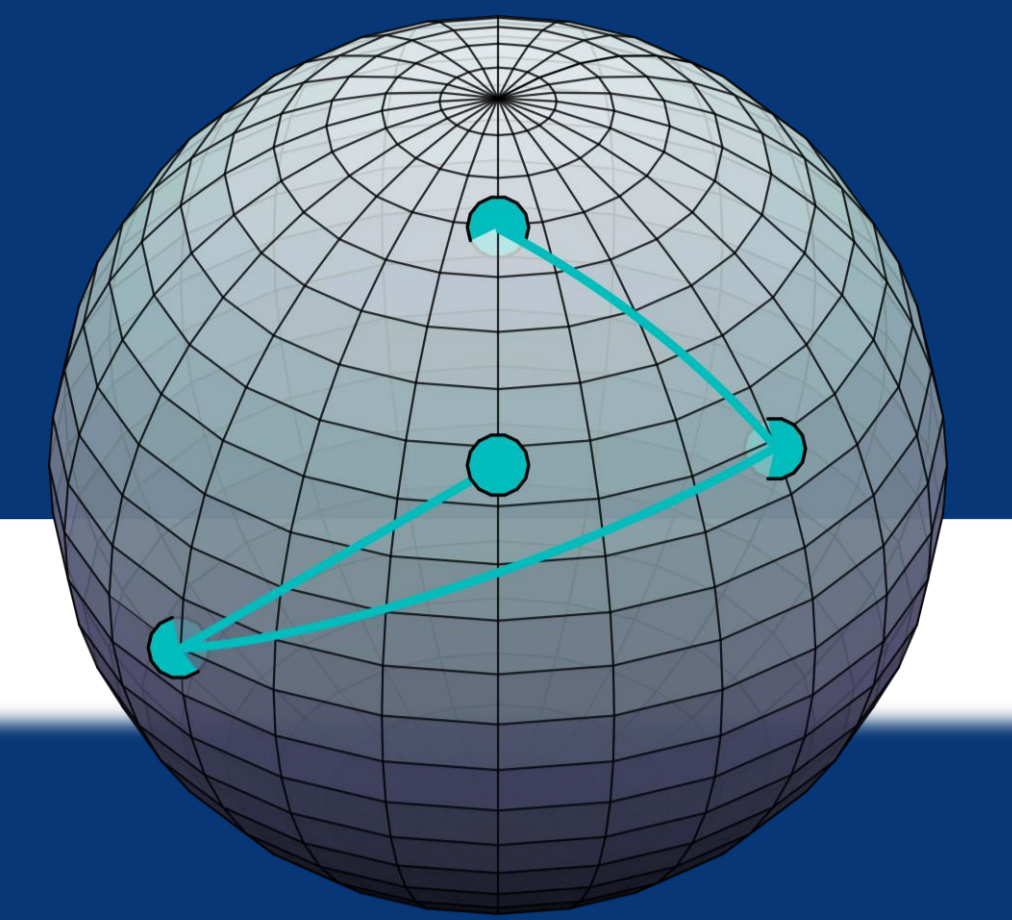




Quaternionic Upsampling: Hyperspherical Techniques for 6 DoF Pose Tracking

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Interpolation and Extrapolation of Pose Data

Fast real-time tracking is an integral component of modern 3D computer vision pipelines. However, data transmission produces latency and optical trackers suffer from defined acquisition rates due to physical limitations.

Quaternionic Upsampling overcomes these problems by

1. Modeling pose parameters as points on multidimensional hyperspheres in (dual) quaternion space
2. Calculating pose samples as points on geodesics and piecewise continuous curves on these manifolds

➤ Online interpolation and extrapolation of poses by continuous linear variation of only one sampling parameter

Upsampling Methods

Euler angle Linear Upsampling

$$\mathbf{a}_i = (\alpha_i^x, \alpha_i^y, \alpha_i^z)$$

$$\text{EuLUp}(\mathbf{a}_1, \mathbf{a}_2, \tau) := (1 - \tau) \mathbf{a}_1 + \tau \mathbf{a}_2$$

Linear Upsampling

$$\mathbf{q}_i \in \mathbb{H}_1$$

$$\text{LUp}(\mathbf{q}_1, \mathbf{q}_2, \tau) := \frac{(1 - \tau) \mathbf{q}_1 + \tau \mathbf{q}_2}{\|(1 - \tau) \mathbf{q}_1 + \tau \mathbf{q}_2\|}$$

Spherical Linear Upsampling [1]

$$\text{SLUp}(\mathbf{q}_1, \mathbf{q}_2, \tau) := \mathbf{q}_1 \cdot (\bar{\mathbf{q}}_1 \cdot \mathbf{q}_2)^\tau$$

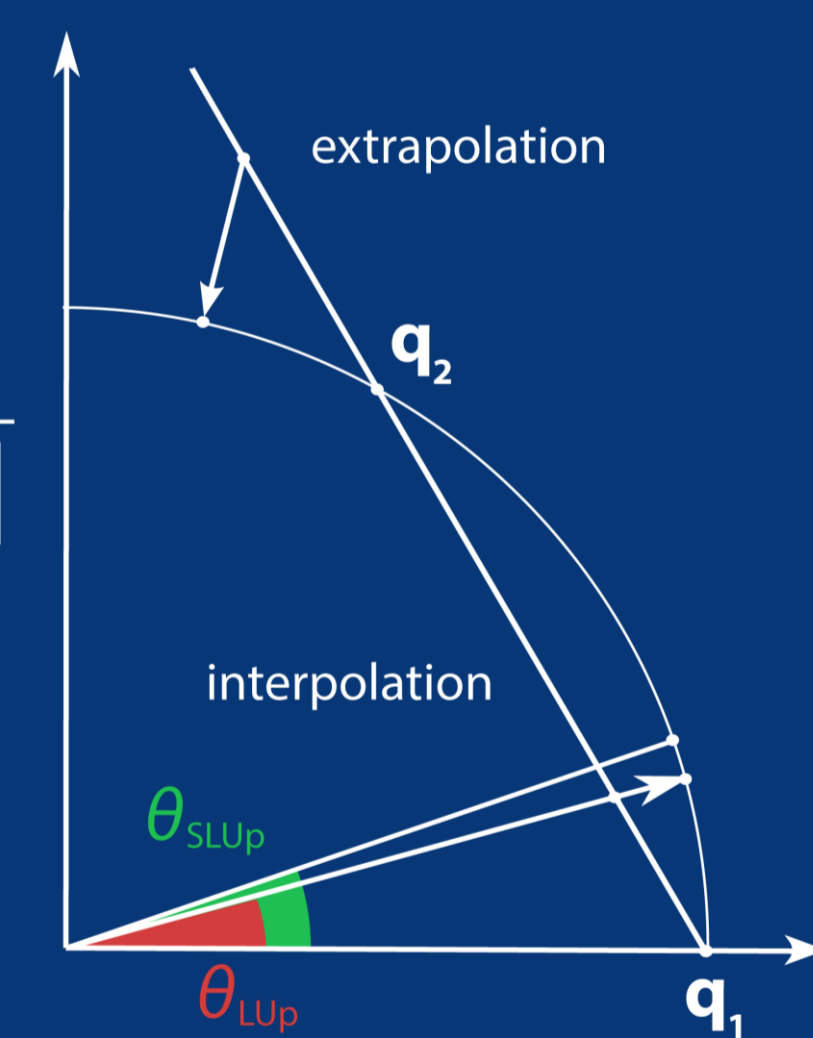
Screw Linear Upsampling

$$\mathbf{Q}_i \in \mathbb{DH}_1$$

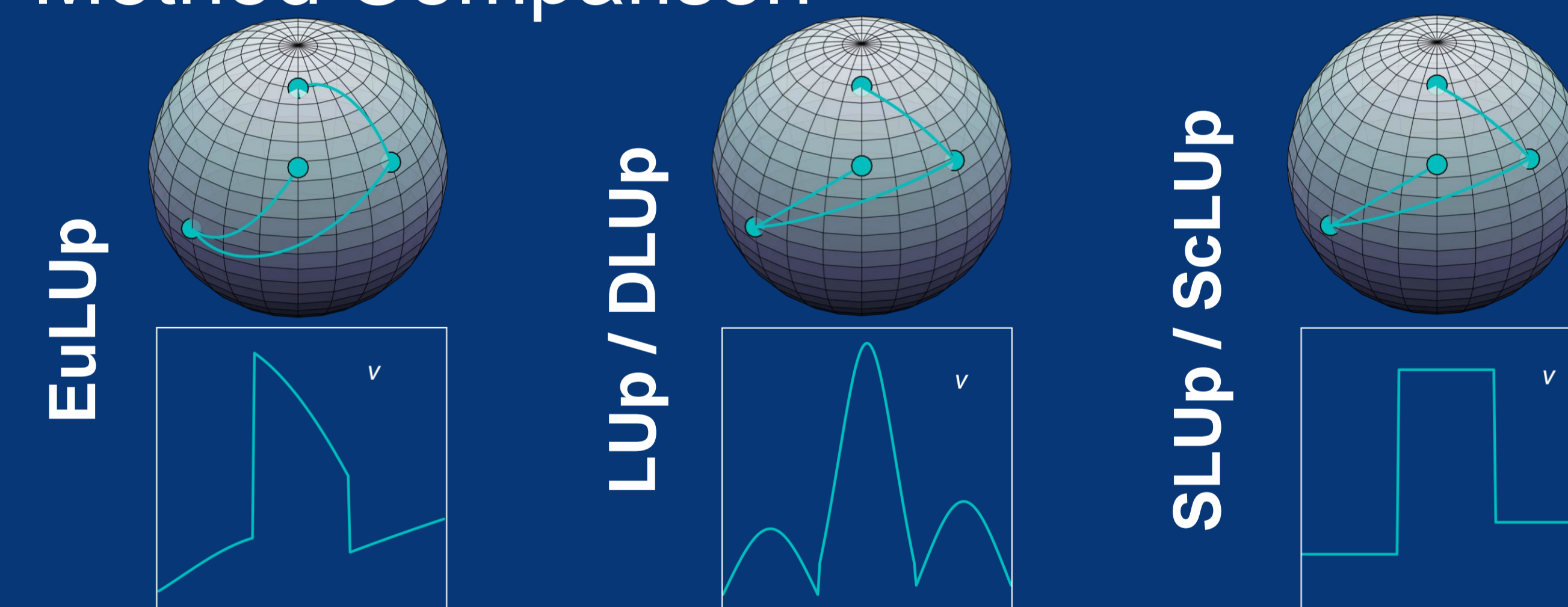
$$\text{ScLUp}(\mathbf{Q}_1, \mathbf{Q}_2, \tau) := \mathbf{Q}_1 \cdot (\mathbf{Q}_1^{-1} \cdot \mathbf{Q}_2)^\tau$$

Dual Linear Upsampling [2]

$$\text{DLUp}(\mathbf{Q}_1, \mathbf{Q}_2, \tau) := \frac{(1 - \tau) \mathbf{Q}_1 + \tau \mathbf{Q}_2}{\|(1 - \tau) \mathbf{Q}_1 + \tau \mathbf{Q}_2\|}$$



Method Comparison



Efficiency Tests [5]

EuLUp	LUp	SLUp	DLUp	ScLUp
469 Hz	306 Hz	236 Hz	4.55 kHz	2.53 kHz

Conclusion

- The method can be used to increase the measurement rate and robustness of any arbitrary 3-DoF or 6-DoF tracker
- Future pose prediction for up to 100 ms with an accuracy of 128 μm and 0.5° in real-time at a rate of above 4 kHz
 - Pose measurement accuracy matters over pose frequency for tracking with restricted object acceleration

General Pose Upsampler

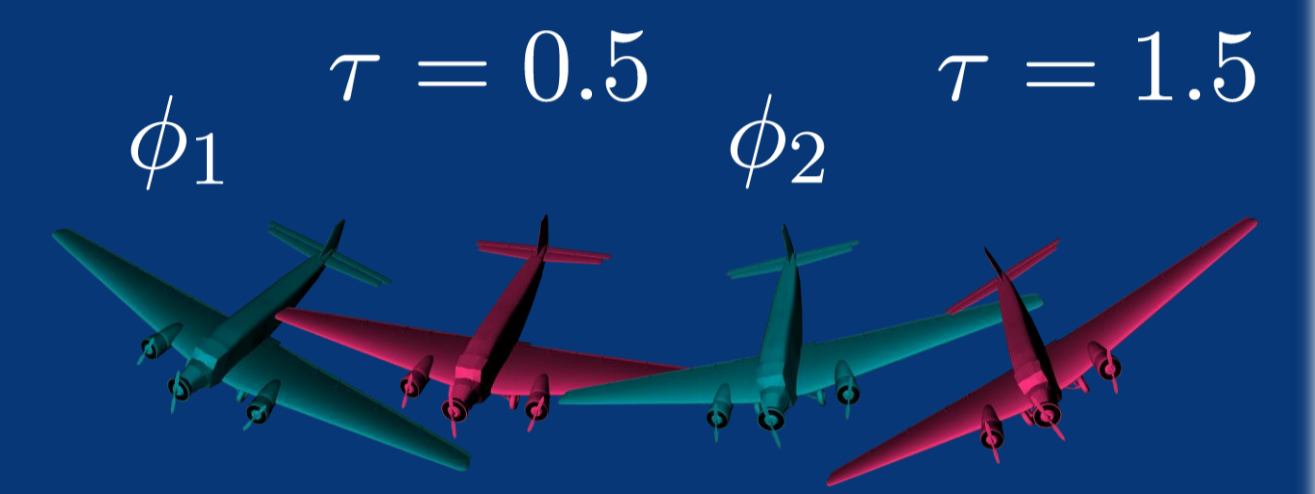
An upsampler $\gamma \in C^0$ from two transformations ϕ_1 and ϕ_2 in pose parameter space \mathbb{Q} is given by

$$\gamma : \mathbb{Q} \times \mathbb{Q} \times \mathbb{R}_0^+ \rightarrow \mathbb{Q}$$

$$(\phi_1, \phi_2, \tau) \mapsto \gamma(\phi_1, \phi_2, \tau)$$

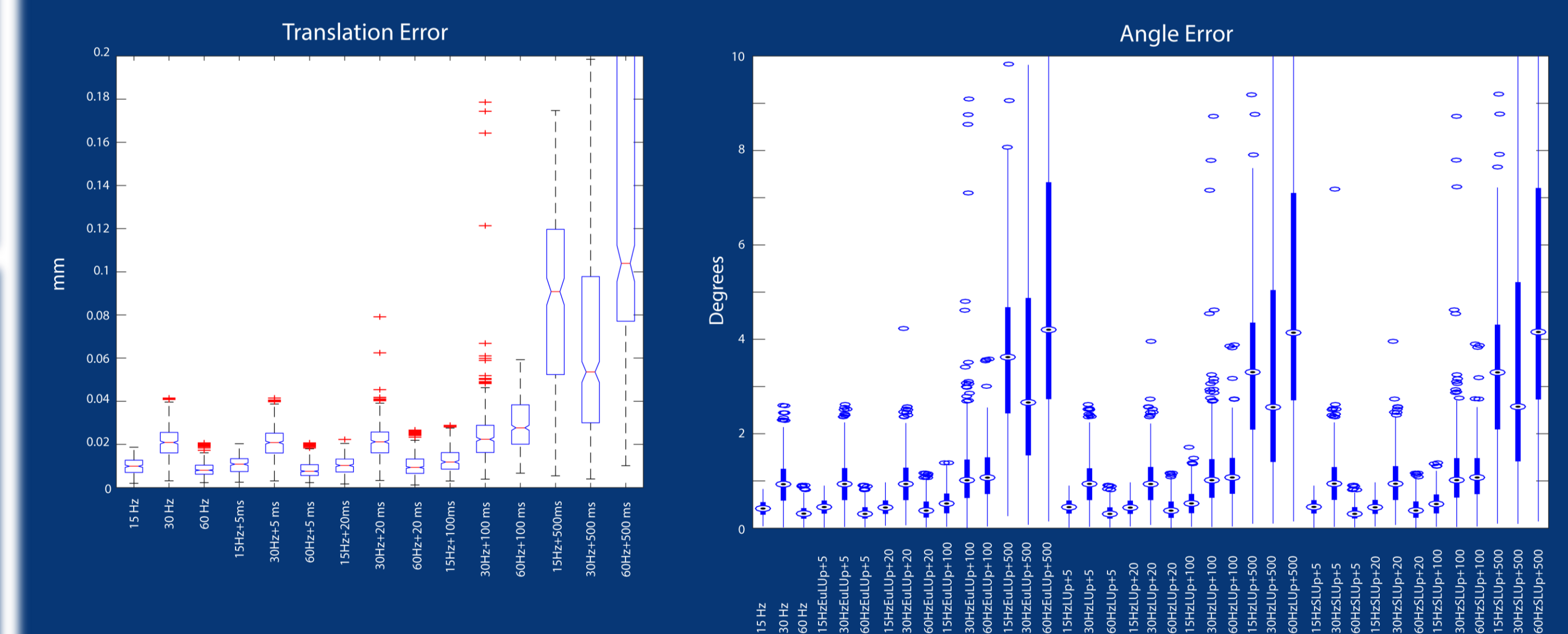
with

$$\gamma(\phi_1, \phi_2, 0) = \phi_1 \quad \text{and} \quad \gamma(\phi_1, \phi_2, 1) = \phi_2$$



Quantitative Experiments

- Natural human hand movements
 - Three different tracking systems [3] at 15, 30, 60 Hz
 - Robotic arm used for ground truth [4]
- Statistical evaluation for quaternionic extrapolation



[1] K. Shoemake. Animating rotation with quaternion curves. SIGGRAPH 1985. Spherical Linear Interpolation [SLERP] is introduced.

[2] X. Feng and W. Wan. Dual quaternion blending algorithm and its application in character animation. Indonesian Journal of Electrical Engineering and Computer Science 2013.

[3] B. Busam et al. A stereo vision approach for cooperative robotic movement therapy. ICCVW 2015.

[4] R. Tsai and R. Lenz. A new technique for fully autonomous and efficient 3D robotics hand/eye calibration. TRA 1989. The eye-on-base variant [implemented in the ViSP library] is used for calibration.

[5] Average result based on 10.000 pose interpolations. All methods have been tested on an Intel Core i7-4770K at 3.5 GHz.