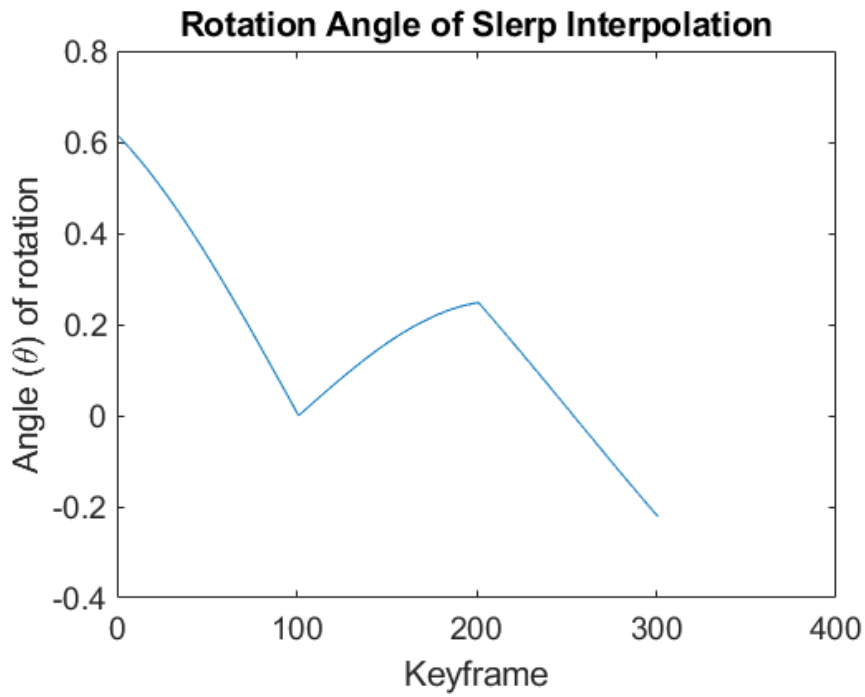
$$SLERP(q_0, q_1, h) = q_0(\overline{q_0} q_1)^h$$





Describing Rotations





Describing Rotations

(Slerp) Quaternion interpolation with rotation over the $x - axis$

Linear Interpolation

- Most simple interpolation is piecewise linear.
- Given a sequence of points $p_i \in \mathbb{R}^n$, we can represent linear interpolation in a cumulative form:

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$$= p_0 + \sum_{i=1}^{n} \alpha_i(t)\Delta p_i$$

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- α_i ramps from 0 to 1 in the interval $i <= t < i + 1$.
- Similarly, we can construct a piece-wise quaternion slerp in a cumulative form:

$$q(t) = q_0 \prod_{i=1}^{n} \omega_i^{\alpha_i(t)}$$

with $\omega_i = log(q_{i-1}^{-1} q_i)$, $q^t := exp(tlog(q))$.

Interpolation Bases

- Several different interpolation schemes can be represented in the cumulative form. $\alpha_i(t)$ is replaced with some basis function $\beta_i(t)$.

$$q(t) = q_0 \prod_{i=1}^{n} exp(\omega_i \beta_i(t))$$

- B-splines
- Bezier curves

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- B-splines
- Bezier curves
- The cumulative form describes an **approximation**. In order to interpolate exactly a non-linear system of equations needs to be solved.

Bezier Curves

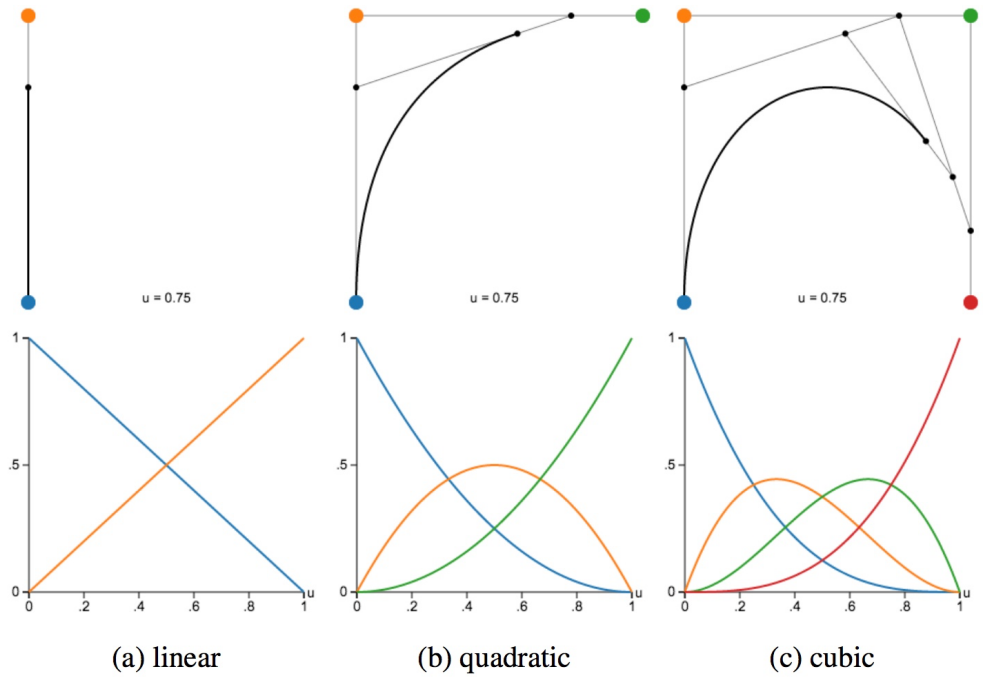
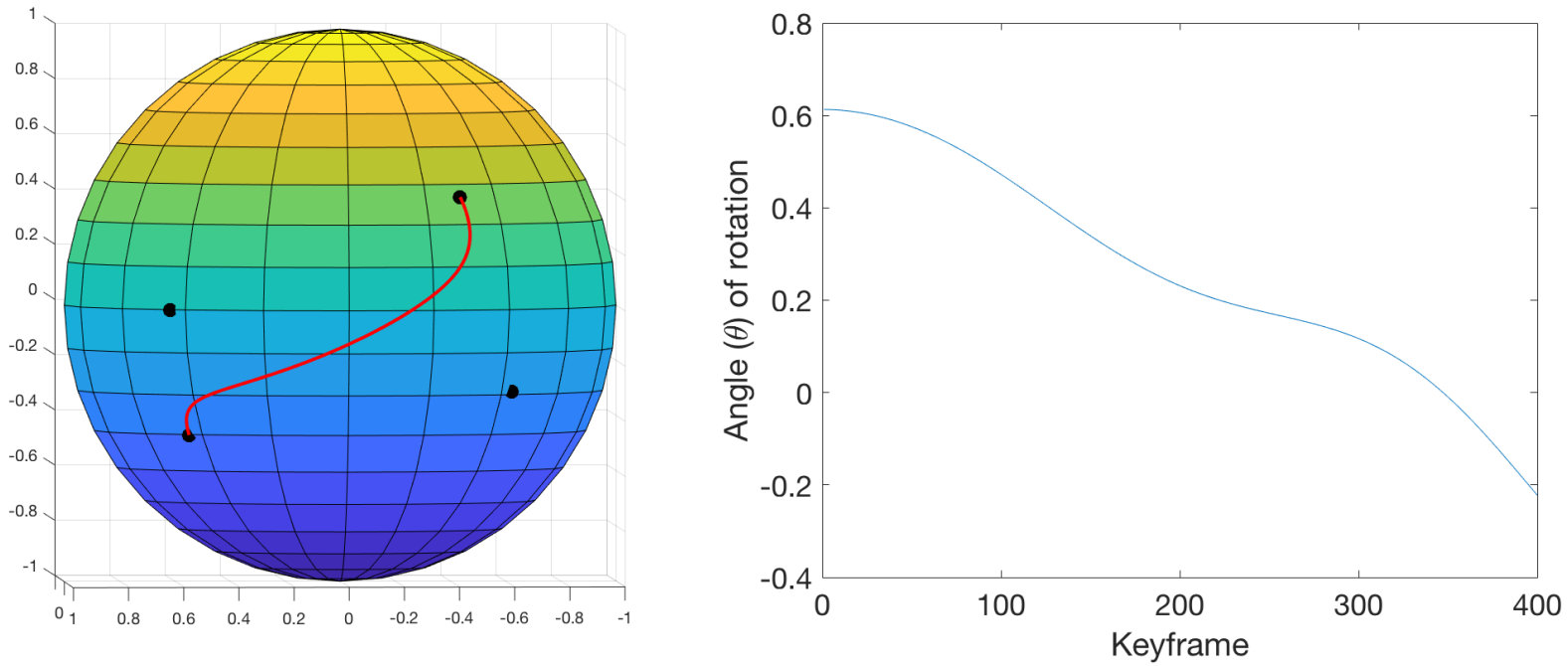


Figure 1: Bezier Curves and their Basis Functions [2]

- global control / very smooth ($\in \mathbf{C^{n-1}}$)
- simple implementation

[2] Anonymous Preprint 2018 [under review]

Bezier Curves



(a) Rotation Axis

(b) Rotation Angle

Figure 2: Rotation Axis and Angle of Bezier Approximation

Bezier Curves

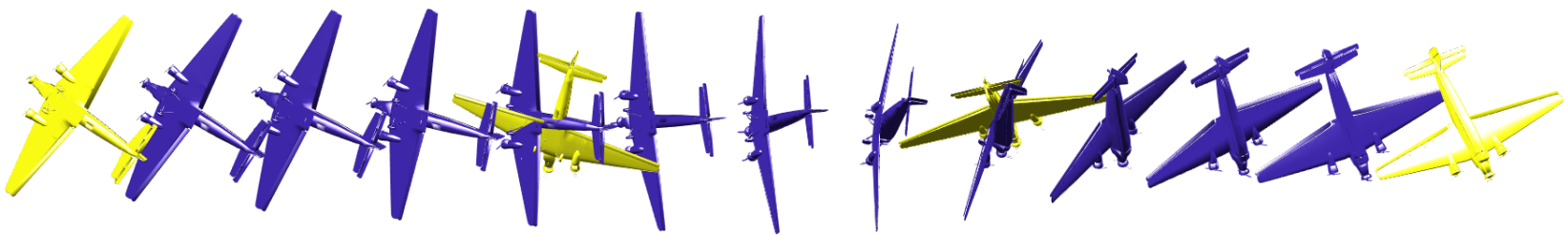


Figure 3: Bezier Curve Interpolation on an Object

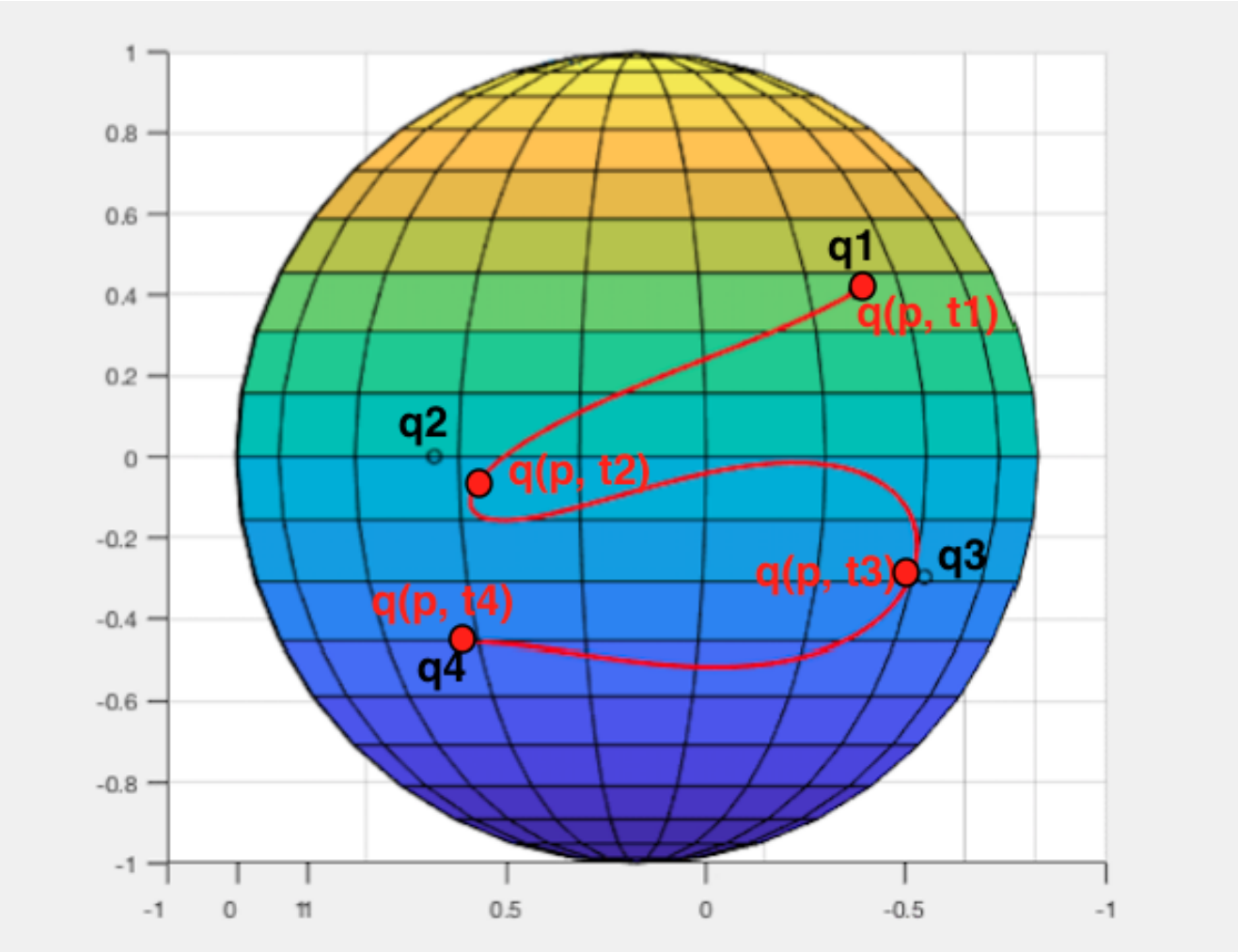
Optimization

- Smoothness

Optimization

- Smoothness
- Accuracy

Optimization



Distance choice

- Goal : evaluate the distance between the initial quaternions $(q_{orig})_{i=1..N}$ and the interpolation path $(q_{calc}(p,t))_{i=1..N}$
- Euclidean distance : $||q_{orig,i} - q_{calc,i}||_2^2$
- Angular distance : $Arccos(2q_{orig,i} \circ q_{calc,i} - 1)$

Optimization

- Smoothness
- Accuracy

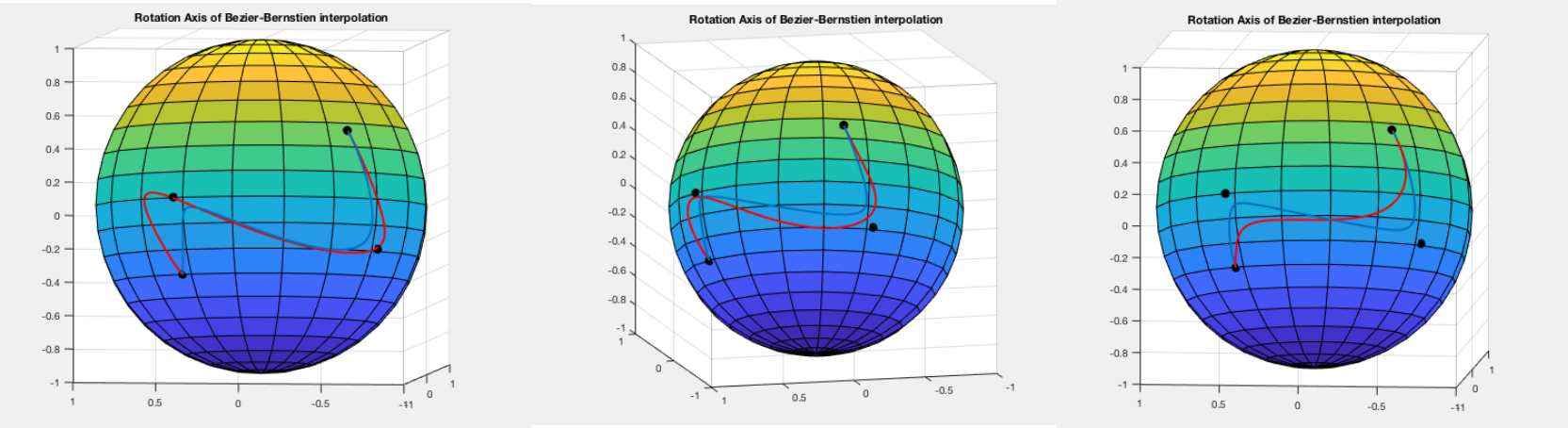
$$\min_{p} \sum dist(q(p, t_i) - q_i) + \lambda \int ||\ddot{q}(p, t)||^2 dt$$

q_i : original quaternions
t_i : times at which the calculated trajectory must fit the original quaternions
p : control points (our variable)
$q(p, t)$: calculated trajectory at time t.

Animation Bezier Curve

Animation Bezier Curve

Trade-off between smoothness and accuracy



(a) $\lambda = 20$ (b) $\lambda = 200$ (c) $\lambda = 2000$

Figure 4: Bezier optimization for different λ

Result of Optimization

Figure 5: Optimized Bezier Interpolation

Solve the problem

$$\min_{p} \sum dist(q(p, t_i) - q_i) + \lambda \int ||\ddot{q}(p, t)||^2 dt$$

- Compute the gradient
- Problem highly non-linear, not convex $\Rightarrow$ Poor convergence
- Use Quasi-Newton method algorithm by providing the gradient

Conclusion : Use Matlab optimization function *fminunc*.

Performance : Running time

- Estimated gradient (numerical) method $\Rightarrow$ not as fast as an optimization with provided gradient.
- Order of magnitude : couple of minutes for 4 or 5 quaternions $\Rightarrow$ not made for real-time data. But the running time growths exponentially if you increase the number of quaternions.

Pose Estimation

Figure 6: ArUco Marker and IMU

IMU and OTS data is noise

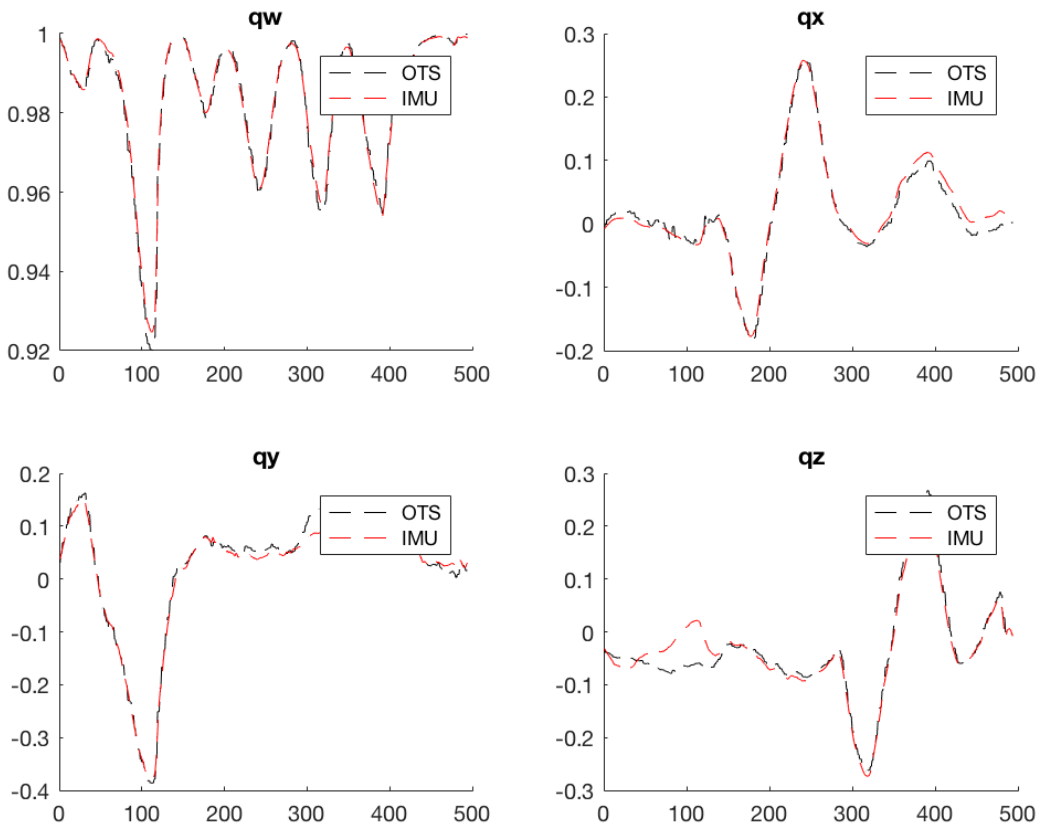


Figure 7: OTS and IMU poses

Undesirable Minima

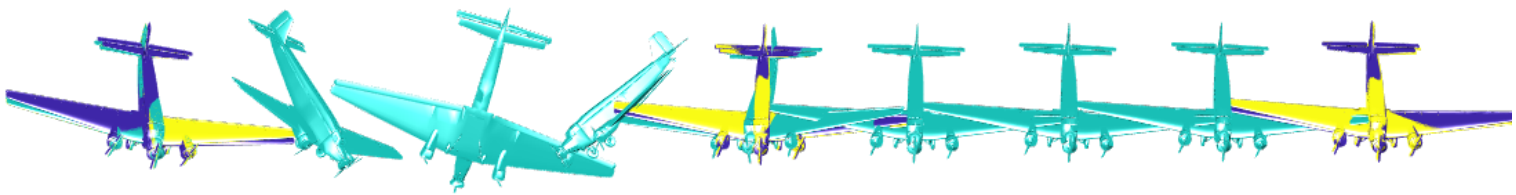


Figure 8: Undesirable minimum found through anti-podal point

Undesirable Minima

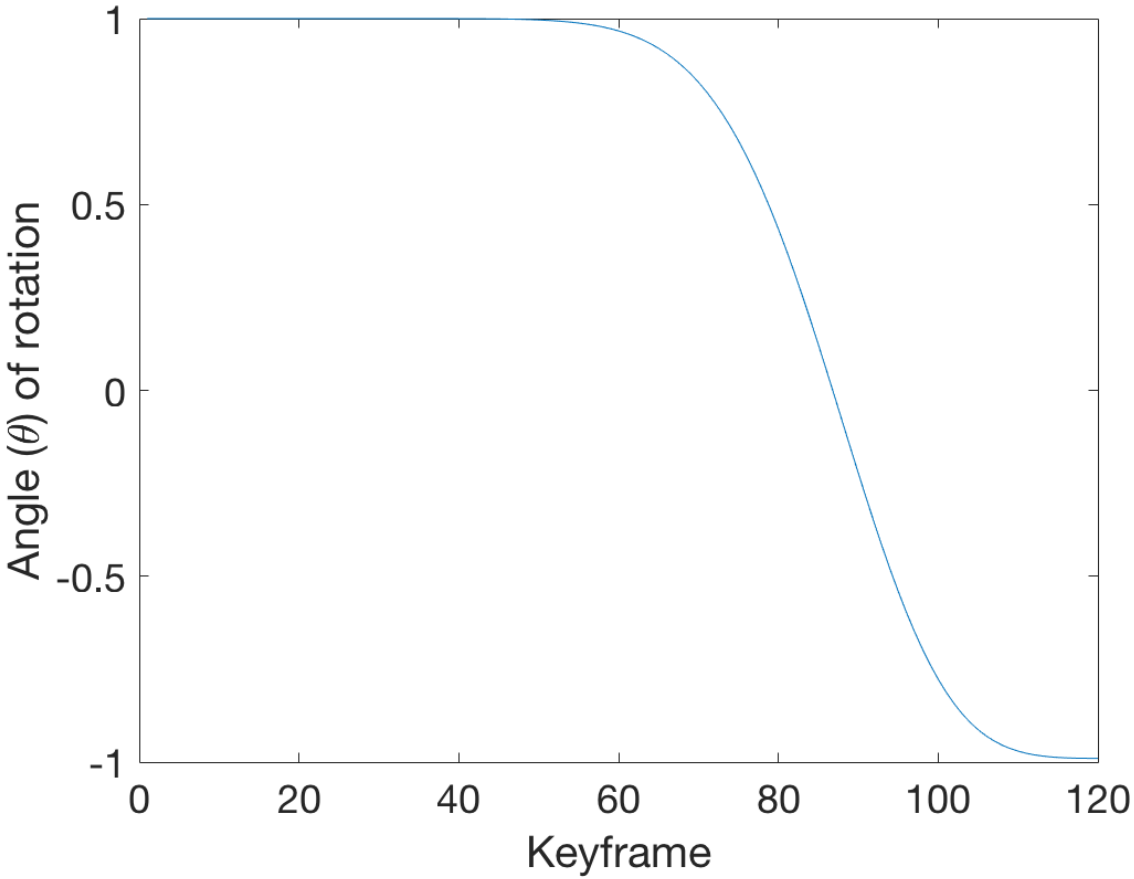


Figure 9: Rotation Angle of undesirable minimum at anti-podal point

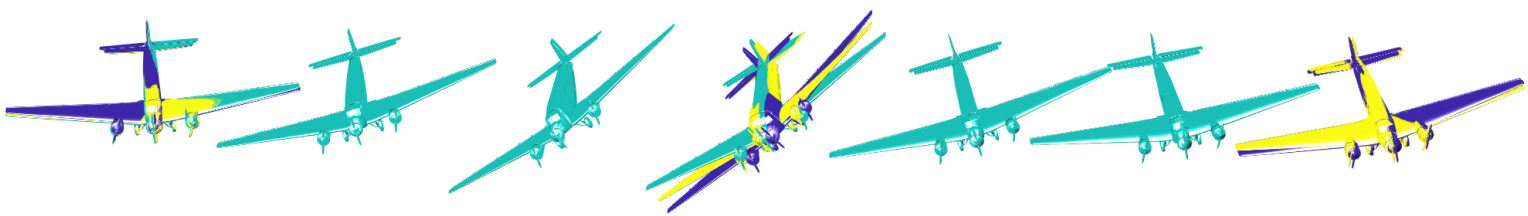
Updated Optimization Model

- J The set of captured OTS poses
- I The set of captured IMU poses
- Quaternions are constrained to Northern Hemisphere

$$\min_{p} \quad \lambda_1 \sum_{i \in I} ||q(p, t_i) - q_i|| + \lambda_2 \sum_{j \in J} ||q(p, t_j) - q_j|| + \int ||\ddot{q}(p, t)||^2 dt$$

$$\text{s.t.} \quad q_0 \geq 0$$

Interpolation of OTS and IMU data



Interpolation of OTS and IMU data

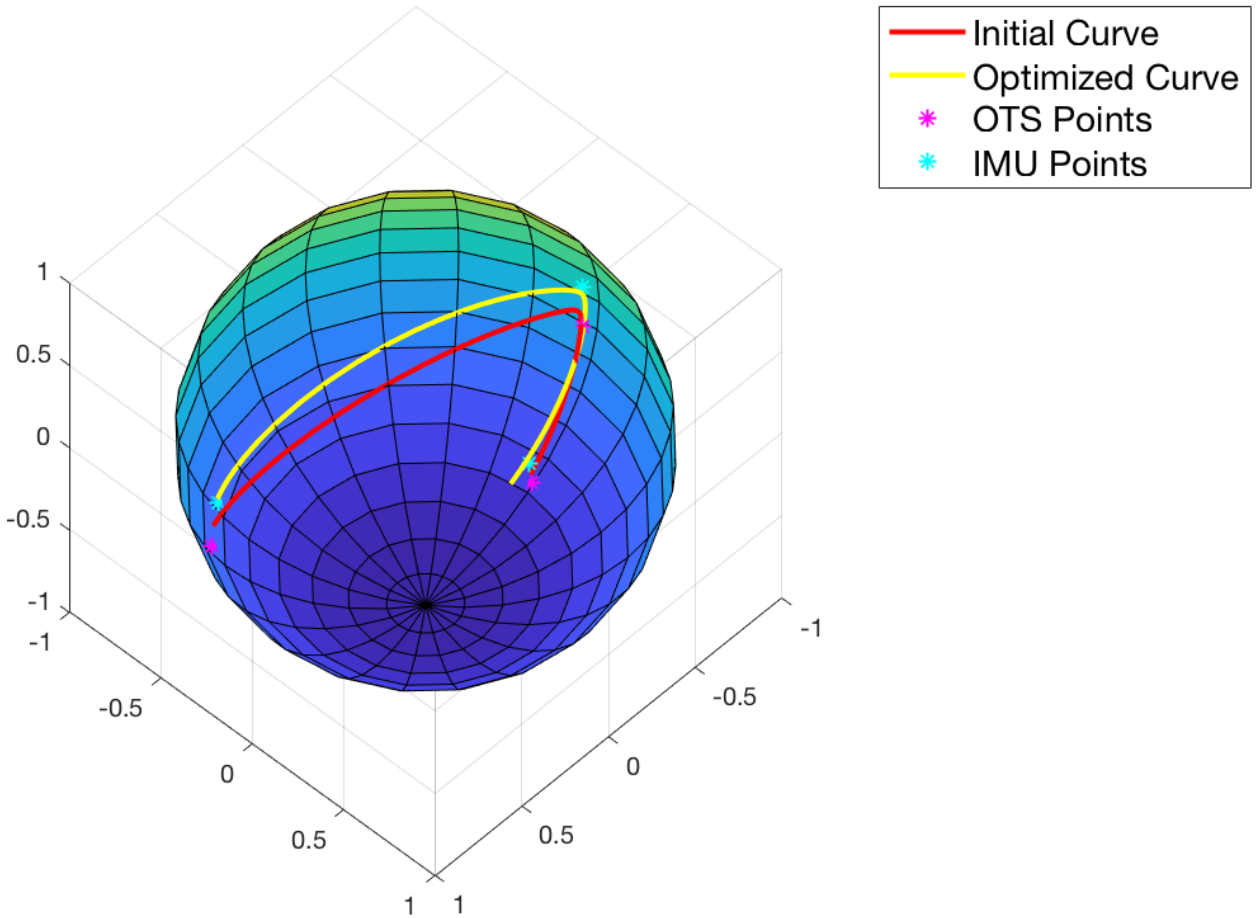


Figure 10: Interpolation Rotation Axis

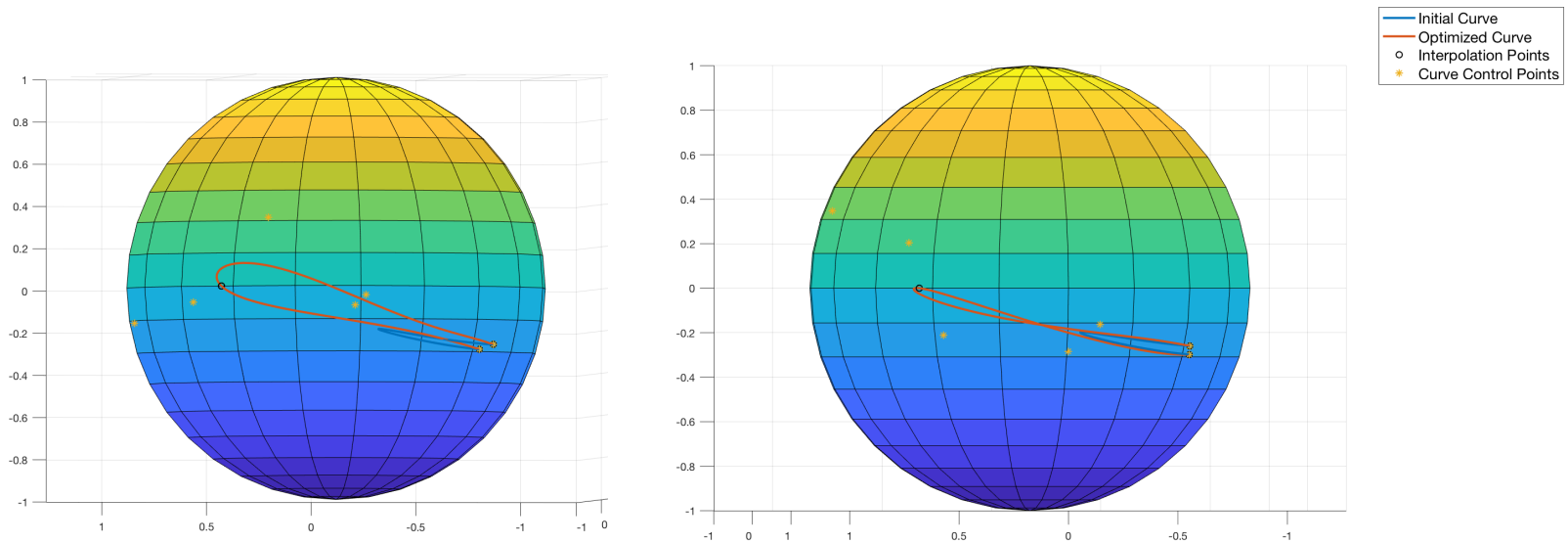
Conclusion

- Implemented Bezier curve interpolation for quaternions
- Designed an Optimization Model to improve interpolation
- Applied it to Sensor Fusion data provided by Framos

Thank you for your attention!

Figure 11: Optimized Bezier Interpolation

Sharp Turn



(a) $\lambda = 3$ (b) $\lambda = 0.1$

Figure 12: Rotation Axis and Angle of Sharp Twist

Sharp Turn

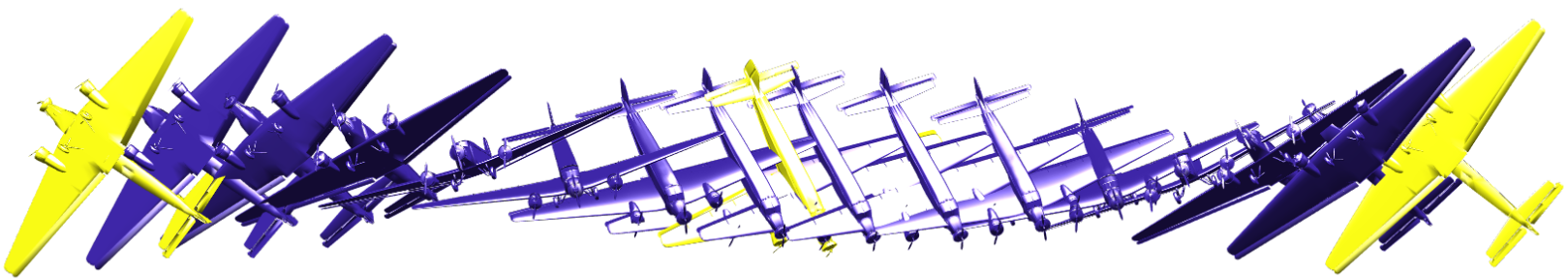


Figure 13: Animation of a Sharp Turn